# Vakcine

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(Lotka-Voltera)

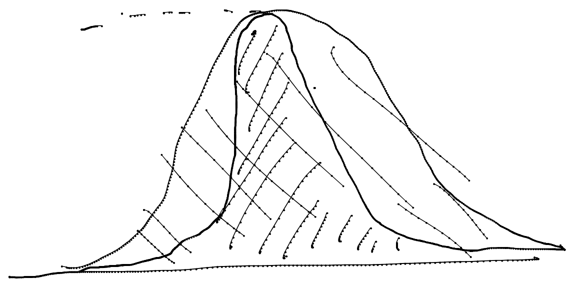
Antitela $\rightsquigarrow$ grabljivac G

Virus $\rightsquigarrow$ plen $\rightsquigarrow$ P

Čoveka $\rightsquigarrow$ trava

$$\frac{dP}{dt} = aP - bPG$$

$$\frac{dG}{dt} = cP$$



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| | | |
|---|---|---|
| Farmeri | (narod) | X |
| Banditi | (mafija) | Y |
| Vlast | (policija) | Z |

- Banditi terorišu F, a V oporezuje F
- V juri B i pogubi ih
- B i V umiru prirodno / odlaze u penziju

$$X' = r \cdot X\left(1 - \frac{X}{K}\right) - h \cdot XZ - \frac{aX}{b+X} \cdot Y$$

$$Y' = e\,\frac{aX}{b+X} \cdot Y - mY - \frac{c \cdot Y}{d+Y} \cdot Z$$

$$Z' = f \cdot \frac{aX}{b+X} \cdot Y - g \cdot Z$$

$$\frac{aX}{b+X}, \quad \begin{array}{l} X \text{ veliko} \rightarrow a \\ X \text{ malo} \rightarrow X \end{array}$$

# Koncentracija leka

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$t=0: \quad b$ (doza leka)

$x(t) = b \cdot e^{-tk}$ , $k$ - stopa raspada spec. leka

$t^*$ - vreme potrebno da se raspadne $\frac{1}{2}$ doze

$$x(t^*) = \underbrace{\frac{b}{2} = b \cdot e^{-t^*k}}$$

$$\frac{1}{2} = e^{-t^*k}$$

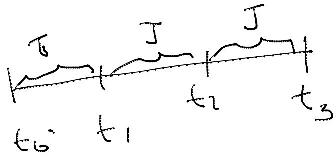
$$\ln\frac{1}{2} = -t^*k$$

$$t^* = \frac{\ln 2}{k}$$

Dodatna doza (b) daje se na svakih J vrem. jed.

$$t = J \;: \quad x(J^+) = \underbrace{b \cdot e^{-kJ}}_{\text{koliko ostalo}} + \underbrace{b}_{\text{nova doza}}$$

$x_0$ - koncentracija leka u telu u trenutku kada se daje I doza




$x_1$ - || - II doza

$x_0 = b$ , $x_1 = x_0 \cdot e^{-kJ} + b$

Nakon još J vremena damo III dozu:

$$x_2 = x_1 \cdot e^{-kJ} + b$$

$\vdots$

Nakon još (n-1)·J vremena damo n-tu dozu

$$x_n = \underbrace{x_{n-1} \cdot e^{-kJ}} + \underbrace{b}$$

$$\boxed{x_{n+1} = x_n \cdot e^{-kJ} + b}$$

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$k, J = \text{const} \Rightarrow e^{-kJ} = \text{const}$ , $a = e^{-kJ}$

$\boxed{x_{n+1} = a \cdot x_n + b}$ lin. diferencna jna I reda

$k>0$ , $J>0 \Rightarrow 0<a<1$

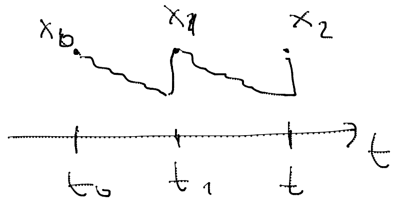
1) (mat.) $a>1$ , $b>0 \Rightarrow x_n \nearrow$

2) $a<1$ , $b<0$ (nije realno za lek) $\Rightarrow x_n \searrow$

3) $a>1$ , $b<0$ ...... ...

