Homework, Foreign language, 3.6. 2026.

ASSIGNMENT 1: Translate this text from English to Serbian.

Terence Tao, Analysis I
Pages 59–62

Chapter 3: Set Theory
Section 3.3: Functions

Definition 3.3.1 (Functions)

In order to do analysis, it is not particularly useful to just have the notion of a set; we also need the notion of a function from one set to another.

Informally, a function

$$f : X \to Y$$

from one set X to another set Y is an operation which assigns to each element (or "input") x in X a single element (or "output") f(x) in Y.

Definition 3.3.1 (Functions).

Let X, Y be sets, and let P(x,y) be a property pertaining to an object x in X and an object y in Y, such that for every x in X there is exactly one y in Y for which P(x,y) is true.

Then we define the function

$$f : X \to Y$$

defined by P on the domain X and range Y to be the object which, given any input x in X, assigns an output f(x) in Y, defined to be the unique object f(x) for which P(x,f(x)) is true.

Thus, for any x in X and y in Y,

$$y = f(x) \iff P(x,y) \text{ is true.}$$

Functions are also referred to as maps or transformations, depending on the context.

Definition 3.3.10 (Composition)

Let

f : X -> Y

and

g : Y -> Z

be two functions.

We define the composition

g o f : X -> Z

by the formula

(g o f)(x) := g(f(x)).

If the range of f does not match the domain of g, we leave the composition g o f undefined.

Example 3.3.11

Let

f : N -> N

be defined by

f(n) := 2n

and let

g : N -> N

be defined by

g(n) := n + 3.

Then

(g o f)(n)
= g(f(n))
= g(2n)
= 2n + 3.

For example,

(g o f)(1) = 5
(g o f)(2) = 7.

Meanwhile,

(f o g)(n)
= f(g(n))
= f(n + 3)
= 2(n + 3)
= 2n + 6.

For example,

(f o g)(1) = 8
(f o g)(2) = 10.

Thus composition is not commutative:

f o g != g o f.

Lemma 3.3.12 (Composition is Associative)

Let

f : X -> Y,
g : Y -> Z,
h : Z -> W

be functions.

Then

f o (g o h) = (f o g) o h.

Proof:

(f o (g o h))(x)
= f((g o h)(x))
= f(g(h(x)))
= (f o g)(h(x))
= ((f o g) o h)(x).

Definition 3.3.14 (Injective Functions)

A function f is one-to-one (injective) if

x != x' ==> f(x) != f(x').

Equivalently,

f(x) = f(x') ==> x = x'.

Example 3.3.15

The function

f : Z -> Z

defined by

f(n) := n^2

is not injective because

f(-1) = f(1) = 1.

However,

g : N -> Z

defined by

g(n) := n^2

is injective.

Definition 3.3.17 (Surjective Functions)

A function

f : X -> Y

is onto (surjective) if

f(X) = Y.

Equivalently,

For every y in Y,
there exists x in X such that

f(x) = y.

Example 3.3.18

The function

f : Z -> Z

defined by

$f(n) := n^2$

is not surjective because negative integers are not squares.

However, if

$A := \{ n^2 : n \in Z \}$

and

$g : Z \to A$

is defined by

$g(n) := n^2$,

then g is surjective.

Definition 3.3.20 (Bijective Functions)

A function

$f : X \to Y$

is bijective if it is both injective and surjective.

Example 3.3.21

Let

$f : \{0,1,2\} \to \{3,4\}$

be defined by

$f(0) := 3$
$f(1) := 3$
$f(2) := 4$.

This function is not bijective because it is not injective.

Let

$g : \{0,1\} \to \{2,3,4\}$

be defined by

$g(0) := 2$
$g(1) := 3$.

This function is not bijective because it is not surjective.

Let

h : {0,1,2} -> {3,4,5}

be defined by

h(0) := 3
h(1) := 4
h(2) := 5.

Then h is bijective because each element of {3,4,5}
comes from exactly one element of {0,1,2}

ASSIGNMENT 2

Translate the following text from English into Serbian.

The text is an excerpt from the story "Gödel's Doom" by Mark Zebrowski (142-143).

Felix, this can't be done." I struggled to free myself, but his strength was that of a true believer.

"Be still, you fool," he said harshly. "Don't you see? This will be the culmination of our careers. We'll never match this no matter how hard we work. Godel is one of the supreme monuments of mathematics, marking the limits of human minds. If we topple him . . ."

"You may not like what you get," I said, twisting my arm. "If his proof is right, then mechanism is false and minds are not machines. They escape the completeness of the purely mechanical. But if Godel is wrong, then we're automatons! I'd rather not know."

He shook his head. "There's even more to it than that, Bruno."

'What?" I was breathing very hard, unable to free myself.

'We're opening up the very vitals of reality."

I had to laugh. "By manipulating man-made symbolic structures? You need a bucket of cold water to soak your head in. Let me go!"

"Completion may be only a few minutes away. Do you want to stop and then wonder what might have been?" He tightened his grip.

"But you can't know how far along it is."

He let go of my hand and seemed to cool down, and I found I didn't have the heart to reach over and stop the run.

"You're right," he said, "I'm sorry. It probably is all for nothing."

I massaged my hand. The AI continued its work run. "Don't feel too bad about it," I managed to say. "It was a nice idea, but it had to confirm Godel. I'm glad we're not machines."

He was shaking his head. "You don't understand. There's no reason to fear that. It's not a problem."

"What isn't?"

"Free will," he said as the AI-5 stopped its run.

Witter and I looked at each other, then at the main screen. It read:

SYSTEM CAPABLE OF GENERATING

ARITHMETIC COMPLETE

"It's a mistake of some kind," I said. Something strange seemed to pass across my eyes. I sat back, expecting to lose consciousness as the tension got to me.

"Maybe," Witter was saying, "but we can test to see if it's a mistake."

"How?" I heard myself ask, even though I knew the answer.

"By trying to make a true statement that is not provable in the system. As long as the AI can show us that we can't make such a statement by proving it, then the system is complete."

The room went black for a second. "But maybe we can't make such a statement," I said.

"We can try," he answered.

We tried for the next twelve hours. I was relieved that our prime AI was no longer running a huge power draw. Witter brought a smaller AI on-line and had it question the alleged complete system achieved by the AI-5.