$x(t), y(t) = ?$

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$(x(t), y(t))$

$(x'(t), y'(t)) = \vec{v}$ brzina

$(x''(t), y''(t)) = \vec{a}$ ubrzanje

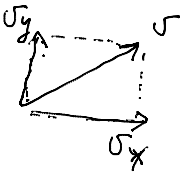
$t_0 = 0$: $(0,0) = (x(0), y(0))$

$x''(t) = 0$ nema horizontalnog ubrzanja

$y''(t) = -g$

$(x''(t), y''(t)) = (0, -g)$

$\vec{v}_0 = (x'(0), y'(0)) = (v_{0x}, v_{0y})$



$y''(t) = -g \quad / \int$

$y'(t) = -g \cdot t + C_1$

$y'(0) = v_{0y} = -g \cdot 0 + C_1 \Rightarrow C_1 = v_{0y}$

$y'(t) = -gt + v_{0y} \quad / \int$

$y(t) = v_{0y} \cdot t - \frac{gt^2}{2} + C_2$

$y(0) = 0 = v_{0y} \cdot 0 - \frac{g \cdot 0^2}{2} + C_2 \Rightarrow C_2 = 0$

$$\boxed{y(t) = v_{0y} t - \frac{gt^2}{2}}$$

$x''(t) = 0 \quad / \int$

$x'(t) = C_1$

$x'(0) = v_{0x} = C_1$

$x'(t) = v_{0x} \quad / \int$

$x(t) = v_{0x} \cdot t + C_2$

$x(0) = 0 = v_{0x} \cdot 0 + C_2 \Rightarrow C_2 = 0$

$$\boxed{x(t) = v_{0x} \cdot t}$$

$y'(0) = v_{0y} = v_0 \cdot \sin\theta$
$x'(0) = v_{0x} = v_0 \cdot \cos\theta$
$\Rightarrow$ $\boxed{y(t) = v_0 \sin(\theta) \cdot t - \frac{g t^2}{2}}$
$\boxed{x(t) = v_0 \cos(\theta) \cdot t}$

(*) t_1 - trenutak kad lopta dostiže max visinu

$y'(t_1) = 0 \Rightarrow v_0 \cdot \sin(\theta) - g \cdot t_1 = 0$

$\boxed{t_1 = \frac{v_0 \sin\theta}{g}}$

(*) Koja je max visina?

$y(t_1) = v_0 \cdot \sin(\theta) \cdot t_1 - \frac{g \cdot t_1^2}{2} = v_0 \sin\theta \cdot \frac{v_0 \sin\theta}{g} - \frac{g \left(\frac{v_0 \sin\theta}{g}\right)^2}{2}$

$= \frac{v_0^2 \sin^2\theta}{g} - \frac{v_0^2 \sin^2\theta}{2g} = \boxed{\frac{v_0^2 \sin^2\theta}{2g} = y(t_1)}$

(*) Udaljenost (x-osa)

$x(t_1) = v_0 \cos\theta \, t_1 = v_0 \cos(\theta) \cdot \frac{v_0 \sin\theta}{g} = \boxed{\frac{v_0^2 \cdot \sin(2\theta)}{2g} = x(t_1)}$

(*) t_2 - trenutak kad pada na zemlju

$y(t_2) = 0 \Rightarrow v_0 \sin\theta \cdot t_2 - \frac{g \cdot t_2^2}{2} = 0$

$t_2 \left(v_0 \sin\theta - \frac{g t_2}{2}\right) = 0$

$t_2 = 0 \vee \boxed{t_2 = \frac{2 v_0 \sin\theta}{g}} = 2 t_1$

(*) $x(t_2)$ - udaljenost od početka u trenutku pada

$x(t_2) = v_0 \cos\theta \cdot t_2 = v_0 \cdot \cos\theta \cdot \frac{2 v_0 \sin\theta}{g}$

$\boxed{x(t_2) = \frac{v_0^2 \cdot \sin(2\theta)}{g}}$

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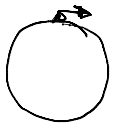
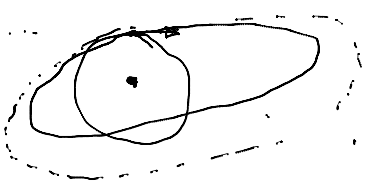
(¥) Za $\pi/4$ najveći domet u slučaju $v_0 = const$

(*) 

Da li će projektil uvek da padne na zemlju?



