

DOMAĆI ZADATAK – MATEMATIKA (Terence Tao, Analysis 2.1)

ZADATAK 2

Zadatak je da se navedeni tekst iz knjige **Terence Tao – Analysis (zadatak 24-27)** prevede na srpski jezik.

Posebna napomena:

- tekst je teorijski (analitički)
- koristi se formalni matematički stil
- svi matematički izrazi (formule, simboli, rekurzije, indukcija) moraju biti prepisani TAČNO, bez menjanja značenja

TEKST ZA PREVOĐENJE

Property $P(n)$ is true for every natural number n . Thus in the rest of this text we will see many proofs which have a form like this:

Proposition 2.1.11. A certain property $P(n)$ is true for every natural number n .

Proof. We use induction. We first verify the base case $n = 0$, i.e., we prove $P(0)$. (Insert proof of $P(0)$ here). Now suppose inductively that n is a natural number, and $P(n)$ has already been proven. We now prove $P(n+1)$. (Insert proof of $P(n+1)$, assuming that $P(n)$ is true, here). This closes the induction, and thus $P(n)$ is true for all numbers n . \square

Of course we will not necessarily use the exact template, wording, or order in the above type of proof, but the proofs using induction will generally be something like the above form.

There are also some other variants of induction:

- backward induction (Exercise 2.2.6)
- strong induction (Proposition 2.2.14)
- transfinite induction (Lemma 8.5.15)

Axioms 2.1–2.5 are known as the Peano axioms for the natural numbers.

We make the assumption:

Assumption 2.6 (Informal):

There exists a number system \mathbb{N} , whose elements are natural numbers, for which Axioms 2.1–2.5 are true.

Remark 2.1.12:

Different representations of natural numbers (Hindu-Arabic, Roman numerals) are isomorphic, because there is a one-to-one correspondence:

$0 \leftrightarrow 0, 1 \leftrightarrow I, 2 \leftrightarrow II, 3 \leftrightarrow III, \dots$

Thus all versions of natural numbers are equivalent.

We do not prove Assumption 2.6 (it will later be included in set theory axioms).

Remark 2.1.13:

Natural numbers are all finite individually, but the set \mathbb{N} is infinite.

- 0 is finite
- if n is finite, then $n++$ is finite
- therefore all natural numbers are finite (by induction)

Infinity is not a natural number.

Remark 2.1.14:

Natural numbers are defined axiomatically, not constructively.

Mathematics studies properties, not physical interpretation.

A number can be:

- beads on an abacus
- bits in memory
- abstract object

As long as axioms are satisfied, it is valid.

Remark 2.1.15:

Historically, numbers were tied to physical meaning (counting, geometry, mass).

This caused problems when introducing:

- negative numbers
- fractions
- irrational numbers
- complex numbers

Modern mathematics resolves this by axiomatic definition.

Recursive definitions:

We define sequences recursively:

$$a_0 = c$$

$$a_{n++} = f_n(a_n)$$

where $fn : N \rightarrow N$.

Proposition 2.1.16 (Recursive definitions):

Given functions $fn : N \rightarrow N$ and initial value c , we can define a unique sequence an such that:

$$a_0 = c$$

$$a_{n+1} = fn(a_n)$$

Proof (informal):

We use induction.

- a_0 is defined as c
- assume a_n is defined
- then $a_{n+1} = fn(a_n)$ is defined

Thus each a_n is uniquely defined.

□

DRUGI ZADATAK:

DOMAĆI ZADATAK – LITERARNI PREVOD (Hofstadter)

ZADATAK

Drugi zadatak je odlomak iz price **Douglas Hofstadter – “The Tale of Hapiton”**.

Zadatak je:

- prevesti tekst na srpski jezik
 - zadržati narativni i dijaloški stil
 - ne izostaviti nijednu informaciju
 - očuvati ton (kolokvijalni + pripovedni + dijalog)
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TEKST ZA PREVOĐENJE

"Bunch of hogwash!" spluttered Curt. "Let's go up there and chuck the whole mess down onto the street and see how far it bounces."

While he was saying this, Janice noticed that there was a smaller note clipped onto the back of the last sheet, and turned it over to read it. It said this:

P.S.—It's really not advisable to try to dismantle my little setup up there in the belfry: I've got a hair trigger linked to the gas pipes, and if anyone tries to dismantle it, psssst! Sorry.

Janice Fleener and Curt Dempster could hardly believe their eyes. What gall!

They got straight on the phone to the police department, and talked to Officer Curran. He sounded poppin' mad when they told him what they'd found, and said he'd do something about it right quick.

So he hightailed it over to the courthouse and ran up those stairs two at a time, and when he reached the top, a-huffin' and a-puffin', he swung open the belfry door and took a look.

To tell the truth, he was a bit ginger in his inspection, because one thing Officer Curran had learned in his many years of police experience is that an ounce of prevention is worth a pound of cure.

So he cautiously looked over the strange contraption, and then he turned around and quite carefully shut the door behind him and went down.

He called up the town sewer department and asked them if they could check out whether there was anything funny going on with the pipes underground.

Well, the long and the short of it is that they verified everything in the Demon's letter, and by the time they had done so, the clock had struck five more times and those five dice had rolled five more times.

Janice Fleener had in fact had her thirteen-year-old daughter Samantha go up and sit in a wicker chair right next to the microcomputer and watch the robot arm throw those dice.

According to Samantha, an occasional seven had turned up now and then, but never had two sevens shown up together, let alone sevens on all five of the weird-looking dice!