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Revised edition

Writing Mathematical Papers in English

a practical guide



European Mathematical Society

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PREFACE

The booklet is intended to provide practical help for authors of mathematical papers. It is written mainly for non-English speaking writers but should prove useful even to native speakers of English who are beginning their mathematical writing and may not yet have developed a command of the structure of mathematical discourse.

The booklet is oriented mainly to research mathematics but applies to almost all mathematics writing, except more elementary texts where good teaching praxis typically favours substantial repetition and redundancy.

There is no intention whatsoever to impose any uniformity of mathematical style. Quite the contrary, the aim is to encourage prospective authors to write structurally correct manuscripts as expressively and flexibly as possible, but without compromising certain basic and universal rules.

The first part provides a collection of ready-made sentences and expressions that most commonly occur in mathematical papers. The examples are divided into sections according to their use (in introductions, definitions, theorems, proofs, comments, references to the literature, acknowledgements, editorial correspondence and referees' reports). Typical errors are also pointed out.

The second part concerns selected problems of English grammar and usage, most often encountered by mathematical writers. Just as in the first part, an abundance of examples are presented, all of them taken from actual mathematical texts.

The author is grateful to Edwin F. Beschler, Daniel Davies, Zofia Denkowska, Zbigniew Lipecki and Zdzisław Skupień for their helpful criticism. Thanks are also due to Adam Mysior and Marcin Adamski for suggesting several improvements, and to Henryka Walas for her painstaking job of typesetting the continuously varying manuscript.

Jerzy Trzeciak

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PART A: PHRASES USED IN MATHEMATICAL TEXTS

ABSTRACT AND INTRODUCTION

We prove that in some families of compacta there are no universal elements.

It is also shown that

Some relevant counterexamples are indicated.

It is of interest to know whether We wish to investigate

We are interested in finding Our purpose is to

It is natural to try to relate to

This work was intended as an attempt to motivate (at motivating)

The aim of this paper is to bring together two areas in which

In		Section 3 the third section [Note: paragraph ≠ section]		we	review some of the standard
					facts on
					have compiled some basic facts
					summarize without proofs the
					relevant material on
					give a brief exposition of
					briefly sketch
					set up notation and terminology.
					discuss (study/treat/examine)
					the case
					introduce the notion of
					develop the theory of
					will look more closely at
					will be concerned with
					proceed with the study of
					indicate how these techniques
					may be used to
					extend the results of to
					derive an interesting formula for
some of the recent results are					
reviewed in a more general setting.					
some applications are indicated.					
our main results are stated and proved.					

Section 4		contains a brief summary (a discussion) of			
		deals with (discusses) the case			
		is intended to motivate our investigation of			
		is devoted to the study of			
		provides a detailed exposition of			
		establishes the relation between			
					represents some preliminaries.

We will		touch only a few aspects of the theory.
		restrict our attention (the discussion/ourselves) to

It is not our purpose to study

No attempt has been made here to develop

It is possible that but we will not develop this point here.

A more complete theory may be obtained by

However, | this topic exceeds the scope of this paper.
 | we will not use this fact in any essential way.

The basic (main) | idea is to apply
 | geometric ingredient is

The crucial fact is that the norm satisfies

Our proof involves looking at

The proof is | based on the concept of
 | similar in spirit to
 | adapted from

This idea goes back at least as far as [7].

We emphasize that

It is worth pointing out that

The important point to note here is the form of

The advantage of using lies in the fact that

The estimate we obtain in the course of proof seems to be of independent interest.

Our theorem provides a natural and intrinsic characterization of

Our proof makes no appeal to

Our viewpoint sheds some new light on

Our example demonstrates rather strikingly that

The choice of seems to be the best adapted to our theory.

The problem is that

The main difficulty in carrying out this construction is that

In this case the method of breaks down.

This class is not well adapted to

Pointwise convergence presents a more delicate problem.

The results of this paper were announced without proofs in [8].

The detailed proofs will appear in [8] (elsewhere/in a forthcoming publication).

For the proofs we refer the reader to [6].

It is to be expected that

One may conjecture that

One may ask whether this is still true if

One question still unanswered is whether

The affirmative solution would allow one to

It would be desirable to but we have not been able to do this.

These results are far from being conclusive.

This question is at present far from being solved.

