## Essential Point

### Center of Gravity

The **center of gravity** of an object is the point at which the entire weight of the object may be concentrated; i.e., the line-of-action of the weight passes through the center of gravity. A single vertically upward directed force, equal in magnitude to the weight of the object and applied through its center of gravity, will keep the object in equilibrium.

# Work, Energy, and Power

## Work

The work done by a force is defined as the product of that force times the parallel distance over which it acts. Consider the simple case of straight-line motion shown in Figure 1-7, where a force $\vec{\mathbf{F}}$ acts on a body that simultaneously undergoes a vector displacement $\vec{\mathbf{s}}$. The component of $\vec{\mathbf{F}}$ in the direction of $\vec{\mathbf{s}}$ is F cos θ. The work W done by the force $\vec{\mathbf{F}}$ is defined to be the component of $\vec{\mathbf{F}}$ in the direction of the displacement, multiplied by the displacement:

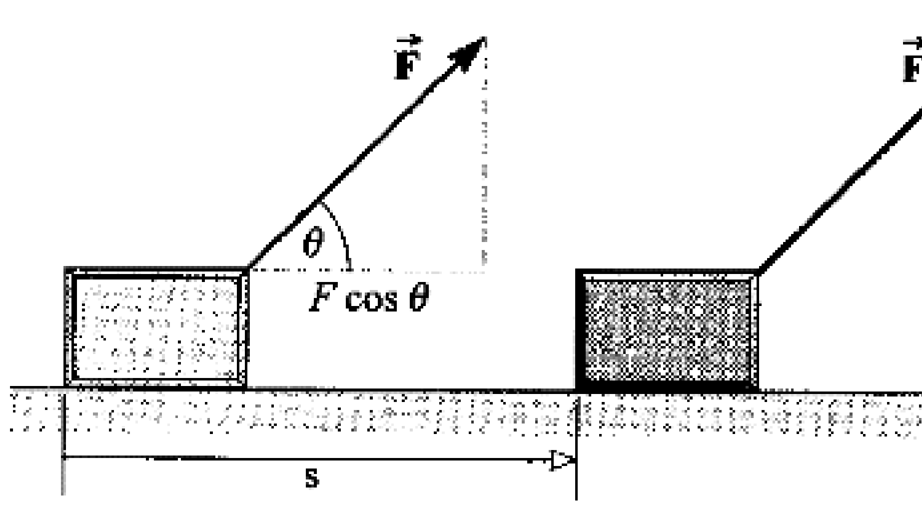



**Figure 1-7**

$W = (F \cos \theta)(s) = Fs \cos \theta$

Notice that θ is the angle between the force and displacement vectors. Work is a scalar quantity. The SI unit of work is the newton-meter, called the joule (J). One joule is the work done by a force of 1 N when it displaces an object 1 m in the direction of the force. Other units sometimes used for work are the erg, where 1 erg = $10^{-7}$ J, and the foot-pound (ft-lb), where 1 ft-lb = 1.355 J.

If $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ are in the same direction, $\cos \theta = \cos 0^\circ = 1$ and W = Fs. But, if $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ are in opposite directions, then $\cos \theta = \cos 180^\circ = -1$ and W = -Fs; the work is negative. Forces such as friction often slow the motion of an object and are then opposite in direction to the displacement. Such forces usually do negative work.

Work is the transfer of energy from one entity to another by way of the action of a force applied over a distance. The point of application of the force must move if work is to be done.

## Energy

**Energy** is a measure of the change imparted to a system. It is given to an object when a force does work on the object. The amount of energy transferred to the object equals the work done. Further, when an object does work, it loses an amount of energy equal to the work it does. Energy and work have the same units, joules. Energy, like work, is a scalar quantity. An object that is capable of doing work possesses energy.