$v_0 < I$ kosmičke v. na zemlji

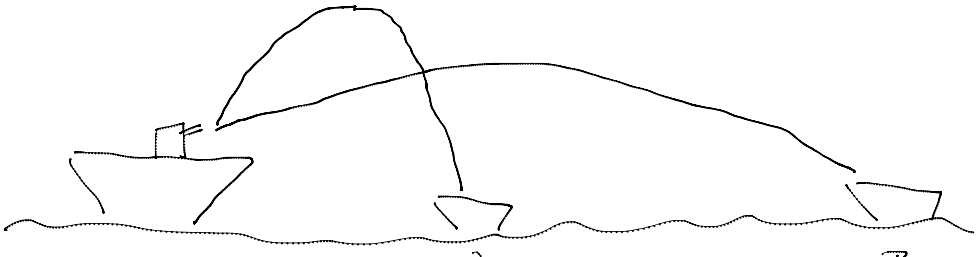
$I < v_0 < \sqrt{2}I$ elipsa (vraća se iza Pere kapota)

$v_0 > \sqrt{2}I$ - izlazi iz ~~orbite~~ zemljinog grav. polja (orbita)

$v_0 > \sqrt{2}I$ odlazi

$v_0 < \sqrt{2}I$ pada na zemlju

(*)



t_A t_B

$v_{0A} \neq v_{0B}$

$$\frac{t_B}{t_A} = \frac{\frac{2v_B \cdot \sin\theta_B}{g}}{\frac{2v_A \cdot \sin\theta_A}{g}} = \frac{v_B \sin\theta_B}{v_A \sin\theta_A}$$

$$t = \frac{2v_0 \sin\theta}{g}$$

$$\frac{y_B}{y_A} = \frac{\frac{v_B^2 \sin^2\theta_B}{2g}}{\frac{v_A^2 \sin^2\theta_A}{2g}} = \frac{v_B^2 \sin^2\theta_B}{v_A^2 \sin^2\theta_A} = \frac{t_B^2}{t_A^2}$$

$$y = \frac{v_0^2 \sin^2\theta}{2g}$$

$$\frac{y_B}{y_A} = \left(\frac{t_B}{t_A}\right)^2$$

$y_B < y_A \Rightarrow t_B < t_A$

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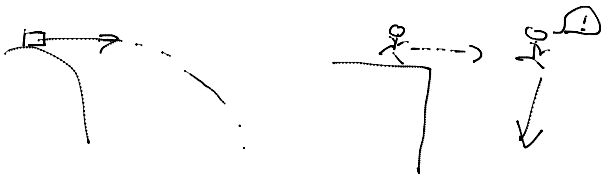

$\theta = 0$

$y''(t) = -g$

$y(0) = h$

$y'(0) = 0$

$x''(t) = 0$

$x(0) = 0$

$x'(0) = v_0$

v_0

$y'(t) = -gt + c_1$

$y'(0) = 0 = -g \cdot 0 + c_1 \Rightarrow c_1 = 0$

$y'(t) = -gt \quad / \int$

$y(t) = -\frac{gt^2}{2} + c_2$

$y(0) = h = -\frac{g \cdot 0^2}{2} + c_2 \Rightarrow c_2 = h$

$$\boxed{y(t) = h - \frac{gt^2}{2}}$$

$x''(t) = 0 \quad / \int$

$x'(t) = c_1$

$x'(0) = v_0 = c_1$

$x'(t) = v_0 \quad / \int$

$x(t) = v_0 \cdot t + c_2$

$x(0) = 0 = v_0 \cdot 0 + c_2$

$\Rightarrow c_2 = 0$

$$\boxed{x(t) = v_0 \cdot t}$$

il

$t = \frac{x}{v_0}$

$$y = h - \frac{g \cdot \frac{x^2}{v_0^2}}{2} = h - \frac{gx^2}{2v_0}$$

parabola

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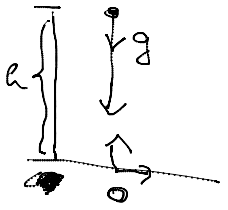
Slobodan pad