Our method has the disadvantage of not being intrinsic.
 The solution falls short of providing an explicit formula.
 What is still lacking is an explicit description of

As for prerequisites, the reader is expected to be familiar with
 The first two chapters of constitute sufficient preparation.
 No preliminary knowledge of is required.

To facilitate access to the individual topics, the chapters are rendered as self-contained as possible.
 For the convenience of the reader we repeat the relevant material from [7] without proofs, thus making our exposition self-contained.

DEFINITION

A set S is *dense* if

A set S is called $\langle$ said to be $\rangle$ *dense* if

We call a set *dense* $\langle$ We say that a set is *dense* $\rangle$ if

We call m the *product measure*. [Note the word order after “we call”.]

The function f is given $\langle$ defined $\rangle$ by $f = \dots$

Let f be given $\langle$ defined $\rangle$ by $f = \dots$

We define T to be $AB + CD$.

This map is defined by $\left\{ \begin{array}{l} \text{requiring } f \text{ to be constant on} \\ \text{the requirement that } f \text{ be constant on} \\ \text{[Note the infinitive.]} \\ \text{imposing the following condition:} \end{array} \right.$

The *length* of a sequence is, by definition, the number of

The *length* of T , denoted by $l(T)$, is defined to be

By the *length* of T we mean

Define $\langle$ Let/Set $\rangle$ $E = Lf$ $\left\{ \begin{array}{l} \text{, where } f \text{ is} \\ \text{we have set } f = \dots \\ \text{, } f \text{ being the solution of} \\ \text{with } f \text{ satisfying} \end{array} \right.$

We will consider $\left\{ \begin{array}{l} \text{the behaviour of the family } g \text{ defined as follows.} \\ \text{the height of } g \text{ (to be defined later) and} \end{array} \right.$

To measure the growth of g we make the following definition.

In this way we obtain what $\left\{ \begin{array}{l} \text{we shall call} \\ \text{will be referred to as} \\ \text{is known as} \end{array} \right.$ the *P-system*.

Since, $\left\{ \begin{array}{l} \text{the norm of } f \text{ is well defined.} \\ \text{the definition of the norm is unambiguous } \langle \text{makes sense} \rangle. \end{array} \right.$

It is immaterial which M we choose to define F as long as M contains x .
This product is independent of which member of g we choose to define it.
It is Proposition 8 that makes this definition allowable.

Our definition agrees $\left| \begin{array}{l} \text{with the one given in [7] if } u \text{ is} \\ \text{with the classical one for} \end{array} \right.$

Note that $\left| \begin{array}{l} \text{this coincides with our previously introduced} \\ \text{terminology if } K \text{ is convex.} \\ \text{this is in agreement with [7] for} \end{array} \right.$

NOTATION

We will denote by Z $\left| \begin{array}{l} \text{Let us denote by } Z \\ \text{Let } Z \text{ denote} \end{array} \right. \left| \begin{array}{l} \text{the set} \\ \text{the set} \end{array} \right. \quad \text{Write } \langle \text{Let/Set} \rangle f = \text{.....}$
 $\left[\text{Not: "Denote } f = \text{....."} \right]$

The closure of A will be denoted by $\text{cl}A$.
We will use the symbol $\langle \text{letter} \rangle k$ to denote
We write H for the value of
We will write the negation of p as $\neg p$.
The notation aRb means that
Such cycles are called homologous (written $c \sim c'$).

Here
Here and subsequently,
Throughout the proof,
In what follows,
From now on,

$\left| \begin{array}{l} K \\ \text{stands for} \end{array} \right| \left| \begin{array}{l} \text{denotes} \\ \text{the map} \end{array} \right.$

We follow the notation of [8] $\langle \text{used in [8]} \rangle$.
Our notation differs $\langle \text{is slightly different} \rangle$ from that of [8].
Let us introduce the temporary notation Ff for gfg .

With the notation $f = \text{.....}$,
With this notation,
In the notation of [8, Ch. 7] $\left| \begin{array}{l} \text{we have} \end{array} \right.$

If f is real, it is customary to write rather than

For simplicity of notation,
To $\langle \text{simplify/shorten} \rangle$ notation,
By abuse of notation,
For abbreviation,

$\left| \begin{array}{l} \text{we} \end{array} \right| \left| \begin{array}{l} \text{write } f \text{ instead of} \\ \text{use the same letter } f \text{ for} \\ \text{continue to write } f \text{ for} \\ \text{let } f \text{ stand for} \end{array} \right.$

We abbreviate $Faub$ to b' .
We denote it briefly by F . $[\text{Not: "shortly"}]$
We write it F for short $\langle \text{for brevity} \rangle$. $[\text{Not: "in short"}]$
The Radon–Nikodym property (RNP for short) implies that
We will write it simply x when no confusion can arise.

