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$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta} \quad \boxed{\begin{array}{l}\theta(0) = \theta_0 \\ v_0(0) = 0 = \frac{d\theta}{dt}(0)\end{array}}$$

za malo θ $(\theta < 15^\circ)$: $\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta, \quad \begin{array}{l}\theta(0) = \theta_0 \\ \frac{d\theta}{dt}(0) = 0\end{array}}$

$\theta(t) = \dots$

$\boxed{T_0 = 2\pi\sqrt{\frac{l}{g}}}$ period oscilovanja

Sa otporom vazduha:

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{b}{m}\cdot\frac{d\theta}{dt} - \frac{g}{l}\sin\theta} \quad \boxed{\begin{array}{l}\theta(0) = \theta_0 \\ v_0 = 0\end{array}}$$

$b = B/l$; $B = 12\pi \cdot r \cdot \mu$

vazduh: $\mu = 1{,}827 \cdot 10^{-5}$ kg/ms

voda: $\mu = 8{,}9 \cdot 10^{-4}$

$$\boxed{\frac{d\theta}{dt} = \pm\sqrt{\frac{2g}{l}(\cos\theta - \cos\theta_0)}}$$

model dobijem $\Delta P + \Delta E = \text{const}$

$$\boxed{T = 4\sqrt{\frac{l}{2g}} \underbrace{\int_0^{\theta_0} \frac{1}{\sqrt{\cos\theta - \cos\theta_0}}\, d\theta}_{}}$$

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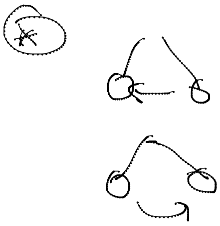


1s
1s

$T = 2s$

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l = \frac{T^2 g}{4\pi^2} = \frac{2^2 \cdot 9{,}81}{4\pi}$

$\boxed{l = 0{,}994 m}$

*

$T_1 : T_2 = 1 : 2$

$\frac{T_1}{T_2} = \frac{1}{2} \Rightarrow \frac{\cancel{2\pi}\sqrt{\frac{l_1}{g}}}{\cancel{2\pi}\sqrt{\frac{l_2}{g}}} = \frac{1}{2} \Rightarrow \frac{l_1}{l_2} = \frac{1}{4}$

*

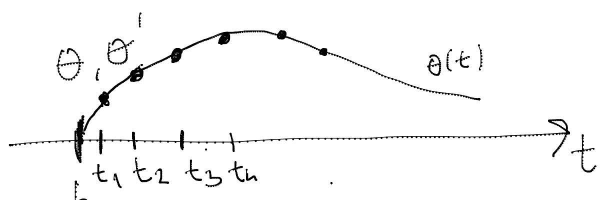
$L = 0{,}5, \quad T = 3{,}51$

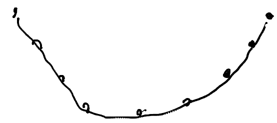
$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = 4\pi^2 \frac{l}{T^2} = 1{,}6022 \, m/s^2$

*

$T = 12s$

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l = 35{,}7826 \, m$





DJ Ireda

Runge-Kutta

$[t, \theta] = ode45(@f(\dots), [t_0, t_n], [\theta_0; \ldots])$

$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \leftarrow$

$x_1(t) = \theta(t)$
$x_2(t) = x_1'(t) = \theta'(t)$

$\Rightarrow$

$x_1'(t) = \theta'(t) = x_2(t)$
$x_2'(t) = \theta''(t) = -\frac{g}{l}\sin\theta = -\frac{g}{l}\sin(x_1(t))$

$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ -g/l \sin(x_2) \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

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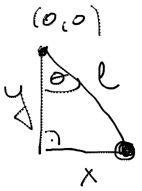


$x = l \cdot \sin\theta$

$y = l \cdot \cos\theta$

$$\frac{d^2\theta}{dt^2} = -\frac{b}{m} \cdot \frac{d\theta}{dt} - \frac{g}{l}\sin\theta \quad \leftarrow$$

$$\left.\begin{array}{l} x_1(t) = \theta(t) \\ x_2(t) = x_1'(t) = \theta'(t) \end{array}\right\} \begin{array}{l} x_1' = \theta' = x_2 \\ x_2' = \theta'' = -\frac{b}{m} \cdot \underbrace{\frac{d\theta}{dt}}_{\theta'} - \frac{g}{l}\sin\theta \\ \quad = -\frac{b}{m} \cdot x_2 - \frac{g}{l}\sin x_1 \end{array}$$

