Odrediti na intervalu $[0,1]$ polinom najbolje ravnomerne aproksimacije prvog stepena za $f(x)=\sqrt{1+x^2}$. Računati na 4 decimale.

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* $Q_0 = c_0 + c_1 x \Rightarrow n=1$, treba nam 3 tačke
* $f(x)$ jeste parna, ali $[0,1]$ nije simetričan
* $f'(x) = \dfrac{2x}{2\sqrt{1+x^2}} = \dfrac{x}{\sqrt{1+x^2}}$

$$f''(x) = \frac{1\cdot\sqrt{1+x^2} - \frac{x\cdot 2x}{2\sqrt{1+x^2}}}{1+x^2} = \dots = \frac{1}{(1+x^2)\sqrt{1+x^2}} > 0$$

$\Rightarrow f$ je $\cup$ $\Rightarrow$ tačke Čeb. alt. su: $x_0 = a = 0$

$x_1 = d = ?$

$x_2 = b = 1$

* Formiramo sistem:

$$\left.\begin{aligned} f(x_0) - Q_0(x_0) &= \alpha L \\ f(x_1) - Q_0(x_1) &= -\alpha L \\ f(x_2) - Q_0(x_2) &= \alpha L \\ (f(x) - Q_0(x))'_{x=d} &= 0 \end{aligned}\right\} \quad \left.\begin{aligned} f(0) - c_0 - c_1\cdot 0 &= \alpha L \\ f(d) - c_0 - c_1\cdot d &= -\alpha L \\ f(1) - c_0 - c_1\cdot 1 &= \alpha L \\ (f(x) - c_0 - c_1 x)'_{x=d} &= 0 \end{aligned}\right\} \quad \underbrace{\begin{aligned} 1 - c_0 &= \alpha L & (1) \\ \sqrt{1+d^2} - c_0 - c_1 d &= -\alpha L & (2) \\ \sqrt{2} - c_0 - c_1 &= \alpha L & (3) \\ \frac{2d}{2\sqrt{1+d^2}} - c_1 &= 0 & (4) \end{aligned}}_{\text{nelinearan sistem}}$$

$\hookrightarrow$ četvrta dodata jednačina

Iz (1) i (3):

$$\left.\begin{aligned} 1 - c_0 &= \alpha L \\ \sqrt{2} - c_0 - c_1 &= \alpha L \end{aligned}\right\} \ominus$$

$$c_1 = \sqrt{2} - 1 = 0.4142$$

Sada (4) postaje: $\dfrac{2d}{2\sqrt{1+d^2}} - 0.4142 = 0$ (jedna nelinearna jna. sa 1 nepoz.)

Rešavamo iterativnom metodom: $d^{(n)} = 0.4142\cdot\sqrt{1+[d^{(n-1)}]^2}$

$d^{(0)} = 4.4142$, $d^{(1)} = 0.4483$, $d^{(2)} = 0.4539$, $d^{(3)} = 0.4549$, $d^{(4)} = 0.4550$

$d^{(5)} = 0.4551$, $d^{(6)} = 0.4551$ (stajemo jer se $d^{(5)}$ i $d^{(6)}$ poklapaju na 4 decimale)

$\Rightarrow x_1 = d = 0.4551 \rightarrow$ to je treća tačka Čeb. alt.

Vratimo d u sistem, on je sada linearan pa se može lako rešiti: $c_0 = 0.9551$, $c_1 = 0.4142$, $L = 0.0449$, $\alpha = 1$