4) (naš slučaj) $a<1$ i $b>0$

$$x = ax+b$$
$$(1-a)x = b$$
$$x = \frac{b}{1-a}$$




$$x_0 = \frac{b}{1-a} \Rightarrow x_n = \frac{b}{1-a} \quad \forall n$$

$$0 < x_0 < \frac{b}{1-a} \Rightarrow x_n \nearrow \frac{b}{1-a}$$

$$x_0 > \frac{b}{1-a} \Rightarrow x_n \searrow \frac{b}{1-a}$$

3 opet) $a>1$, $b<0$

$$x_0 = \frac{b}{1-a} \Rightarrow x_n = \frac{b}{1-a} \quad \forall n$$

$$0 < x_0 < \frac{b}{1-a} \Rightarrow x_n \searrow -\infty$$

$$x_0 > \frac{b}{1-a} \Rightarrow x_n \nearrow +\infty$$

# Romeo & Julija

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$$\left.\begin{aligned} R_{n+1} &= a\cdot R_n + b\cdot J_n \\ J_{n+1} &= c\cdot R_n + d\, J_n \end{aligned}\right\}$$ lin. sistem dif. j-na $\quad J_0 \quad R_0$

$a>0,\ b>0 \qquad a<0,\ b>0$

$a>0,\ b<0 \qquad a<0,\ b<0$

$$\begin{bmatrix} R_{n+1} \\ J_{n+1} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} R_n \\ J_n \end{bmatrix}$$

#1) Ljubav zavisi isključivo od onog drugog

$(a=d=0) \qquad M = \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix}$

$R_{n+1} = bJ_n \Rightarrow R_n = bJ_{n-1}$ $\qquad \left.\begin{matrix} R_0 \\ J_0 \end{matrix}\right\}$ dato

$\underline{J_{n+1} = cR_n}$

$J_{n+1} = c\cdot(bJ_{n-1})$

$J_{n+1} - cbJ_{n-1} = 0$

$\lambda^2 - cb = 0 \Rightarrow \lambda_{1,2} = \pm\sqrt{cb}\ ,\quad c,b \gtrless 0$

$J_n = C_1\cdot\lambda_1^n + C_2\cdot\lambda_2^n = C_1(\sqrt{cb})^n + C_2(-\sqrt{cb})^n$

$J_0,\ J_1 = c\cdot R_0$

$n=0: \quad J_0 = C_1\cdot\lambda_1^0 + C_2\cdot\lambda_2^0 = C_1 + C_2 \qquad /\sqrt{cb}$

$n=1: \quad \underline{J_1 = c\cdot R_0} = C_1\cdot(\sqrt{cb})^1 + C_2(-\sqrt{cb})^1$

$J_0\cdot\sqrt{cb} = C_1\sqrt{cb} + C_2\cancel{\sqrt{cb}}$

$c\cdot R_0 = C_1\sqrt{cb} - C_2\cancel{\sqrt{cb}} \quad \oplus$

$J_0\sqrt{cb} + c\cdot R_0 = 2C_1\sqrt{cb} \Rightarrow \boxed{C_1 = \dfrac{J_0\sqrt{cb} + cR_0}{2\sqrt{cb}}}$

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$$C_2 = J_0 - C_1 = \frac{2\sqrt{cb}\cdot J_0 - J_0\sqrt{cb} - cR_0}{2\sqrt{cb}}$$

$$\boxed{C_2 = \frac{J_0\sqrt{cb} - cR_0}{2\sqrt{cb}}}$$

PR: $J_n = \ldots$

$$\begin{bmatrix} R_{n+1} \\ J_{n+1} \end{bmatrix} = M^n \cdot \begin{bmatrix} R_0 \\ J_0 \end{bmatrix}$$

$M\cancel{v} = \underset{\sim}{\lambda} v \quad , v \neq 0$

$M \to \lambda_1 \quad V_1$

$\lambda_2 \quad V_2$

$$\begin{bmatrix} R_0 \\ J_0 \end{bmatrix} = a_1 V_1 + a_2 V_2$$

eig( )

$R_0 = a_1 \cdot V(1,1) + a_2 \cdot V(1,2)$

$J_0 = a_1 \cdot V(2,1) + a_2 \cdot V(2,2)$

$$V \backslash \begin{bmatrix} R_0 \\ J_0 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\begin{bmatrix} R_n \\ J_n \end{bmatrix} = M^n \cdot (a_1 V_1 + a_2 V_2)$$

$$= a_1 M^n \cdot V_1 + a_2 M^n V_2$$

$$= a_1 \cdot \lambda_1^n V_1 + a_2 \lambda_2^n \cdot V_2$$

$M^n V = M^{n-1} \cdot \underbrace{M \cdot V}_{\lambda V}$

$= M^{n-2} \cdot \underbrace{M \cdot \lambda V}_{\lambda V}$

$= \lambda^2 M^{n-3} \cdot M \cdot V$

$\vdots$

$= \lambda^n \cdot V$