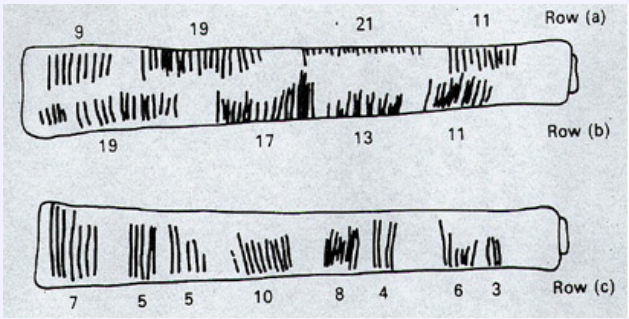
OMM - Uvod

February 25, 2021

modellus (lat) - prikazati realnost

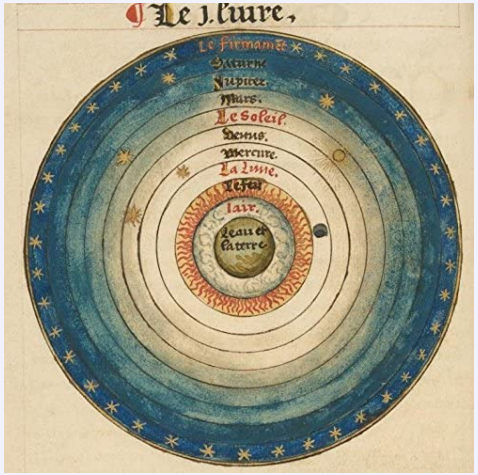
Modeliranje?

Matematičko modeliranje?

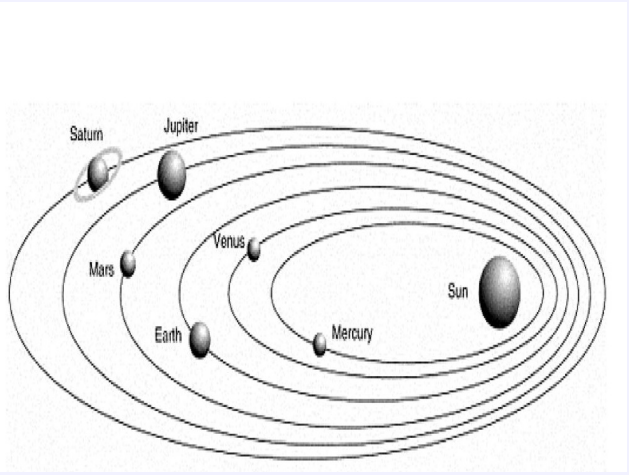


"All models are wrong, but some are useful". George E. P. Box

Ptolomejev model (150 gne)



Keplerov model (1619)

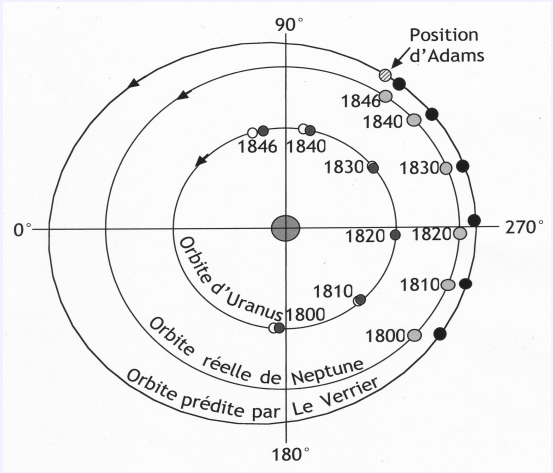


Njutn (Zakon gravitacije)

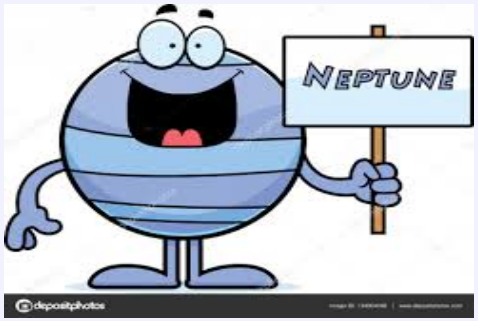
Obračunava gravitaciju ne samo na Zemlji
Zašto meseci kruže oko svojih planeta?
Zašto se komete ponavljaju?
Plima/oseka?

Uran se ponaša čudno?