$v_0 = 0$

$y(0) = h$

$y'(0) = 0$

$y''(t) = -g \quad / \int$

$y'(t) = -gt + C_1$

$y'(0) = 0 = C_1$

$y'(t) = -gt \quad / \int$

$y(t) = -\frac{gt^2}{2} + C_2$

$y(0) = h = C_2$

$$\boxed{y(t) = h - \frac{gt^2}{2}}$$

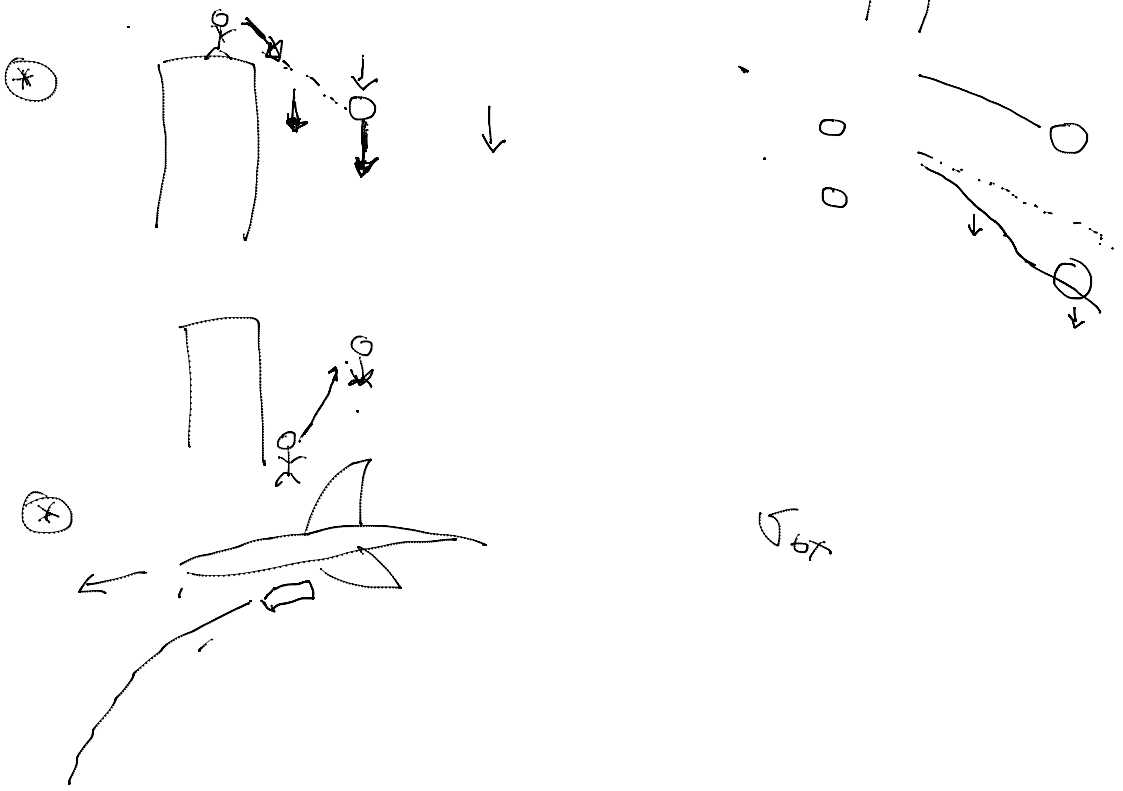
$x(0) = 0$

$x'(0) = 0$

$x''(0) = 0$

$$\boxed{x(t) = 0}$$

Bacanje paje i kuglice



v_{0x}

(*)

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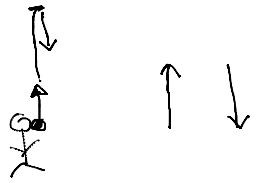
Koliko vremena ima Pera Kojot da pobegne nakon što ispali projektil u vis brzinom $v_0 = 60\ m/s$?



