sitely directed, force on some other body interacting with it. This is often called the *Law of Action and Reaction*.

The Law of Universal Gravitation

When two masses m and m´ gravitationally interact, they attract each other with forces of equal magnitude. For point masses (or spherically symmetric bodies), the attractive force F_G is given by

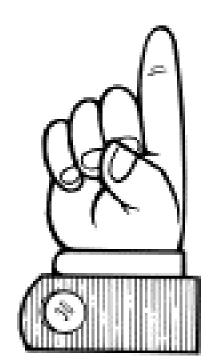
$$F_G = G\frac{mm'}{r^2}$$

where r is the distance between mass centers, and where $G = 6.67 \times 10^{-11}$ N m^2/kg^2 when F_G is in newtons, m and m´ are in kilograms, and r is in meters.

Weight

The weight of an object (F_W) is the gravitational force acting downward on the object. On the Earth, it is the gravitational force exerted on the object by the planet. Its units are newtons (in the SI) and pounds (in the British system). An object of mass m falling freely toward the Earth is subject to only one force—the pull of gravity, which we call the weight F_W of the object. The object's acceleration due to F_W is the free-fall acceleration g.

Therefore, $\mathbf{F} = m\vec{\mathbf{a}}$ provides us with the relation between $F = F_W$, $a = g$, and m; it is $F_W = mg$. Because, on average, $g = 9.8\ m/s^2$ on Earth, a 1.0-kg object weighs 9.8 N at the Earth's surface.



Specific Types of Forces

The **tensile force** ($\vec{\mathbf{F}}_T$) acting on a string, chain, or tendon is the applied force tending to stretch it. The magnitude of the tensile force is the **tension** (F_T).

The **normal force** ($\vec{\mathbf{F}}_N$) on an object that is being supported by a surface is the component of the supporting force that is perpendicular to the surface.

The **friction force** ($\vec{\mathbf{F}}_f$) is a tangential force acting on an object that opposes the sliding of that object on an adjacent surface with which it is in contact. The friction force is parallel to the surface and opposite to the direction of motion or of impending motion. Only when the applied force exceeds the maximum static friction force will an object begin to slide.

The **coefficient of kinetic friction** (μ_k), defined for the case in which one surface is sliding across another at constant speed, is

$$\mu_k = \frac{\text{friction force}}{\text{normal force}} = \frac{F_f}{F_N}$$

The **coefficient of static friction** (μ_s), defined for the case in which one surface is just on the verge of sliding across another surface, is

$$\mu_s = \frac{\text{maximum friction force}}{\text{normal force}} = \frac{F_f(\max)}{F_N}$$

where the maximum friction force occurs when the object is just on the verge of slipping but is nonetheless at rest.

Dimensional Analysis

All mechanical quantities, such as acceleration and force, can be expressed in terms of three fundamental dimensions: length L, mass M, and time T. For example, acceleration is a length (a distance) divided by (time)^2; we say it has the dimensions L/T^2, which we write as [LT^{-2}].

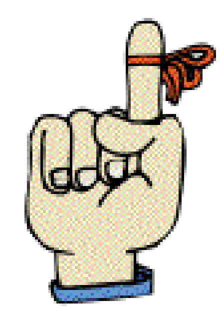
The dimensions of volume are [L^3], and those of velocity are [LT^{-1}]. Because force is mass multiplied by acceleration, its dimensions are [MLT^{-2}]. Dimensions are helpful in checking equations, since each term of an equation must have the same dimensions. For example,

$$s = v_1 t + \frac{1}{2} at^2$$

$$[L] \rightarrow [LT^{-1}][T] + [LT^{-2}][T^2]$$

so each term has the dimensions of length. As examples, an equation cannot have a volume [L^3] added to an area [L^2], or a force [MLT^{-2}] subtracted from a velocity [LT^{-1}]; these terms do not have the same dimensions.

> Remember, all terms in an equation must have the same dimensions.



