A **vector** is a quantity that possesses both magnitude and direction. Examples of vector quantities are displacement, velocity, acceleration, and force. A vector quantity can be represented by an arrow drawn to scale. The length of the arrow is proportional to the magnitude of the vector quantity. The direction of the arrow represents the direction of the vector quantity.

The Components of a Vector

Before we define the components of a vector, we first must introduce the elementary relationships between trigonometric functions. The trigonometric functions are defined in relation to a right angle. For the right triangle shown in Figure 1-1, by definition

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{B}{C}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{C}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{B}{A}$$

We often use these in the forms

$B = C \sin\theta$; $A = C \cos\theta$; $B = A \tan\theta$

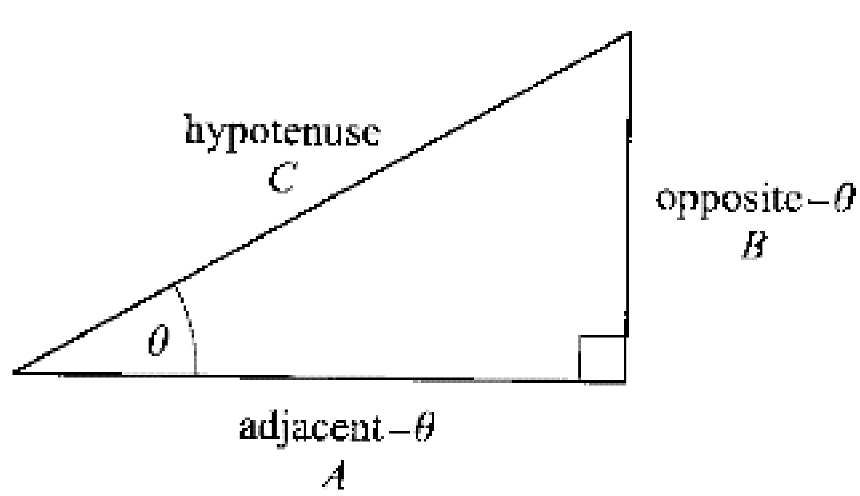
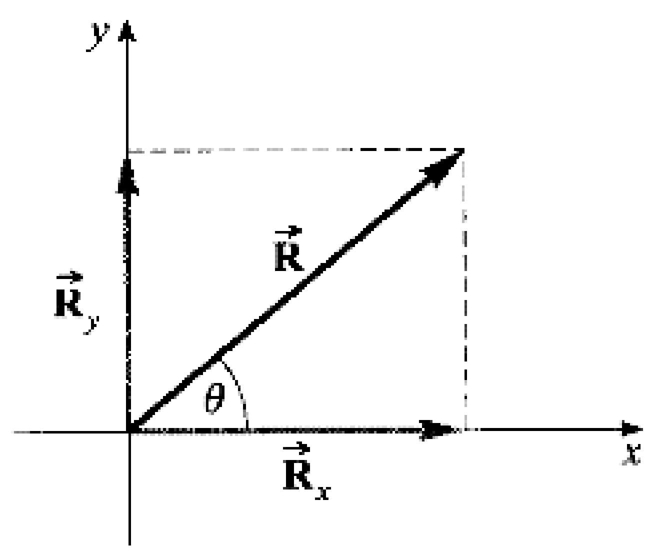


Figure 1-1

A component of a vector is its effective value in a given direction. For example, the x-component of a displacement is the displacement parallel to the x-axis caused by the given displacement. A vector in three dimensions may be considered as the resultant of its component vectors resolved along any three *mutually perpendicular* directions. Similarly, a vector in two dimensions may be resolved into two component vectors acting along any two mutually perpendicular directions. Figure 1-2 shows the vector $\vec{\mathbf{R}}$ and its x and y vector components, $\vec{\mathbf{R}}_x$, $\vec{\mathbf{R}}_y$, which have magnitudes

$$|\vec{\mathbf{R}}_x| = |\vec{\mathbf{R}}| \cos\theta \quad \text{and} \quad |\vec{\mathbf{R}}_y| = |\vec{\mathbf{R}}| \sin\theta$$