**Kinetic energy** (KE) is the energy possessed by an object because it is in motion. If an object of mass m is moving with a speed v, it has translational KE given by

$$KE = \frac{1}{2}mv^2$$

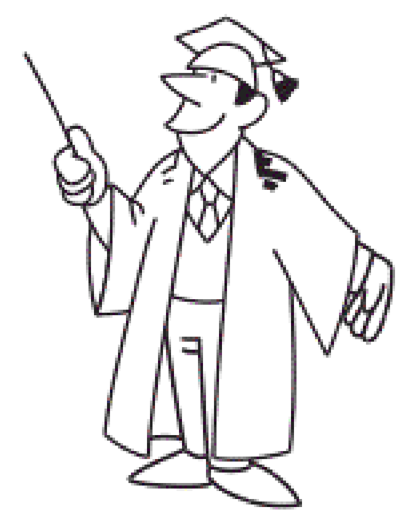
When m is in kg and v is in m/s, the units of KE are joules.

**Gravitational potential energy** (GPE) is the energy possessed by an object because of the gravitational interaction. As mass falls through a vertical distance h, a gravitational force can do work in the amount mgh. We define the GPE of an object relative to an arbitrary zero level, often the Earth's surface. If the object is at a height h above the zero (or reference) level, its GPE is

$$GPE = mgh$$

where g is the acceleration due to gravity. Notice that mg is the weight of the object. The units of GPE are joules when m is in kg, g is in $m/s^2$, and h is in m.

## Work-Energy Theorem



When work is done on a point mass or a rigid body, and there is no change in PE, the energy imparted can only appear as KE. Insofar as a body is not totally rigid, however, energy can be transferred to its parts and the work done on it will not precisely equal its change in KE.

## Conservation of Energy

Energy can be neither created nor destroyed, but only transformed from one kind to another.

## Power

**Power** is the time rate of doing work:

$$\text{Average power} = \frac{\text{work done by a force}}{\text{time taken to do this work}} = \text{force} \times \text{speed}$$

where the speed is measured in the direction of the force applied to the object. More generally, power is the rate of transfer of energy. In the SI, the unit of power is the watt (W), and 1 W = 1 J/s. Another unit of power often used is the horsepower: 1 hp = 746 W.

# Impulse and Momentum

## Linear Momentum

The **linear momentum** ($\vec{\mathbf{p}}$) of a body is the product of its mass (m) and velocity ($\vec{\mathbf{v}}$):

Linear momentum = (mass of body)(velocity of body)

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Momentum is a vector quantity whose direction is that of the velocity. The SI units of momentum are kg • m/s.

## Impulse

An **impulse** is the product of a force ($\vec{\mathbf{F}}$) and the time interval ($\Delta t$) over which the force acts:

Impulse = (force)(length of time the force acts)

Impulse is a vector quantity whose direction is that of the force. Its SI units are N • s.

An impulse causes a change in momentum. The change of momentum produced by an impulse is equal to the impulse in both magnitude and direction. Thus, if a constant force $\vec{\mathbf{F}}$ acting for a time $\Delta t$ on a body of mass m changes its velocity from an initial value $\vec{\mathbf{v}}_i$ to a final value $\vec{\mathbf{v}}_f$, then

Impulse = change in momentum

$$\vec{\mathbf{F}}\,\Delta t = m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right)$$

**Remember!**

**Conservation of Linear Momentum**

If the net external force acting on a system of objects is zero, the vector sum of the momenta of the objects will remain constant.

Newton's Second Law was actually given as $\vec{\mathbf{F}}=\frac{\Delta\vec{\mathbf{p}}}{\Delta t}$ from which it follows that $\vec{\mathbf{F}}\Delta t=\Delta\vec{\mathbf{p}}$. Moreover, $\vec{\mathbf{F}}\Delta t=\Delta(m\vec{\mathbf{v}})$, and if m is constant $\vec{\mathbf{F}}\Delta t=m(\vec{\mathbf{v}}_f-\vec{\mathbf{v}}_i)$.