(1781)

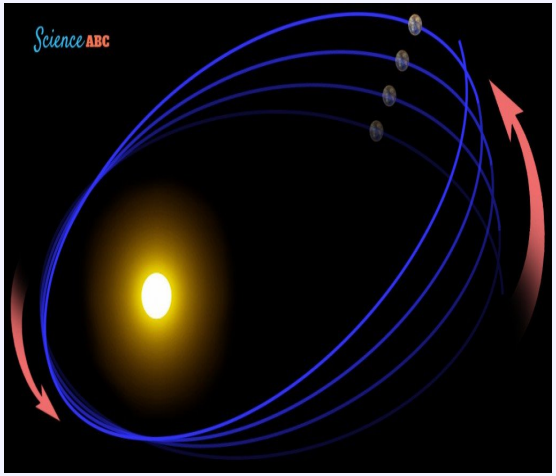


Urbain Le Verrier (1811-1877)

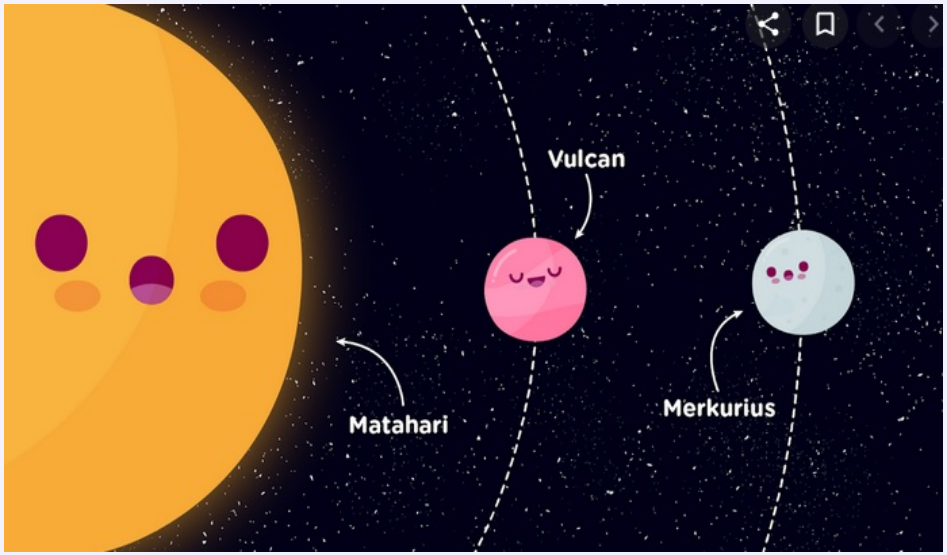


(1846)

Merkur se ponaša čudno?