Mathematical Operations with Units

In every mathematical operation, the units terms (for example, lb, cm, ft^3, mi/h, m/s^2) must be carried along with the numbers and must undergo the same mathematical operations as the numbers.

Quantities cannot be added or subtracted directly unless they have the same units (as well as the same dimensions). For example, if we are to add algebraically 5 m (length) and 8 cm (length), we must first convert m to cm or cm to m. However, quantities of any sort can be combined in multiplication or division, in which the units as well as the numbers obey the algebraic laws of squaring, cancellation, etc.

Equilibrium under the Action of Concurrent Forces

Concurrent Forces

Concurrent forces are forces whose lines of action all pass through a common point. The forces acting on a point object are concurrent because they all pass through the same point, the point object.

Equilibrium

An object is in **equilibrium** under the action of concurrent forces provided it is not accelerating. A condition for equilibrium under concurrent forces is the requirement that $\Sigma \mathbf{F} = 0$ or, in component form,

$$\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$$

That is, the resultant of all external forces acting on the object must be zero.

Problem Solution Method (Concurrent Forces)

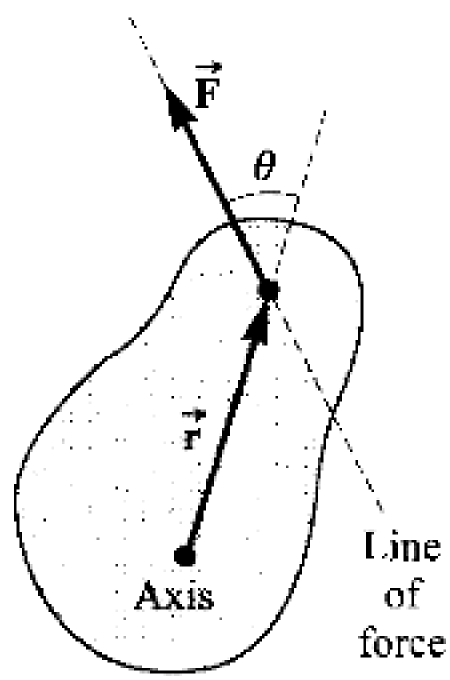
(1) Isolate the object for discussion.
(2) Show the forces acting on the isolated object in a diagram (the *free-body* diagram).
(3) Find the rectangular components of each force.
(4) Write the condition for equilibrium in equation form.
(5) Solve for the required quantities.

Equilibrium of a Rigid Body Under Coplanar Forces

The **torque (or moment)** about an axis, due to a force, is a measure of the effectiveness of the force in producing rotation about that axis. It is defined in the following way:

Torque = τ = rF sin θ

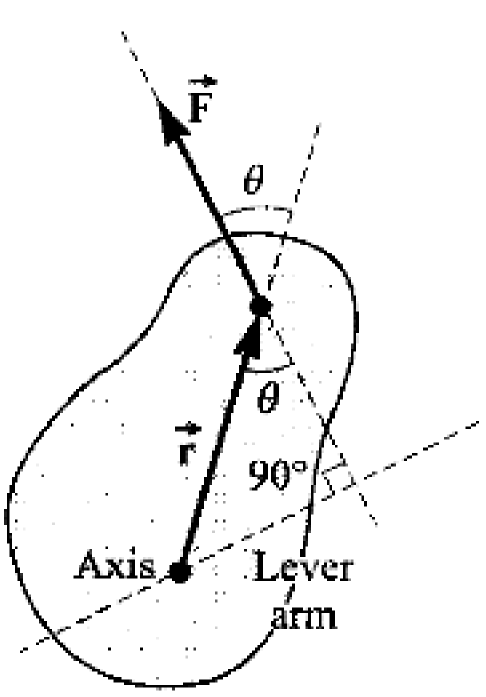
where r is the radial distance from the axis to the point of application of the force, and θ is the acute angle between the lines-of-action of $\vec{r}$ and $\vec{F}$



, as shown in Figure 1-5.