## Collisions and Explosions

In **collisions and explosions**, the vector sum of the momenta just before the event equals the vector sum of the momenta just after the event. The vector sum of the momenta of the objects involved does not change during the collision or explosion. Thus, when two bodies of masses $m_1$ and $m_2$ collide,

Total momentum before impact = total momentum after impact

$$m_1\vec{\mathbf{u}}_1+m_2\vec{\mathbf{u}}_2=m_1\vec{\mathbf{v}}_1+m_2\vec{\mathbf{v}}_2$$

where $\vec{\mathbf{u}}_1$ and $\vec{\mathbf{u}}_2$ are the velocities before impact, and $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ are the velocities after impact. In one dimension, in component form,

$m_1u_{1x} + m_2u_{2x} = m_1v_{1x} + m_2v_{2x}$

and similarly for the y- and z-components. Remember that vector quantities are always boldfaced and velocity is a vector. On the other hand,

$u_{1x}$, $u_{2x}$, $v_{1x}$, and $v_{2x}$ are the scalar values of the velocities (they can be positive or negative). A positive direction is initially selected and vectors pointing opposite to this have negative numerical scalar values.

A **perfectly elastic collision** is one in which the sum of the translational KEs of the objects is not changed during the collision. In the case of two bodies,

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$$

## Coefficient of Restitution

For any collision between two bodies in which the bodies move only along a single straight line (e.g., the x-axis), a **coefficient of restitution**, **e**, is defined. It is a pure number given by

$$e=\frac{v_{2x}-v_{1x}}{u_{1x}-u_{2x}}$$

where $u_{1x}$ and $u_{2x}$ are values before impact, and $v_{1x}$ and $v_{2x}$ are values after impact. Notice that $|u_{1x} - u_{2x}|$ is the relative speed of approach and $|v_{2x} - v_{1x}|$ is the relative speed of recession.

For a perfectly elastic collision, e = 1. For inelastic collisions, e < 1. If the bodies stick together after collision, e = 0.

## Center of Mass

The **center of mass** of an object (of mass m) is the single point that moves in the same way as a point mass (of mass m) would move when subjected to the same external forces that act on the object. That is, if the $\vec{\mathbf{F}}$ resultant force acting on an object (or system of objects) of mass m is , the acceleration of the center of mass of the object (or system) is given by

$$\vec{\mathbf{a}}_{cm} = \frac{\vec{\mathbf{F}}}{m}$$

If the object is considered to be composed of tiny masses $m_1$, $m_2$, $m_3$, and so on, at coordinates $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$, and so on, then the coordinates of the center of mass are given by

$$x_{cm} = \frac{\Sigma x_i m_i}{\Sigma m_i},\ y_{cm} = \frac{\Sigma y_i m_i}{\Sigma m_i},\ z_{cm} = \frac{\Sigma z_i m_i}{\Sigma m_i}$$

where the sums extend over all masses composing the object. In a uniform gravitational field, the center of mass and the center of gravity coincide.

# Angular Motion in a Plane

## Angular Displacement

Angular displacement ($\theta$) is usually expressed in radians, in degrees, or in revolutions:

$1 \text{ rev} = 360^\circ = 2\pi \text{ rad}$ or $1 \text{ rad} = 57.3^\circ$

One radian is the angle subtended at the center of a circle by an arc equal in length to the radius of the circle. Thus an angle $\theta$ in radians is given in terms of the arc length $l$ it subtends on a circle of radius r by:

$$\theta = \frac{l}{r}$$

The radian measure of an angle is a dimensionless number. Radians, like degrees, are not a physical unit—the radian is not expressible in meters, kilograms, or seconds. Nonetheless, we will use the abbreviation rad to remind us that we are working with radians.

## Angular Speed

The angular speed ($\omega$) of an object whose axis of rotation is fixed is the rate at which its angular coordinate, the angular displacement $\theta$, changes with time. If $\theta$ changes from $\theta_i$ to $\theta_f$ in a time t, then the *average angular speed* is

$$\omega_{av} = \frac{\theta_f - \theta_i}{t}$$

The units of $\omega_{av}$ are exclusively rad/s. Since each complete turn or cycle of a revolving system carries it through $2\pi$ rad,

$$\omega = 2\pi f$$

where f is the **frequency** in revolutions per second, rotations per second, or cycles $\vec{\omega}$ per second. Accordingly, $\omega$ is also called the **angular frequency**. We can associate a direction with $\omega$ and thereby create a vector quantity $\vec{\omega}$. Thus, if the fingers of the right hand curve around in the direction of rotation, the thumb points along the axis of rotation in the direction of , the **angular velocity** vector.