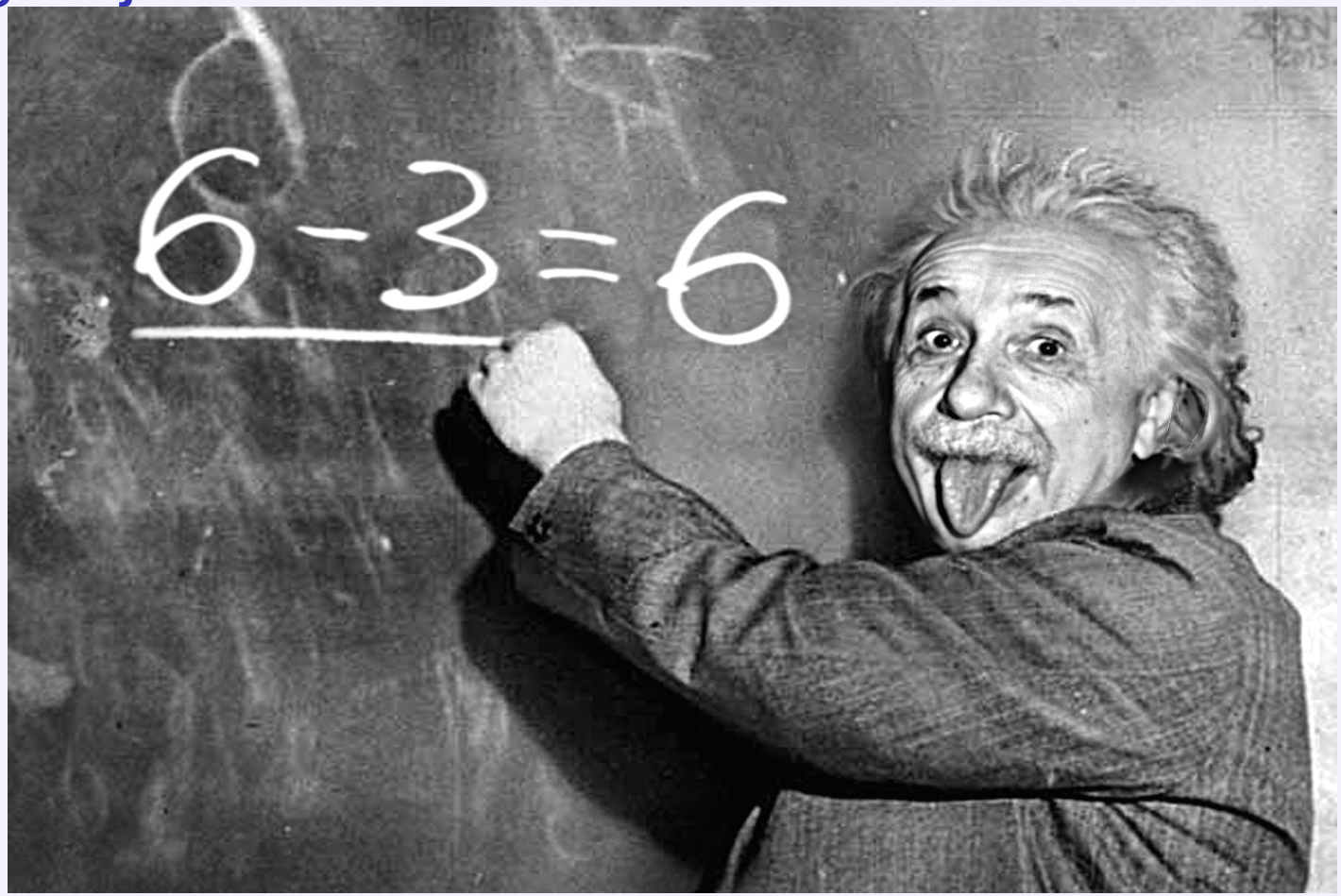


Merkur 



 Vulkan!

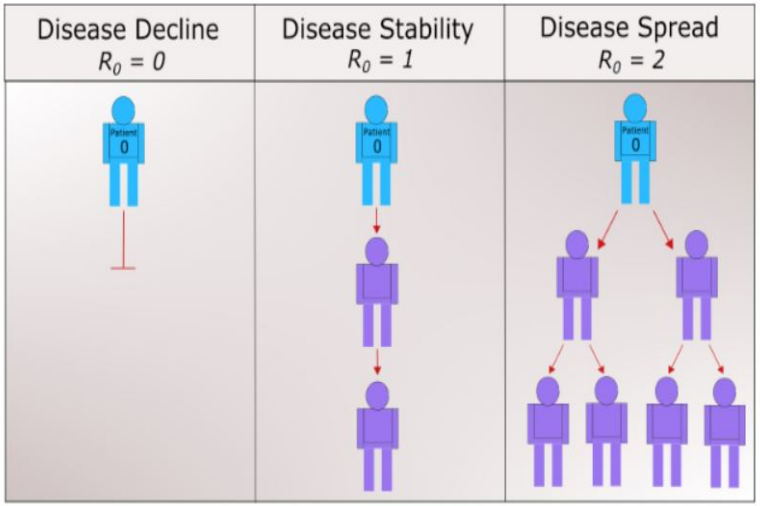
A gde je Vulkan?



(1915)