or equivalently,



$$R_x = R\cos\theta \quad \text{and} \quad R_y = R\sin\theta$$

Figure 1-2

Unit Vectors

Unit vectors have a magnitude of one and are represented by a boldface symbol topped with a caret. The special unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are assigned to the x-, y-, and z-axes, respectively. A vector 3 $\hat{\mathbf{i}}$ represents a three-unit vector in the +x direction, while -5 $\hat{\mathbf{k}}$ represents a five-unit vector in the -z direction. A vector $\vec{\mathbf{R}}$ that has scalar x-, y-, and z-components R_x, R_y, and R_z, respectively, can be written as

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$$

When an object moves from one point in space to another, the **displacement** is the vector from the initial location to the final location.

Important Point!

Vector Addition

The **resultant**, or sum, of a number of vectors of a particular type (force vectors, for example) is that single vector that would have the same effect as all the original vectors taken together.

It is independent of the actual distance traveled.

Graphical Addition of Vectors (Polygon Method)

This method for finding the resultant $\vec{\mathbf{R}}$ of several vectors ($\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$) consists in beginning at any convenient point and drawing (to scale and in the proper directions) each vector arrow in turn. They may be taken in any order of succession:

$$\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}=\vec{\mathbf{C}}+\vec{\mathbf{A}}+\vec{\mathbf{B}}=\vec{\mathbf{R}}$$

The tail end of each arrow is positioned at the tip end of the preceding one, as shown in Figure 1-3.

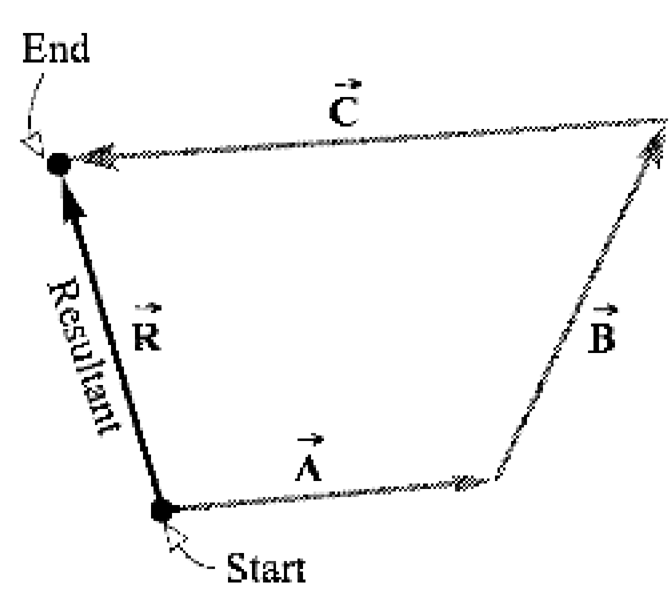


Figure 1-3

The resultant is represented by an arrow with its tail end at the starting point and its tip end at the tip of the last vector added. If $\vec{\mathbf{R}}$ is the resultant, R = $|\vec{\mathbf{R}}|$ is the size or **magnitude** of the resultant.

Parallelogram Method for Vector Addition

The resultant of two vectors acting at any angle may be represented by the diagonal of a parallelogram. The two vectors are drawn as the sides of the parallelogram and the resultant is its diagonal, as shown in Figure 1-4. The direction of the resultant is away from the origin of the two vectors.