## Angular Acceleration

The angular acceleration ($\alpha$) of an object whose axis of rotation is fixed is the rate at which its angular speed changes with time. If $\omega$ changes uniformly from $\omega_i$ to $\omega_f$ in a time t, then the *angular acceleration* is constant and

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

The units of $\alpha$ are typically rad/s$^2$, rev/min$^2$, and such.

## Equations for Uniformly Accelerated Angular Motion

The equations for uniformly accelerated angular motion are exactly

| Linear | Angular |
|---|---|
| $v_{av} = \frac{1}{2}(v_i + v_f)$ | $\omega_{av} = \frac{1}{2}(\omega_i + \omega_f)$ |
| $s = v_{av}t$ | $\theta = \omega_{av}t$ |
| $v_f = v_i + at$ | $\omega_f = \omega_i + \alpha t$ |
| $v_f^2 = v_i^2 + 2as$ | $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ |
| $s = v_i t + \frac{1}{2}at^2$ | $\theta = \omega_i t + \frac{1}{2}\alpha t^2$ |

analogous to those for uniformly accelerated linear motion. In the usual notation, we have:

Taken alone, the second of these equations is just the definition of average speed, so it is valid whether the acceleration is constant or not.

## Relation Between Angular and Tangential Quantities

When a wheel of radius r rotates about an axis whose direction is fixed, a point on the rim of the wheel is described in terms of the circumferential distance $l$ it has moved, its tangential speed v, and its tangential acceleration, $a_T$. These quantities are related to the angular quantities $\theta$, $\omega$, and $\alpha$, which describe the rotation of the wheel, through the relations

$$l = r\theta \qquad v = r\omega \qquad a_T = r\alpha$$

provided radian measure is used for $\theta$, $\omega$, and $\alpha$. By simple reasoning, $l$ can be shown to be the length of belt wound on the wheel or the distance the wheel would roll (without slipping) if free to do so. In such cases, v and $a_T$ refer to the tangential speed and acceleration of a point on the belt or of the center of the wheel.

## Centripetal Acceleration

A point mass m moving with constant speed v around a circle of radius r is undergoing acceleration. Although the magnitude of its linear velocity is not changing, the direction of the velocity is continually changing. This change in velocity gives rise to an acceleration $a_C$ of the mass, directed toward the center of the circle. We call this acceleration the **centripetal acceleration**; its magnitude is given by

$$a_C = \frac{(\text{tangential speed})^2}{\text{radius of circular path}} = \frac{v^2}{r}$$

where v is the speed of the mass around the perimeter of the circle.

Because $v = r\omega$, we also have $a_C = r\omega^2$, where $\omega$ must be in rad/s. Notice that the word "acceleration" is commonly used in physics as either a scalar or a vector quantity. Fortunately, there is usually no ambiguity.

## Centripetal Force $\vec{\mathbf{F}}_C$

The **centripetal force** ( ) is the force that must act on a mass m moving in a circular path of radius r to give it the centripetal acceleration $v^2/r$. From F = ma, we have

$$F_C = \frac{mv^2}{r} = mr\omega^2$$

where $\vec{\mathbf{F}}_C$ is directed toward the center of the circular path.

# Rigid-Body Rotation

## Moment of Inertia

The **moment of inertia** (I) of a body is a measure of the rotational inertia of the body. If an object that is free to rotate about an axis is difficult to set into rotation, its moment of inertia about that axis is large. An object with small I has little rotational inertia.

If a body is considered to be made up of tiny masses $m_1$, $m_2$, $m_3$, .