"A model should be as simple as it can be, but no simpler." Albert Einstein

Korona



$R0$ - osnovna reproduktivna stopa virusa

$R_0 = c\beta$

c - broj osoba sa kojim zaraženi stupa u kontakt

β - verovatnoća infekcije pri kontaktu

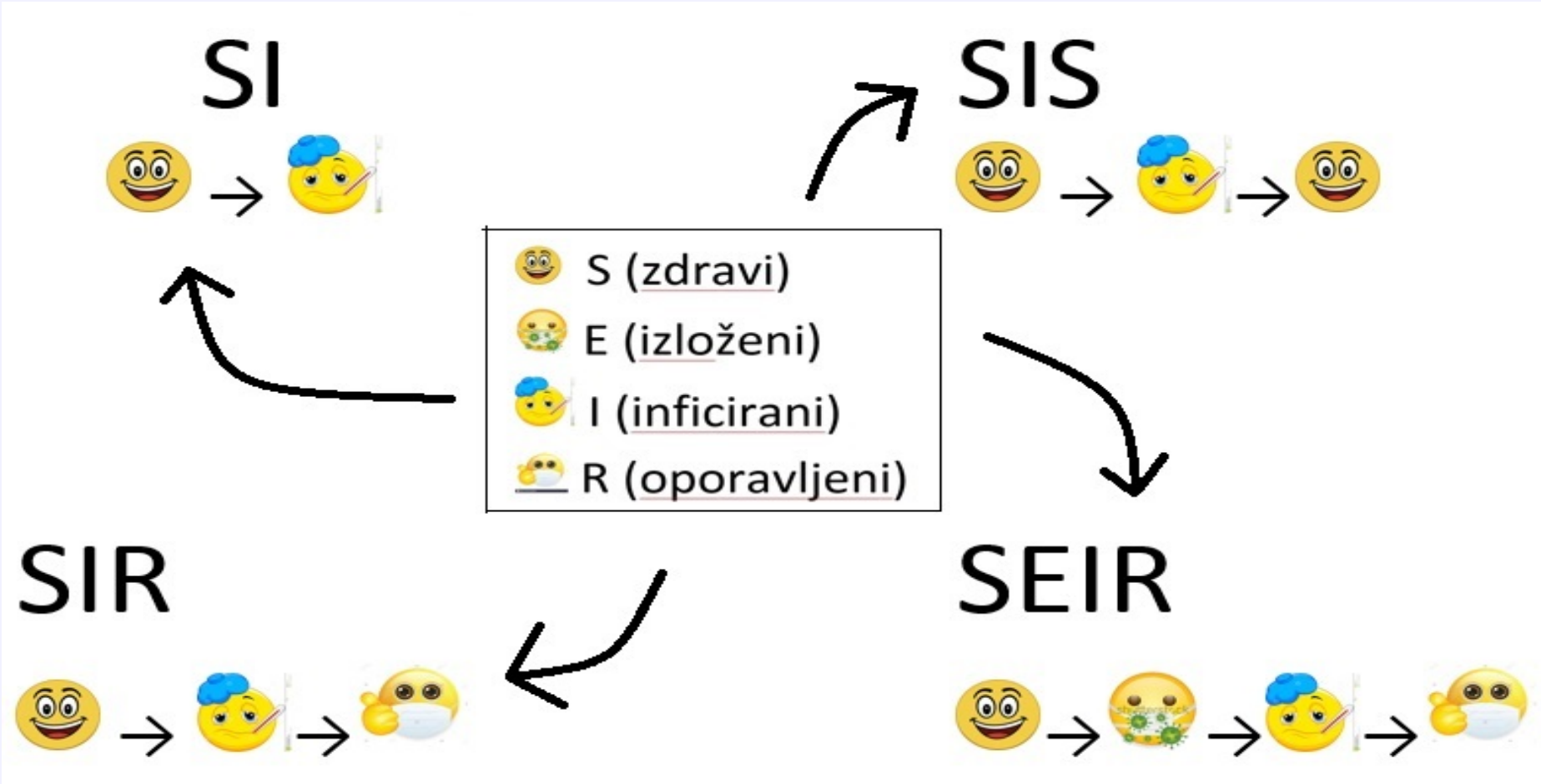
p - deo populacije koji je imun/vakcinisan

$R_p = \beta(c - cp) = \beta c - \beta cp = R_0 - R_0 p = R_0(1 - p)$

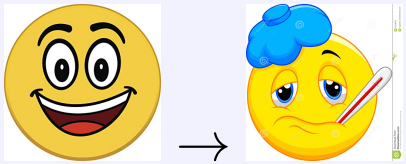
Cilj $R_p < 1 \Rightarrow R_0(1 - p) < 1 \Rightarrow p > 1 - \frac{1}{R_0}$

$1 - \frac{1}{R_0}$ - prag neophodan za sprečavanje širenja bolesti.