It will cause no confusion if we use the same letter to designate a member of A and its restriction to K .

We shall write the above expression as
 The above expression may be written as
 We can write (4) in the form

$$t = \dots$$

The Greek indices label components of sections of E .

Print terminology:

The expression in italics (in italic type), in large type, in bold print;
 in parentheses () (= round brackets),
 in brackets [] (= square brackets),
 in braces { } (= curly brackets), in angular brackets $\langle \rangle$;
 within the norm signs

Capital letters = upper case letters; small letters = lower case letters;
 Gothic (German) letters; script (calligraphic) letters (e.g. $\mathcal{F}$, $\mathcal{G}$);
 special Roman (blackboard bold) letters (e.g. $\mathbb{R}$, $\mathbb{N}$)

Dot $\cdot$, prime $'$, asterisk = star $*$, tilde $\sim$, bar $\bar{}$ [over a symbol], hat $\hat{}$,
 vertical stroke (vertical bar) $|$, slash (diagonal stroke/slant) $/$,
 dash $—$, sharp $\#$

Dotted line $\dots\dots$, dashed line $-----$, wavy line $\sim\sim\sim\sim$

PROPERTY

The $\langle \text{An} \rangle$ element	such that $\langle \text{with the property that} \rangle \dots$ [Not: “such an element that”] with the following properties: $\dots$ satisfying $Lf = \dots$ with $Nf = 1$ (with coordinates x, y, z) of norm 1 (of the form $\dots$) whose norm is $\dots$ all of whose subsets are $\dots$ by means of which g can be computed for which this is true at which g has a local maximum described by the equations $\dots$ given by $Lf = \dots$ depending only on $\dots$ (independent of $\dots$) not in A so small that (small enough that) $\dots$ as above (as in the previous theorem) so obtained occurring in the cone condition [Note the double “r”.] guaranteed by the assumption $\dots$
---	--

The $\langle \text{An} \rangle$ element	we have just defined we wish to study $\langle \text{we used in Chapter 7} \rangle$ to be defined later [= which will be defined] in question under study $\langle \text{consideration} \rangle$
---	--

....., the constant C being independent of [= where C is]
, the supremum being taken over all cubes
, the limit being taken in L .

....., where C	is so chosen that is to be chosen later. is a suitable constant. is a conveniently chosen element of involves the derivatives of ranges over all subsets of may be made arbitrarily small by
------------------	--

The operators A_i	have $\langle \text{share} \rangle$ many of the properties of have still better smoothness properties. lack $\langle \text{fail to have} \rangle$ the smoothness properties of still have norm 1. not merely symmetric but actually self-adjoint. not necessarily monotone. both symmetric and positive-definite. not continuous, nor do they satisfy (2). [Note the inverse word order after “nor”.] are neither symmetric nor positive-definite. only nonnegative rather than strictly positive, as one may have expected. any self-adjoint operators, possibly even unbounded. still $\langle \text{no longer} \rangle$ self-adjoint. not too far from being self-adjoint.
---------------------	--

The	preceding theorem indicated set above-mentioned group resulting region required $\langle \text{desired} \rangle$ element	[But adjectival clauses with prepositions come <i>after</i> a noun, e.g. “the group defined in Section 1”.]
-----	--	---

Both X and Y are finite.

Neither X nor Y is finite.

Both X and Y are countable, but neither is finite.

Neither of them is finite. [*Note*: “Neither” refers to *two* alternatives.]

None of the functions F_i is finite.

The set X is not finite; nor $\langle \text{neither} \rangle$ is Y .

Note that X is not finite, nor is Y countable. [Note the inversion.]

We conclude that X is empty $\Big|$; so also is Y .
 $\Big|$, but Y is not.

Hence X belongs to Y $\Big|$, and so does Z .
 $\Big|$, but Z does not.

ASSUMPTION, CONDITION, CONVENTION

We will make $\langle \text{need} \rangle$ the following assumptions:

From now on we make the assumption:

The following assumption will be needed throughout the paper.

Our basic assumption is the following.

Unless otherwise stated $\langle \text{Until further notice} \rangle$ we assume that

In the remainder of this section we assume $\langle \text{require} \rangle g$ to be

In order to get asymptotic results, it is necessary to put some restrictions
on f .

