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7:51 AM

(*) $\varepsilon = 10^{-3}$

$$A = \begin{pmatrix} 1.12 & 4.45 & 0.38 \\ 4.45 & 1.31 & 0.56 \\ 0.38 & 0.56 & 0.52 \end{pmatrix}$$

tragovima λ_1, x_1 ?

$$A^2 = \begin{pmatrix} 21.2013 & & * \\ & 21.8322 & \\ * & & 0.7284 \end{pmatrix}$$

$$|\lambda_1| \approx \sqrt[2]{|tr(A^2)|} = \sqrt{43.7619} = 6.6153$$

$$A^4 = \begin{pmatrix} 580.7789 & & * \\ & 605.5998 & \\ * & & 17.6106 \end{pmatrix}$$

$$|\lambda_1| \approx \sqrt[4]{|tr(A^4)|} = \sqrt[4]{\dots} = 5.8905$$

$A^8 = (\dots)$ $\quad |\lambda_1| \approx \sqrt[8]{|tr(A^8)|} = 5.7581$

$A^{16} = (\dots)$ $\quad |\lambda_1| \approx \sqrt[16]{|tr(A^{16})|} = 5.7509$

$A^{32} = (\dots)$ $\quad |\lambda_1| \approx \sqrt[32]{|tr(A^{32})|} = 5.7508$

$$|5.7509 - 5.7508| < 10^{-3}$$

$|\lambda_1| = 5.7508$

$A x_1 = \lambda_1 x_1$

$x_1 = A^{32} \cdot v \cdot \delta = (0.693, 0.710, 0.126)^T$

v ↓ proizv. $[1, 0, 0]^T$

δ ↓ $\dfrac{1}{\text{norma}(A^{32} \cdot v)}$

$$A \cdot x_1 = \lambda_1 x_1$$

$$\begin{pmatrix} + \\ + \\ + \end{pmatrix} = \underbrace{\lambda_1}_{+} \cdot \begin{pmatrix} + \\ + \\ + \end{pmatrix}$$

$\lambda_1 = +5.7508$

$$\begin{pmatrix} - \\ - \end{pmatrix} = \underbrace{\lambda_1}_{<0} \begin{pmatrix} + \\ + \end{pmatrix}$$

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(*) $\varepsilon = 5 \cdot 10^{-3}$ traganja odrediti najmanju po modulu

(pr. vekt, sk. proizvod)

$$A = \begin{pmatrix} 4 & 0 & 3 \\ 2 & 2 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

$Ax_i = \lambda_i x_i \rightarrow \lambda_1$ najveća za A, λ_n najmanja za A

$A^{-1} x_i = \frac{1}{\lambda_i} x_i \rightarrow \frac{1}{\lambda_1}$ najmanja od A^{-1}, $\frac{1}{\lambda_n}$ najveća za A^{-1}

$$B = A^{-1} = \begin{pmatrix} -0.1250 & 1.125 & -0.75 \\ -0.6250 & 1.625 & -0.75 \\ 0.5 & -1.5 & 1 \end{pmatrix} \qquad \lambda_1(B) = ?$$

$B^2 = (\dots)$, $\sqrt{|tr(B^2)|} = 1.9365$

$B^4 = (\dots)$, $\sqrt[4]{|tr(B^4)|} = 1.8684$

$B^8 = (\dots)$, $\sqrt[8]{|tr(B^8)|} = 1.8660$ $\quad < 5 \cdot 10^{-3}$

$|\lambda_1(B)| = 1.8660$

$x_1 = B^8 \cdot V \cdot 5 = (-0.5477, -0.5477, 0.635)^T$

$Bx_1 = \lambda_1 \cdot x_1$

$(\quad) = \underbrace{\lambda_1}_{>0} (\quad)$ $\Rightarrow \lambda_1(B) = 1.866$

$\lambda_3(A) = \frac{1}{\lambda_1(B)} = \frac{1}{1.866} = 0.5359$

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Metoda iscrpljivanja

$|\lambda_1| > \dots > |\lambda_n|$, $x_1, \dots, x_n$

u matlab ne == !? | levi - desni | < 1e-4

$A: \lambda_1, x_1$

$A^*: \overline{\lambda_1}, y_1$

x_1, y_1 normirani $(x_1, y_1) \approx 1$

$(x_i, y_j) = \delta_{ij}$

$$\boxed{A_1 = A - \lambda_1 \cdot x_1 \cdot y_1^*}$$

$\lambda_1 [\,]$

$\lambda(A_1) = \{0, \lambda_2(A), \dots, \lambda_n(A)\}$

$\lambda_1(A_1) = \lambda_2(A)$

$$\begin{aligned}
A_1 x_1 &= (A - \lambda_1 x_1 y_1^*) \cdot x_1 \\
&= A x_1 - \lambda_1 x_1 \underbrace{y_1^* x_1} \\
&= A x_1 - \lambda_1 x_1 \underbrace{(x_1, y_1)}_{\approx 1} \\
&= \cancel{A x_1 - \lambda_1 x_1} = \lambda_1 x_1 - \lambda_1 x_1 (x_1, y_1) \\
&= (\lambda_1 - \lambda_1 \underbrace{(x_1, y_1)}_{1}) \cdot x_1 \\
&= \underset{\lambda_1(A_1) = 0}{\underline{0}} \cdot x_1
\end{aligned}$$

$A x_1 = 0$

$\underbrace{A x_1}_{0} = \lambda_1 \underset{\neq 0}{\underline{x_1}}$

$$\begin{aligned}
A_1 \cdot x_i &= (A - \lambda_1 x_1 y_1^*) x_i \qquad i = 2, 3, \dots, n \\
&= A x_i - \lambda_1 x_1 y_1^* \cdot x_i \\
&= \lambda_i x_i - \lambda_1 x_1 \underbrace{(x_i, y_1)}_{0} \\
&= \lambda_i \cdot x_i
\end{aligned}$$

$\lambda_i(A_1) = \lambda_i(A)$, $i = 2, \dots, n$

---

* nekom preth. met $\to \lambda_1(A)$, tol

$A_1 = \dots$ nekom preth. met $\xrightarrow{tol} \lambda_1(A_1) = \lambda_2(A)$

$\lambda \in \{0, 0, \lambda_3(A), \dots, \lambda_n(A)\}$

$A_2 = \dots$ nekom preth. met $\to \lambda_1(A_2) = \lambda_3(A)$

$\vdots$

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(*) Primenom met. iscrpljivanja i metode tragova odrediti drugu po veličini modula s. vred. ..... sa tačnošću $5 \cdot 10^{-3}$

(sk. proizvod prozv. vekt)

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

1) tragovima $\lambda_1(A)$, x_1 $\longrightarrow$ $\lambda_1 = 4.414$

$x_1 = (0.5, 0.5, 0.707)^T$

$\dfrac{x_1}{\|x_1\|} = x_1$

$\lambda_1(A) = \lambda_1(A^T)$

$x_1 = y_1$

1*) tragovima sa tol $\lambda_1(A^*) = \overline{\lambda_1}$, y_1

2) $A_1 = A - \lambda_1 x_1 \cdot x_1^T = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$

3) tragovima sa tol $\lambda_1(A_1) = \lambda_2(A)$