Modeliranje zaraznih bolesti



Model SI



$S(t) + I(t) = 1$ - konstantan broj jedinki
λ - koeficijent zaraze
Δt - vremenski interval
$\lambda SI\Delta t$ - verovatnoća da dodje do infekcije za Δt
$S_{t+\Delta t} = S_t - \lambda SI\Delta t$ - promena broja jedinki u S za vreme Δt
$-\lambda SI = \frac{S_{t+\Delta t} - S_t}{\Delta t}$
$\Delta t \to 0 \Rightarrow \frac{dS}{dt} = -\lambda SI$

$I_{t+\Delta t} = I_t + \lambda SI\Delta t$ - promena broja jedinki u I za vreme Δt
$\lambda SI = \frac{I_{t+\Delta t} - I_t}{\Delta t}$
$\Delta t \to 0 \Rightarrow \frac{dI}{dt} = -\lambda SI$

Zamenom $S(t) + I(t) = 1 \Rightarrow \frac{dI(t)}{dt} = \lambda I(t)(1 - I(t))$ (DJ sa razdvojenim promenljivim)

Model SIS

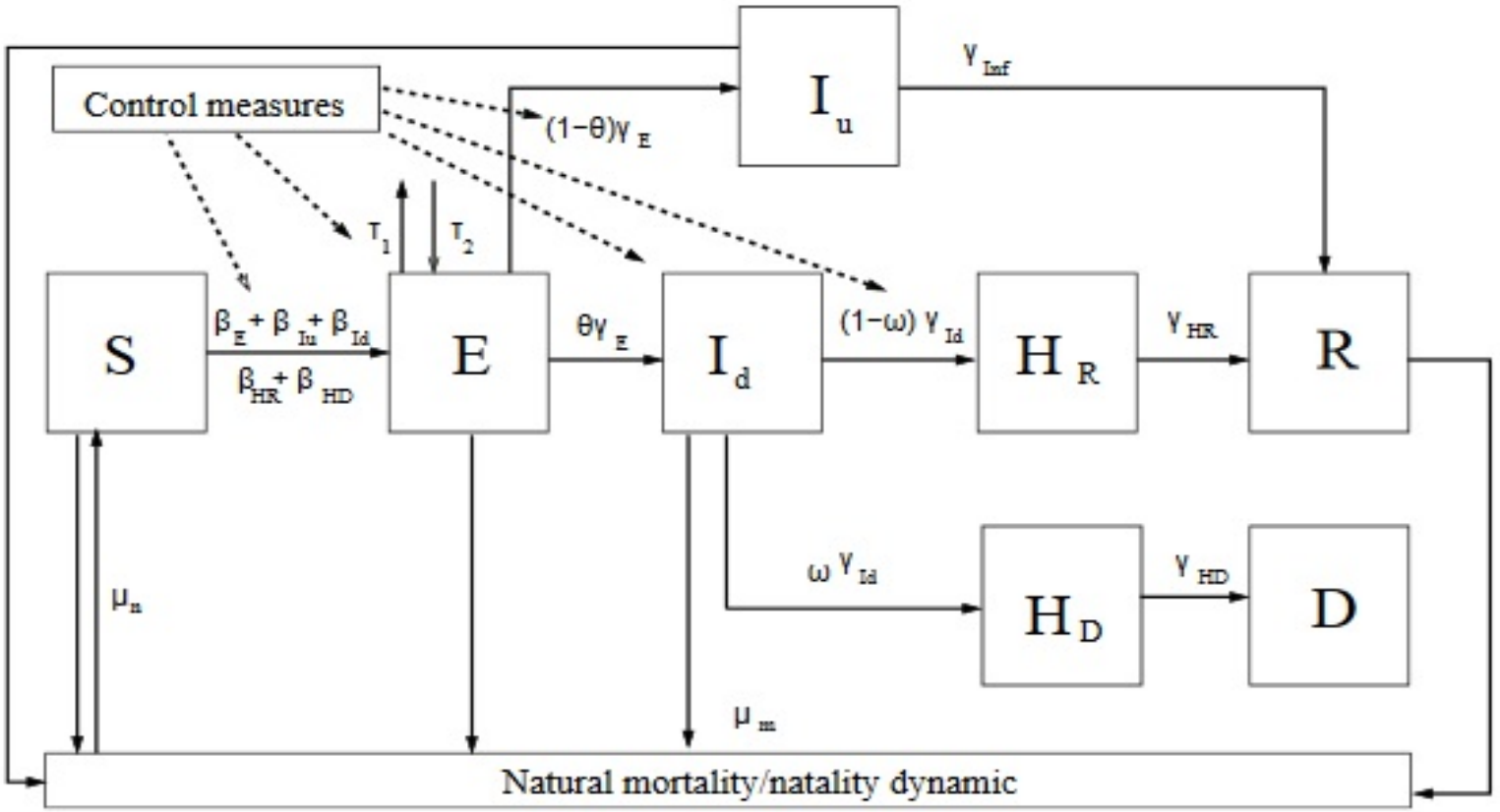
λ - koeficijent zaraze
γ - koeficijent oporavka

$$\frac{dS(t)}{dt} = -\lambda S(t)I(t) + \gamma I(t)$$

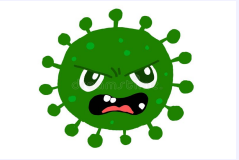
$$\frac{dI(t)}{dt} = \lambda S(t)I(t) - \gamma I(t)$$

(sistem DJ)

Model za koronu???