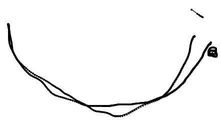
$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{b}{m}x_2 - \frac{g}{l}\sin x_1 \end{bmatrix}$$

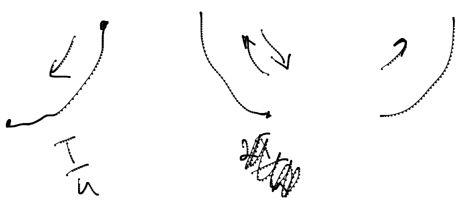
$t_0 = 0$: KE = 0 , PE max

$t_1 > t_0$: PE ↘ , KE ↗

$t_2 = \frac{T}{4}$: PE min   KE max

$\frac{T}{2} > t_3 > \frac{T}{4}$: PE ↗ , KE ↘





integral(f, a, b)

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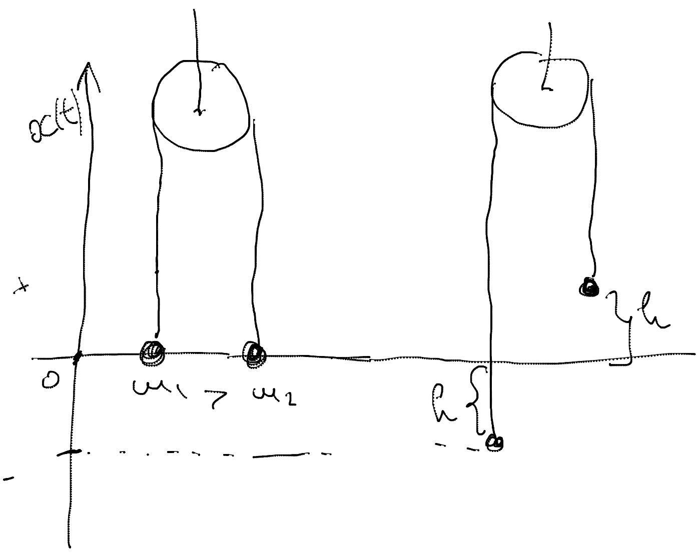
(*) Neka su 2 tega masa m_1 i m_2 međusobno povezana kanapom zanemarljive težine okačena preko jednog kotura (zanemarljive težine). Odrediti jednačinu kretanja tegova.



$x(t) = ?$

$\Delta P = mgh$

$\Delta P = -m_1 g x(t) + m_2 g \cdot x(t)$

$= -g\, x(t)\,(m_1 - m_2)$

$\Delta K = \frac{1}{2} m v^2 \qquad v_1 = v_2$

$\Delta K = \frac{1}{2} m_1 \cdot \underbrace{v^2}_{\frac{dx}{dt}} + \frac{1}{2} m_2 v^2$

$= \frac{1}{2} m_1 \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} m_2 \left(\frac{dx}{dt}\right)^2$

$= \frac{1}{2} \left(\frac{dx}{dt}\right)^2 (m_1 + m_2)$

$\Delta P + \Delta E = \text{const}$

$\frac{1}{2} \left(\underbrace{\frac{dx}{dt}}\right)^2 (m_1 + m_2) = g\, x(t)\,(m_1 - m_2)$

$\frac{dx}{dt} = \sqrt{\frac{2g(m_1 - m_2)}{m_1 + m_2}} \cdot \sqrt{x}$ → DJ sa razdv. prom.

$\frac{dx}{\sqrt{x}} = \sqrt{\qquad} \, dt \quad / \int$

$2\sqrt{x}\Big|_0^t = 2\sqrt{x(t)} - \underbrace{2\sqrt{x(0)}}_{0}$

$2\sqrt{x(t)} = \sqrt{\qquad} \cdot t \quad /^2$

$$\boxed{x(t) = \frac{1}{\cancel{4}\,2} \frac{2g(m_1 - m_2)}{m_1 + m_2} \cdot t^2}$$