t_1

$y(0)=0 \quad x(0)=0 \} x(t)=0$

$y'(0)=v_0 \quad x'(0)=0$

$\theta = \pi/2$

$t_1 = \frac{v_0 \sin\theta}{g} = \ldots = 6,1162\ s$

$h = y(t_1) = \frac{v_0^2 \sin^2\theta}{2g} = 183,4862\ m$

t_2

$2 \cdot t$

$y(t_2)=0$

$h = 183,4862\ m$

$y(t_2) = h - \frac{gt_2^2}{2} = 0$

$t_2 = 6,1162\ s$

$t = t_1 + t_2 = 2t_1 = 12,2324\ s$

(*) $v_0 = 45\ m/s$

$\theta = 60^\circ = \pi/3$

$h = 265\ m$

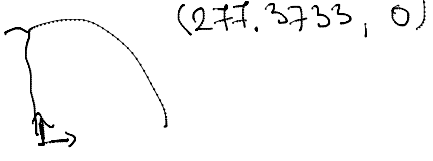
$y(t_2) = v_0 \sin\theta \cdot t - \frac{gt^2}{2} = -265$

$45 \cdot \sin\pi/3 \cdot t - \frac{9.81 t^2}{2} + 265 = 0$

$t_{1,2} \nearrow \ldots$

$\rightarrow 12,3277$ trenutak pada na tenu

$x(t_2) = v_0 \cos\theta \cdot t_2 = 277,3733\ m$



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$t_0 = 0$

(*) $\theta = \pi/4$

$L = 2{,}5\ m$

$d_0 = 5\ m$

$V = 9\ m/s$

$v_0 = ?$

$y(t_0) = y(t_{end}) = 0$

$x(t_0) = 0$

$\rightarrow x(t_{end}) = d_0 + V \cdot t_{end}$ (min)

$= d_0 + V \cdot t_{end} + L$ (max)

Min: $x(t_{end}) = v_0 \cdot \cos(\theta) \cdot t_{end} = d_0 + V \cdot t_{end}$

$y(t_{end}) = v_0 \cdot \sin(\theta) \cdot t_{end} - \frac{g \cdot t_{end}^2}{2} = 0$

$v_0, t_{end} = ?$

$v_0 = \frac{d_0 + V \cdot t}{t \cdot \cos(\theta)}$

$\frac{d_0 + Vt}{t \cdot \cos\theta} \cdot \sin\theta \cdot t - \frac{g \cdot t^2}{2} = 0$

$(d_0 + Vt) \cdot \tan\theta - \frac{g \cdot t^2}{2} = 0$

$(5 + 9t) \cdot 1 - \frac{9{,}81 \cdot t^2}{2} = 0$

$t_{1,2} \nearrow \searrow 2{,}2816\ s = t_{end}$

$v_0 = \frac{d_0 + V \cdot t_{end}}{t_{end} \cdot \cos\theta} = \ldots = 15{,}8271\ m/s$

Max:

$x(t_{end}) = v_0 \cos\theta \cdot t_{end} = d_0 + L + V \cdot t_{end} \Rightarrow v_0 = \ldots$

$y(t_{end}) = \ldots$

$t_{1,2} \nearrow \searrow 2{,}4571\ s = t_{end}$

$v_0 = 17{,}0446\ m/s$

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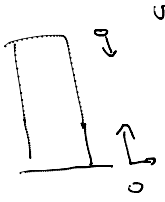
⑧ Da li novčić ~~bačen~~ ispušten sa Burdž kalifa može da ubije nekoga na ulici?

$h = 830\,m$

$y'(t) = -gt$



$y(t_1) = 0 \iff h - \frac{gt_1^2}{2} = 0 \iff t_1 = \sqrt{\frac{2h}{g}}$

$t_1 = \sqrt{\frac{2 \cdot 830}{9.81}} = 13,0083\,s$

$y'(t_1) = -g \cdot t_1 = -127,6111\,m/s \rightsquigarrow 459,36\,km/h$

$\sim 300\,m/s \quad (1000\,m/s)$