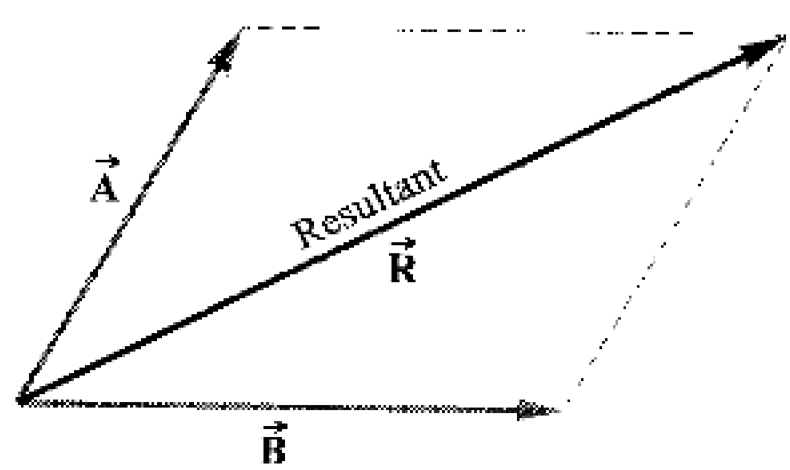


Figure 1-4

Component Method for Vector Addition

Each vector is resolved into its x-, y-, and z-components, with negatively directed components taken as negative. The scalar x-component, R_x, of the resultant $\vec{\mathbf{R}}$ is the algebraic sum of all the scalar components. The scalar y- and z-components of the resultant are found in a similar way. With the components known, the magnitude of the resultant is given by

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

In two dimensions, the angle of the resultant with the x-axis can be found from the relation

$$\tan\theta = \frac{R_y}{R_x}$$

Vector Subtraction

To subtract a vector $\vec{\mathbf{B}}$ from a vector $\vec{\mathbf{A}}$, reverse the direction of $\vec{\mathbf{B}}$ and add individually to vector $\vec{\mathbf{A}}$, that is $\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$.

Uniformly Accelerated Motion

Speed is a scalar quantity. If an object takes a time interval t to travel a distance d, then

$$\text{Average speed} = \frac{\text{total distance traveled}}{\text{time taken}}$$

or

$$v_{av} = \frac{d}{t}$$

Here the distance is the total (along the path) length traveled.

Velocity is a vector quantity. If an object undergoes a vector displacement, $\vec{\mathbf{s}}$, in a time interval, t, then

$$\text{Average velocity} = \frac{\text{vector displacement}}{\text{time taken}}$$

$$\vec{\mathbf{v}}_{av} = \frac{\vec{\mathbf{s}}}{t}$$

The direction of the velocity vector is the same as that of the displacement vector. The units of velocity (and speed) are those of distance divided by time, such as m/s or km/h.

Acceleration, also a vector quantity, measures the time rate-of-change of velocity:

$$\text{Average acceleration} = \frac{\text{change in velocity vector}}{\text{time taken}}$$

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t}$$

where $\vec{\mathbf{v}}_i$ is the initial velocity, $\vec{\mathbf{v}}_f$, is the final velocity, and t is the time interval over which the change occurred. The units of acceleration are those of velocity divided by time. A typical example is (m/s)/s (or m/s^2).

Uniform Motion along a Straight Line

This represents an important situation. In this case, the *acceleration vector is constant* and lies along the line of the displacement vector, so that the directions of $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ can be specified with plus and minus signs. If we represent the displacement by s (positive if in the positive direction, and negative if in the negative direction), then the motion can be described with the five equations for uniformly accelerated motion:

$$s = v_{av} t$$

$$s = v_i t + \frac{1}{2} a t^2$$

$$v_f^2 = v_i^2 + 2as$$

$$a = \frac{v_f - v_i}{t}$$

$$v_{av} = \frac{v_f + v_i}{2}$$

Direction is important, and a positive direction must be chosen when analyzing motion along a line. Either direction may be chosen as positive. If a displacement, velocity, or acceleration is in the opposite direction, it must be taken as negative.

Instantaneous Velocity

Instantaneous velocity is the average velocity evaluated for a time interval that approaches zero. Thus, if an object undergoes a change in displacement $\Delta\vec{s}$ over a time interval Δt, then for that object the **instantaneous velocity** is

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{s}}{\Delta t}$$

where the notation means that the ratio $\frac{\Delta\vec{s}}{\Delta t}$ is to be evaluated for a time interval Δt that approaches zero.

You Need to Know ✓

Graphical Interpretations

Graphical interpretations for motion along a straight line (the x-axis) are as follows:

- The *instantaneous velocity* of an object at a certain time is the slope of the displacement versus time graph at that time. It can be positive, negative, or zero.
- The *instantaneous acceleration* of an object at a certain time is the slope of the velocity versus time graph at that time.
- For constant-velocity motion, the x-versus-t graph is a straight line. For constant-acceleration motion, the v-versus-t graph is a straight line.
- In general (i.e., one- , two-, or three-dimensional motion), the slope at any moment of the distance-versus-time graph is the speed.