We shall make two standing assumptions on the maps under consideration.

It is required $\langle \text{assumed} \rangle$ that

The requirement on g is that

....., where g $\Big|$ is subject to the condition $Lg = 0$.
 $\Big|$ satisfies the condition $Lg = 0$.
 $\Big|$ is merely required to be positive.

Let us orient M by $\Big|$ the requirement that g be positive.
 $\Big|$ [Note the infinitive.]
 $\Big|$ requiring g to be
 $\Big|$ imposing the condition:

Now, (4) holds $\Big|$ for $\langle \text{provided/whenever/only in case} \rangle p \neq 1$.
 $\Big|$ unless $p = 1$.
 $\Big|$ under $\Big|$ the condition $\langle \text{hypothesis} \rangle$ that
 $\Big|$ the more general assumption that
 $\Big|$ some further restrictions on
 $\Big|$ additional $\langle \text{weaker} \rangle$ assumptions.

$\Big|$ satisfies $\langle \text{fails to satisfy} \rangle$ the assumptions of
 $\Big|$ has the desired $\langle \text{asserted} \rangle$ properties.
 $\Big|$ provides the desired diffeomorphism.
It $\Big|$ still satisfies $\langle \text{need not satisfy} \rangle$ the requirement that
 $\Big|$ meets this condition.
 $\Big|$ does not necessarily have this property.
 $\Big|$ satisfies all the other conditions for membership of X .

There is no loss of generality in assuming

Without loss $\langle \text{restriction} \rangle$ of generality we can assume

This involves no loss of generality.

We can certainly assume that $\left\{ \begin{array}{l} \text{, since otherwise} \\ \text{, for [= because]} \\ \text{, for if not, we replace} \\ \text{. Indeed,} \end{array} \right.$

Neither the hypothesis nor the conclusion is affected if we replace

By choosing $b = a$ we may actually assume that

If $f = 1$, which we may assume, then

For simplicity (convenience) we ignore the dependence of F on g .

[E.g. in notation]

It is convenient to choose

We can assume, by decreasing k if necessary, that

Thus F meets S transversally, say at $F(0)$.

There exists a minimal element, say n , of F .

Hence G acts on H as a multiple (say n) of V .

For definiteness (To be specific), consider

This condition $\left\{ \begin{array}{l} \text{is not particularly restrictive.} \\ \text{is surprisingly mild.} \\ \text{admits (rules out/excludes) elements of} \\ \text{is essential to the proof.} \\ \text{cannot be weakened (relaxed/improved/omitted/} \\ \text{dropped).} \end{array} \right.$

The theorem is true if “open” is deleted from the hypotheses.

The assumption is superfluous (redundant/unnecessarily restrictive).

We will now show how to dispense with the assumption on

Our lemma does not involve any assumptions about curvature.

We have been working under the assumption that

Now suppose that this is no longer so.

To study the general case, take

For the general case, set

The map f will be viewed (regarded/thought of) as $\left\{ \begin{array}{l} \text{a functor} \\ \text{realizing} \end{array} \right.$

From now on we $\left\{ \begin{array}{l} \text{think of } L \text{ as being constant.} \\ \text{regard } f \text{ as a map from} \\ \text{tacitly assume that} \end{array} \right.$

It is understood that $r \neq 1$.

We adopt (adhere to) the convention that $0/0=0$.

THEOREM: GENERAL REMARKS

This theorem	is	an extension ⟨a fairly straightforward generalization/a sharpened version/a refinement⟩ of
		an analogue of
		a reformulation ⟨restatement⟩ of
		in terms of
		analogous to
		a partial converse of
		an answer to a question raised by
		deals with
		ensures the existence of
		expresses the equivalence of
		provides a criterion for
		yields information about
		makes it legitimate to apply

The theorem states ⟨asserts/shows⟩ that

Roughly ⟨Loosely⟩ speaking, the formula says that

When f is open, (3.7) just amounts $\left| \begin{array}{l} \text{to saying that} \\ \text{to the fact that} \end{array} \right.$

Here is another way of stating (c):

Another way of stating (c) is to say:

An equivalent formulation of (c) is:

