

$$\left. \begin{aligned} g_1 &= g_1(x_1, \dots, x_m) \\ &\vdots \\ g_m &= g_m(x_1, \dots, x_m) \end{aligned} \right\}$$

- Ako je  $g$  diferencijabilna u  $S(x_0, r)$ :

\* slučaj  $m=1$ :  $x, y \in S(x_0, r)$

$$g(x) - g(y) = g'(\xi)(x - y)$$

$$|g(x) - g(y)| = |g'(\xi)| \cdot |x - y|$$

$$\leq \max_{x \in [x_0-r, x_0+r]} |g'(x)| \cdot |x - y|$$

$$\underbrace{\left[ \underset{x \in S(x_0, r)}{q} = \max |g'(x)| \right]}_{\text{koef. konte.}}$$

\* slučaj  $m > 1$ :  $x, y \in S(x_0, r)$  ( $m$ -dimenzioni vektori)

$$g_i(x) - g_i(y) = \sum_{j=1}^m \frac{\partial g_i(\xi_i)}{\partial x_j} (x_j - y_j) \quad , \quad \begin{matrix} \xi_i \in S(x_0, r) \\ i = 1, \dots, m \end{matrix}$$

$$|g_i(x) - g_i(y)| \leq \sum_{j=1}^m \left| \frac{\partial g_i(\xi_i)}{\partial x_j} \right| \cdot |x_j - y_j|$$

$$\|g(x) - g(y)\|_\infty = \max_{1 \leq i \leq m} |g_i(x) - g_i(y)|$$

$$= \max_{1 \leq i \leq m} \left| \sum_{j=1}^m \frac{\partial g_i(\xi_i)}{\partial x_j} (x_j - y_j) \right|$$

$$\leq \max_{1 \leq i \leq m} \sum_{j=1}^m \left| \frac{\partial g_i(\xi_i)}{\partial x_j} \right| \cdot |x_j - y_j|$$

$$\leq \underbrace{\max_{1 \leq j \leq m} |x_j - y_j|}_{\|x - y\|_\infty} \cdot \max_{1 \leq i \leq m} \sum_{j=1}^m \left| \frac{\partial g_i(\xi_i)}{\partial x_j} \right|$$

$$= \|x - y\|_\infty \cdot \max_{1 \leq i \leq m} \sum_{j=1}^m \left| \frac{\partial g_i(\xi_i)}{\partial x_j} \right|$$

$$= \|x - y\|_\infty \cdot \max_{z \in S(x_0, r)} \|J(z)\|_\infty$$

$$\underbrace{\left[ \underset{z \in S(x_0, r)}{q} = \max \|J(z)\|_\infty \right]}$$

$$J(x) = \begin{pmatrix} \frac{\partial g_1(x)}{\partial x_1} & \dots & \frac{\partial g_1(x)}{\partial x_m} \\ \vdots & & \vdots \\ \frac{\partial g_m(x)}{\partial x_1} & \dots & \frac{\partial g_m(x)}{\partial x_m} \end{pmatrix}$$