Acceleration Due to Gravity (g)

The acceleration of a body moving only under the force of gravity is g, the gravitational (or free-fall) acceleration, which is directed vertically downward. On Earth, $g = 9.8\ m/s^2$ (i.e., $32\ ft/s^2$). On the Moon, the free-fall acceleration is $1.6\ m/s^2$.

Velocity Components

Suppose that an object moves with a velocity $\vec{v}$ at some angle θ up from the x-axis, as would initially be the case with a ball thrown into the air. That velocity then has x and y vector components of $\vec{v}_x$ and $\vec{v}_y$. The corresponding scalar components of the velocity are:

$v_x = v \cos\theta$ and $v_y = v \sin\theta$

Projectile Problems

Projectile problems can be solved easily if air friction can be ignored. One simply considers the motion to consist of two independent parts: horizontal motion with $a = 0$ and $v_f = v_i = v_{av}$ (i.e., constant speed), and vertical motion with $a = g = 9.8\ m/s^2$ downward.

Newton's Laws

Mass

The **mass** of an object is a measure of the inertia of the object. **Inertia** is the tendency of a body at rest to remain at rest, and of a body in motion to continue moving with unchanged velocity.

Force

Force, in general, is the agency of change. In mechanics, it is a push or a pull that changes the velocity of an object. Force is a vector quantity, having magnitude and direction. An external force is one whose source lies outside of the system being considered. The net or resultant external force acting on an object causes the object to accelerate in the direction of that force. The acceleration is proportional to the force and inversely proportional to the mass of the object. The **newton** is the SI unit of force. One newton (1 N) is that resultant force which will give a 1-kg mass an acceleration of 1 m/s^2. The pound is 4.45 N.

Newton's Laws

Newton's First Law: *An object at rest will remain at rest; an object in motion will continue in motion with constant velocity unless acted on by an external force.* Force is the changer of motion.

Newton's Second Law: *If the resultant or net force $\vec{\mathbf{F}}$ acting on an object of mass m is not zero, the object accelerates in the direction of the force. The acceleration $\vec{\mathbf{a}}$ is proportional to the force and inversely proportional to the mass of the object. With $\vec{\mathbf{F}}$ in newtons, m in kilograms, and $\vec{\mathbf{a}}$ in m/s^2, this can be written as*

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m} \quad \text{or} \quad \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

The acceleration $\vec{\mathbf{a}}$ has the same direction as the resultant force $\vec{\mathbf{F}}$.

The vector equation $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ can be written in terms of components as

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z$$

where the forces are the components of the external forces acting on the object.

Newton's Third Law: Matter *interacts* with matter—forces come in pairs. *For each force exerted on one body, there is an equal, but oppo-*

sitely directed, force on some other body interacting with it. This is often called the *Law of Action and Reaction*.

The Law of Universal Gravitation

When two masses m and m´ gravitationally interact, they attract each other with forces of equal magnitude. For point masses (or spherically symmetric bodies), the attractive force F_G is given by

$$F_G = G\frac{mm'}{r^2}$$

where r is the distance between mass centers, and where $G = 6.67 \times 10^{-11}$ N m²/kg² when F_G is in newtons, m and m´ are in kilograms, and r is in meters.

Weight

The weight of an object (F_W) is the gravitational force acting downward on the object. On the Earth, it is the gravitational force exerted on the object by the planet. Its units are newtons (in the SI) and pounds (in the British system). An object of mass m falling freely toward the Earth is subject to only one force—the pull of gravity, which we call the weight F_W of the object. The object's acceleration due to F_W is the free-fall acceleration g.

Therefore, $\mathbf{F} = m\vec{\mathbf{a}}$ provides us with the relation between $F = F_W$, $a = g$, and m; it is $F_W = mg$. Because, on average, $g = 9.8$ m/s² on Earth, a 1.0-kg object weighs 9.8 N at the Earth's surface.