Model za koronu???



$$
\begin{aligned}
\frac{\mathrm{d}S^{(i)}}{\mathrm{d}t}(t) &= -\frac{S^{(i)}(t)}{N^{(i)}}\left(m_E^{(i)}(t)\beta_E^{(i)}E^{(i)}(t) + m_{I_u}^{(i)}(t)\beta_{I_u}^{(i)}I_{\mathrm{u}}^{(i)}(t) + m_{I_d}^{(i)}(t)\beta_{I_d}^{(i)}I_{\mathrm{d}}^{(i)}(t)\right) \\
&\quad -\frac{S^{(i)}(t)}{N^{(i)}}\left(m_{H_R}^{(i)}(t)\beta_{H_R}^{(i)}H_R^{(i)}(t) + m_{H_D}^{(i)}(t)\beta_{H_D}^{(i)}H_D^{(i)}(t)\right) \\
&\quad -\mu_m^{(i)}S^{(i)}(t) + \mu_n^{(i)}\left(S^{(i)}(t) + E^{(i)}(t) + I_{\mathrm{u}}^{(i)}(t) + I_{\mathrm{d}}^{(i)}(t) + H_R^{(i)}(t) + R^{(i)}(t)\right), \\
\frac{\mathrm{d}E^{(i)}}{\mathrm{d}t}(t) &= \frac{S^{(i)}(t)}{N^{(i)}}\left(m_E^{(i)}(t)\beta_E^{(i)}E^{(i)}(t) + m_{I_u}^{(i)}(t)\beta_{I_u}^{(i)}I_{\mathrm{u}}^{(i)}(t) + m_{I_d}^{(i)}(t)\beta_{I_d}^{(i)}I_{\mathrm{d}}^{(i)}(t)\right) \\
&\quad +\frac{S^{(i)}(t)}{N^{(i)}}\left(m_{H_R}^{(i)}(t)\beta_{H_R}^{(i)}H_R^{(i)}(t) + m_{H_D}^{(i)}(t)\beta_{H_D}^{(i)}H_D^{(i)}(t)\right) \\
&\quad -\mu_m^{(i)}E^{(i)}(t) - \gamma_E E^{(i)}(t) + \tau_1^{(i)}(t) - \tau_2^{(i)}(t), \\
\frac{\mathrm{d}I_{\mathrm{d}}^{(i)}}{\mathrm{d}t}(t) &= \theta^{(i)}(t)\gamma_E E^{(i)}(t) - (\mu_m^{(i)} + \gamma_{I_d}^{(i)}(t))I_{\mathrm{d}}^{(i)}(t), \\
\frac{\mathrm{d}I_{\mathrm{u}}^{(i)}}{\mathrm{d}t}(t) &= (1-\theta^{(i)}(t))\gamma_E E^{(i)}(t) - (\mu_m^{(i)} + \gamma_{\mathrm{Inf}})I_{\mathrm{u}}^{(i)}(t), \\
\frac{\mathrm{d}H_R^{(i)}}{\mathrm{d}t}(t) &= (1-\omega^{(i)}(t))\gamma_{I_d}^{(i)}(t)I_{I_d}^{(i)}(t) - \gamma_{H_R}H_R^{(i)}(t), \\
\frac{\mathrm{d}H_D^{(i)}}{\mathrm{d}t}(t) &= \omega^{(i)}(t)\gamma_{I_d}^{(i)}(t)I_{I_d}^{(i)}(t) - \gamma_{H_D}^{(i)}(t)H_D^{(i)}(t), \\
\frac{\mathrm{d}R^{(i)}}{\mathrm{d}t}(t) &= \gamma_{H_R}^{(i)}(t)H_R^{(i)}(t) - \mu_m^{(i)}R^{(i)}(t), \\
\frac{\mathrm{d}D^{(i)}}{\mathrm{d}t}(t) &= \gamma_{H_D}^{(i)}(t)H_D^{(i)}(t),
\end{aligned}
$$