Theorems 2 and 3 may be summarized by saying that

Assertion (ii) is nothing but the statement that

Geometrically speaking, the hypothesis is that; part of the conclusion is that

The interest	$\left \begin{array}{l} \text{of the lemma is} \end{array} \right $	$\left \begin{array}{l} \text{in the assertion} \\ \text{that it allows one to} \end{array} \right.$
The principal significance		
The point		

The theorem gains in interest if we realize that

The theorem $\left| \begin{array}{l} \text{is still true} \\ \text{still holds} \end{array} \right|$ if $\left| \begin{array}{l} \text{we drop the assumption} \\ \text{it is just assumed that} \end{array} \right.$

If we take $f = \dots$, $\left| \begin{array}{l} \text{we recover} \end{array} \right|$ the standard lemma
 Replacing f by $-f$, $\left| \begin{array}{l} \text{we recover} \end{array} \right|$ [7, Theorem 5].

This specializes to the result of [7] if $f = g$.

This result will $\left| \begin{array}{l} \text{be needed in} \\ \text{prove extremely useful in} \\ \text{not be needed until} \end{array} \right|$ Section 8.

THEOREM: INTRODUCTORY PHRASE

We have thus proved Summarizing, we have	We can now	rephrase Theorem 8 as follows. state the analogue of formulate our main results.
---	------------	--

We are thus led to the following strengthening of Theorem 6:
 The remainder of this section will be devoted to the proof of

The continuity of A is established by our next theorem.
 The following result may be proved in much the same way as Theorem 6.
 Here are some elementary properties of these concepts.
 Let us mention two important consequences of the theorem.
 We begin with a general result on such operators.

[Note: Sentences of the type “We now have the following lemma”,
 carrying no information, can in general be cancelled.]

THEOREM: FORMULATION

If and if, then

Let M be	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"> Suppose that Assume that Write </td> <td style="padding-left: 10px;"> Then, </td> </tr> </table>	Suppose that Assume that Write	Then,	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"> provided $m \neq 1$. unless $m = 1$. with g a constant satisfying </td> </tr> </table>	provided $m \neq 1$. unless $m = 1$. with g a constant satisfying
Suppose that Assume that Write	Then,				
provided $m \neq 1$. unless $m = 1$. with g a constant satisfying					

Furthermore (Moreover),
 In fact, [= To be more precise]
 Accordingly, [= Thus]

Given any $f \neq 1$ suppose that Then

Let P satisfy	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"> the hypotheses of the above assumptions. $N(P) = 1$. </td> <td style="padding-left: 10px;"> Then </td> </tr> </table>	the hypotheses of the above assumptions. $N(P) = 1$.	Then
the hypotheses of the above assumptions. $N(P) = 1$.	Then		

Let assumptions 1–5 hold. Then
 Under the above assumptions,
 Under the same hypotheses,
 Under the conditions stated above,
 Under the assumptions of Theorem 2 with “convergent”
 replaced by “weakly convergent”,
 Under the hypotheses of Theorem 5, if moreover

Equality holds in (8) if and only if
 The following conditions are equivalent:

[Note: Expressions like “the following inequality holds” can in
 general be dropped.]

PROOF: BEGINNING

We
Let us

first	prove ⟨show/recall/observe⟩ that prove a reduced form of the theorem. outline ⟨give the main ideas of⟩ the proof. examine Bf .
-------	---

But $A = B$.

To see ⟨prove⟩ this, let $f = \dots$ We prove this as follows. This is proved by writing $g = \dots$
--

We first compute If .

To this end, consider [= For this purpose; <i>not</i> : “To this aim”] To do this, take For this purpose, we set

To deduce (3) from (2), take

We claim that Indeed,

We begin by proving ⟨by recalling the notion of⟩

Our proof starts with the observation that

The procedure is to find

The proof consists in the construction of

The proof is

straightforward ⟨quite involved⟩. by induction on n . left to the reader. based on the following observation.
--

The main ⟨basic⟩ idea of the proof is to take

The proof

falls naturally into three parts. will be divided into three steps.
--

We have divided the proof into a sequence of lemmas.

Suppose

the assertion of the lemma is false. , contrary to our claim, that

Conversely ⟨To obtain a contradiction⟩,

suppose that

On the contrary,
Suppose the lemma were false. Then we could find

If

there existed an x, x were not in B , it were true that,	we would have there would be
--	---

Assume the formula holds for degree k ; we will prove it for $k + 1$.

Assuming (5) to hold for k , we will prove it for $k + 1$.

We give the proof only for the case $n = 3$; the other cases are left to the reader.