Figure 1-5

Often this definition is written in terms of the lever arm of the force,



which is the perpendicular distance from the axis to the line of the force, as shown in Figure 1-6.

Figure 1-6

Because the lever arm is simply r sin θ, the torque becomes

$\tau = (F)(\text{lever arm})$

The units of torque are newton-meters (N • m). Plus and minus signs can be assigned to torques; for example, a torque that tends to cause counterclockwise rotation about the axis is positive, whereas one causing clockwise rotation is negative.

Conditions for Equilibrium

The two conditions for equilibrium of a rigid object under the action of *coplanar forces* are:

(1) As listed above, the first condition for equilibrium is the ***force condition.***

The vector sum of all forces acting on the body must be zero:

$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$

where the plane of the coplanar forces is taken to be the xy-plane.

(2) The second condition for equilibrium is the ***torque condition.***

Take an axis perpendicular to the plane of the coplanar forces. Call the torques that tend to cause clockwise rotation about the axis negative, and counterclockwise torques positive; then the sum of all the torques acting on the object must be zero:

$\Sigma\tau = 0$

If the sum of the torques is zero about one axis for a body that obeys the force condition, it is zero about all other axes parallel to the first. We can choose the axis in such a way that the line of an unknown force passes through the intersection of the axis and the plane of the forces. The angle θ between $\mathbf{r}$ and $\mathbf{F}$ is then zero; hence, that particular unknown force exerts zero torque and therefore does not appear in the torque equation.

Essential Point

Center of Gravity

The **center of gravity** of an object is the point at which the entire weight of the object may be concentrated; i.e., the line-of-action of the weight passes through the center of gravity. A single vertically upward directed force, equal in magnitude to the weight of the object and applied through its center of gravity, will keep the object in equilibrium.

Work, Energy, and Power

Work

The work done by a force is defined as the product of that force times the parallel distance over which it acts. Consider the simple case of straight-line motion shown in Figure 1-7, where a force $\vec{\mathbf{F}}$ acts on a body that simultaneously undergoes a vector displacement $\vec{\mathbf{s}}$. The component of $\vec{\mathbf{F}}$ in the direction of $\vec{\mathbf{s}}$ is F cos θ. The work W done by the force $\vec{\mathbf{F}}$ is defined to be the component of $\vec{\mathbf{F}}$ in the direction of the displacement, multiplied by the displacement:

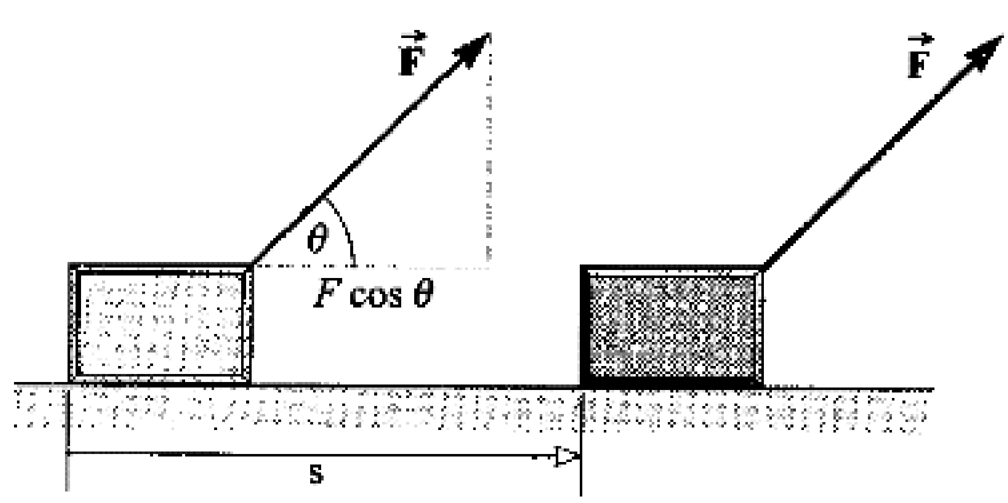


Figure 1-7