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$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\sin\theta$$

$$v = \frac{ds}{dt}, \quad \boxed{s = \ell\cdot\theta(t)}$$

$$v = \ell\cdot\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{1}{\ell}v$$

$$\frac{dv}{dt} = \ell\cdot\frac{d^2\theta}{dt^2} = \ell\cdot\left(-\frac{g}{\ell}\sin\theta\right) = -g\cdot\sin\theta$$

$$\left.\begin{array}{l} x_1 = \theta \\ x_2 = v \end{array}\right\} \Rightarrow \begin{array}{l} x_1' = \theta' = \frac{1}{\ell}v = \frac{1}{\ell}x_2 \\ x_2' = v' = -g\cdot\sin\theta = -g\cdot\sin(x_1) \end{array}$$

$$\frac{d^2\theta}{dt^2} = -\frac{b}{m}\cdot\frac{d\theta}{dt} - \frac{g}{\ell}\sin\theta$$

$$\frac{d\theta}{dt} = \frac{1}{\ell}v$$

$$\frac{dv}{dt} = \ell\cdot\frac{d\theta^2}{dt^2} = \ell\cdot\left(-\frac{b}{m}\cdot\underbrace{\frac{d\theta}{dt}}_{\frac{1}{\ell}v} - \frac{g}{\ell}\sin\theta\right)$$

$$\frac{dv}{dt} = -\frac{b}{m}\cdot v - g\cdot\sin\theta$$

$$\left.\begin{array}{l} x_1 = \theta \\ x_2 = v \end{array}\right\} \begin{array}{l} x_1' = \theta' = \frac{1}{\ell}v = \frac{1}{\ell}x_2 \\ x_2' = v' = -\frac{b}{m}v - g\cdot\sin\theta = -\frac{b}{m}x_2 - g\cdot\sin(x_1) \end{array}$$

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$\frac{dN(t)}{dt} = p \cdot N(t)$, $p = n - m$, $N(0) = N_0$ } MALTHUSOV (eksponencijalni) model

$N(t) = N(0) \cdot e^{pt}$

$\frac{dN(t)}{dt} = r \cdot N(t) \cdot \left(1 - \frac{N(t)}{K}\right)$, $N(0) = N_0$, $K \rightarrow$ max kapacitet, $r = p(0)$ } VERHULSTOV (logistički) model

$N(t) = \frac{K}{1 + \left(\frac{K}{N_0} - 1\right) \cdot e^{-rt}}$

$N(t)$ - broj atoma C14

K - stopa raspada

$\boxed{\frac{N(t)}{dt} = -K \cdot N(t)}$

$\frac{dN}{N} = -K\,dt$

$\ln|N| = -K \cdot t + C$

$N = e^{-kt} \cdot e^{C}$

$N(0) = e^{C} = N_0$

$\boxed{N(t) = e^{-kt} \cdot N_0}$

C14 : C12 = 92 : 100

tj. $N(C14) = N(C12)$

$\frac{N(t)}{N_0} = e^{-kt}$

$0.92 = e^{-k \cdot t}$, $t = ?$

$\Rightarrow t = 688{,}2\ldots$

$t = 5730$ god

$N(5730) = \frac{N_0}{2}$

$N(5730) = e^{-k \cdot 5730} \cdot N_0 = \frac{N_0}{2}$

$e^{-k \cdot 5730} = \frac{1}{2}$

$\vdots$

$K = 0{,}000121179$

t	$N(t)$	$\frac{N(t+\Delta t)-N(t)}{\Delta t}$
0	1.99	0.69
1	2.68	0.95
2	3.63	1.26
3	4.89	1.74
4	6.63	2.30
5	8.93	3.17
6	12.10	

$$N(t+\Delta t) = N(t) + p \cdot \Delta t \cdot N(t)$$

$$\frac{N(t+\Delta t) - N(t)}{\Delta t} = p \cdot N(t)$$

$\Delta t = 1$

$t=0$: $0{,}69 = p \cdot 1{,}99$

$t=1$: $0{,}95 = p \cdot 2.68$

$t=2$: $1{,}26 = p \cdot 3{,}63$

$\vdots$

$t=5$: $3{,}17 = p \cdot 8{,}93$

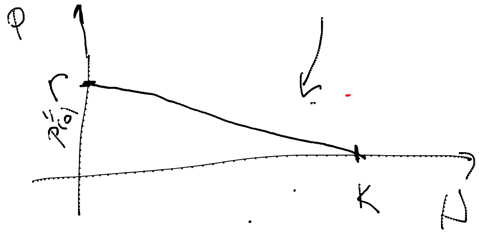
$p_0, \ldots, p_5$

$p_{sr} = 0{,}351$

$$\frac{dN(t)}{dt} = 0.3510 \cdot N(t) \qquad N(0) = 1{,}99$$

$$N(t) = N_0 \cdot e^{pt} = 1.99 \cdot e^{0{,}351 \cdot t}$$

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) \qquad \frac{dN}{dt} = p \cdot N(t)$$



Gompertz (19. vek)

(deo preostale moći čoveka da izbegne smrt sa godinama)

Gompertzov model (modelovanje raka)

$\rightarrow$ stopa smrtnosti

$\rightarrow$ broj živih u odnosu na starost

$$\begin{cases} \frac{dp}{dt} = -\alpha\, p(t) \\ \frac{dN}{dt} = p(t)\, N(t) \end{cases}, \quad N_0 = N(0)$$

$$p(t) = p_0 \cdot e^{-\alpha t}$$

$p(t) = \ldots$

$$\frac{dN}{dt} = p_0 \cdot e^{-\alpha t} \cdot N(t)$$

$$\frac{dN}{N} = p_0 \cdot e^{-\alpha t}\, dt$$

$$\ln|N| = -\frac{p_0}{\alpha} \cdot e^{-\alpha t} + C$$

$$N = e^{-\frac{p_0}{\alpha} \cdot e^{-\alpha t}} \cdot e^{C}$$

$$N(0) = N_0 = e^{-\frac{p_0}{\alpha}} \cdot e^{C} \Rightarrow e^{C} = \frac{N_0}{e^{-\frac{p_0}{\alpha}}} = N_0 \cdot e^{\frac{p_0}{\alpha}}$$

$$N = e^{-\frac{p_0}{\alpha} \cdot e^{-\alpha t}} \cdot N_0 \cdot e^{\frac{p_0}{\alpha}}$$

$$\boxed{N(t) = N_0 \cdot e^{\frac{p_0}{\alpha}(1 - e^{-\alpha t})}}$$

$$\lim_{t \to \infty} N(t) = \lim_{t \to \infty} N_0\, e^{\frac{p_0}{\alpha}(1 - e^{-\alpha t})} = N_0 \cdot e^{\frac{p_0}{\alpha}} \quad (\equiv K)$$

($e^{-\alpha t} \to 0$)