We give only the main ideas of the proof.

PROOF: ARGUMENTS

By	definition, the definition of assumption, the compactness of	But $f = g$	, which follows from as was described ⟨shown/mentioned/ noted⟩ in
	Taylor's formula, a similar argument, the above, the lemma below, continuity,		shows that yields ⟨gives/ implies⟩ $f = \dots$ leads to $f = \dots$

Theorem 4 now

Since f is compact, $\left[\begin{array}{l} Lf = 0. \quad [Not: \text{“Since } \dots, \text{ then } \dots\text{”}] \\ \text{we have } Lf = 0. \\ \text{it follows that } Lf = 0. \\ \text{we see } \langle \text{conclude} \rangle \text{ that } Lf = 0. \end{array} \right.$

But $Lf = 0$ since f is compact.

We have $Lf = 0$, because [+ a longer explanation]

We must have $Lf = 0$, for otherwise we can replace

As f is compact we have $Lf = 0$.

Therefore $Lf = 0$ by Theorem 6.

That $Lf = 0$ follows from Theorem 6.

From	$\left[\begin{array}{l} \text{this} \\ (5) \\ \text{what has already} \\ \text{been proved,} \end{array} \right.$	$\left[\begin{array}{l} \text{we conclude } \langle \text{deduce/see} \rangle \text{ that } \dots \\ \text{we have } \langle \text{obtain} \rangle M = N. \\ [Note: \text{without “that”}] \\ \text{it follows that } \dots \\ \text{it may be concluded that } \dots \end{array} \right.$
------	--	---

According to ⟨On account of⟩ the above remark, we have $M = N$.

It follows that
Hence ⟨Thus/Consequently,/Therefore⟩ $\left| M = N. \right.$

[hence = from this; thus = in this way; therefore = for this reason;
it follows that = from the above it follows that]

This gives $M = N$. We thus get $M = N$. The result is $M = N$. Now (3) becomes $M = N$. This clearly forces $M = N$.	It is compact,	$\left[\begin{array}{l} \text{and so } M = N. \\ \text{and consequently } M = N. \\ \text{and, in consequence, } M = N. \\ \text{and hence bounded.} \\ \text{which gives } \langle \text{implies/} \\ \text{yields} \rangle M = N. \\ [Not: \text{“what gives”}] \end{array} \right.$
---	----------------	---

Now $F = G = H$, $\left| \begin{array}{l} \text{the last equality being a consequence of Theorem 7.} \\ \text{which is due to the fact that } \dots \end{array} \right.$

Since, (2) shows that, by (4).

We conclude from (5) that, hence that, and finally that

The equality $f=g$, which is part of the conclusion of Theorem 7, implies that

As in the proof of Theorem 8, equation (4) gives

Analysis similar to that in the proof of Theorem 5 shows that [Not: “similar as in”]

A passage to the limit similar to the above implies that

Similarly (Likewise),

Similar arguments apply
The same reasoning applies | to the case

The same conclusion can be drawn for

This follows by the same method as in

The term Tf can be handled in much the same way, the only difference being in the analysis of

In the same manner we can see that

The rest of the proof runs as before.

We now apply this argument again, with I replaced by J , to obtain

PROOF: CONSECUTIVE STEPS

Consider	Define	$f = \dots$	Let us	evaluate
Choose	Let			compute
Fix	Set			apply the formula to
				suppose for the moment
				regard s as fixed and

[Note: The imperative mood is used when you *order* the reader to do something, so you should not write e.g. “Give an example of” if you mean “We give an example of”]

Adding g to the left-hand side		yields (gives) $h = \dots$
Subtracting (3) from (5)		we obtain (get/have) $f = g$
Writing (Taking) $h = Hf$		[Note: without “that”]
Substituting (4) into (6)		we conclude (deduce/see) that
Combining (3) with (6)		we can assert that
Combining these		we can rewrite (5) as
[E.g. these inequalities]		
Replacing (2) by (3)		
Letting $n \rightarrow \infty$		
Applying (5)		
Interchanging f and g		

[Note: The ing-form is either the subject of a sentence (“Adding gives”), or requires the subject “we” (“Adding we obtain”); so do *not* write e.g. “Adding the proof is complete.”]

We continue in this fashion obtaining (to obtain) $f = \dots$

We may now integrate k times to conclude that

Repeated application of Lemma 6 enables us to write

We now proceed by induction.

We can now proceed analogously to the proof of

We next $\left| \begin{array}{l} \text{claim } \langle \text{show/prove that} \rangle \text{} \\ \text{sharpen these results and prove that} \end{array} \right.$

Our next $\left| \begin{array}{l} \text{claim is that} \\ \text{goal is to determine the number of} \\ \text{objective is to evaluate the integral } I. \\ \text{concern will be the behaviour of} \end{array} \right.$

We now turn to the case $f \neq 1$.

We are now in a position to show [= We are able to]

We proceed to show that

The task is now to find

Having disposed of this preliminary step, we can now return to

We wish to arrange that f be as smooth as possible.
 [Note the infinitive.]

We are thus looking for the family

We have to construct

In order to get this inequality, it $\left| \begin{array}{l} \text{will be necessary to} \\ \text{is convenient to} \end{array} \right.$

To deal with If ,
 To estimate the other term, $\left| \begin{array}{l} \text{we note that} \end{array} \right.$
 For the general case,

PROOF: "IT IS SUFFICIENT TO"

It $\left| \begin{array}{l} \text{suffices} \\ \text{is sufficient} \end{array} \right. \left| \begin{array}{l} \text{to} \\ \text{show } \langle \text{prove} \rangle \text{ that} \\ \text{make the following observation.} \\ \text{use (4) together with the observation that} \end{array} \right.$

We need only consider three cases:

We only need to show that

It remains to prove that $\langle$ to exclude the case when $\rangle$

What is left is to show that

We are reduced to proving (4) for

We are left with the task of determining

The only point remaining concerns the behaviour of

The proof is completed by showing that

We shall have established the lemma if we prove the following:

If we prove that, the assertion follows.

The statement $O(g)=1$ will be proved once we prove the lemma below.

PROOF: “IT IS EASILY SEEN THAT”

It is

clear $\langle$ evident/immediate/obvious $\rangle$ that	easily seen that
	easy to check that
	a simple matter to

We see $\langle$ check $\rangle$ at once that, which is clear from (3).
They are easily seen to be smooth., as is easy to check.

It follows easily $\langle$ immediately $\rangle$ that

Of course $\langle$ Clearly/Obviously $\rangle$,

The proof is straightforward $\langle$ standard/immediate $\rangle$.

An easy computation $\langle$ A trivial verification $\rangle$ shows that

(2) makes it obvious that [= By (2) it is obvious that]

The factor Gf poses no problem because G is

PROOF: CONCLUSION AND REMARKS

....., which

proves the theorem.	
completes the proof.	
establishes the formula.	
is the desired conclusion.	
is our claim $\langle$ assertion $\rangle$.	[Not: “thesis”]
gives (4) when substituted in (5) $\langle$ combined with (5) $\rangle$.	

[Not: “what”]
This

....., and

the proof is complete.	
this is precisely the assertion of the lemma.	
the lemma follows.	
(3) is proved.	
$f = g$ as claimed $\langle$ required $\rangle$.	

This contradicts our assumption $\langle$ the fact that $\rangle$.

....., contrary to (3).

....., which is impossible. [Not: “what is”]

....., which contradicts the maximality of

....., a contradiction.

The proof for G is similar.

The map G may be handled in much the same way.

Similar considerations apply to G .

The same proof

works $\langle$ remains valid $\rangle$ for	
obtains $\langle$ fails $\rangle$ when we drop the assumption	

The method of proof carries over to domains

The proof above gives more, namely f is

A slight change in the proof actually shows that

Note that we have actually proved that
 [= We have proved more, namely that]

We have used | only the fact that
 the existence of only the right-hand derivative.

For $f=1$ | it is no longer true that
 the argument breaks down.

The proof strongly depended on the assumption that

Note that we did not really have to use; we could have applied

For more details we refer the reader to [7].

The details are left to the reader.

We leave it to the reader to verify that [Note: the “it” is necessary]
 This finishes the proof, the detailed verification of (4) being left to the reader.

REFERENCES TO THE LITERATURE

(see for instance [7, Th. 1]) (see [7] and the references given there)

(see [Ka2] for | more details)
 the definition of
 the complete bibliography)

The best general reference here				was proved by Lax [8].
The standard work on		is		can be found in
The classical work here				Lax [7, Ch. 2].

This construction		is due to Strang [8].
		goes back to the work of
		as far as [8].
		was motivated by [7].
		generalizes that of [7].
		follows [7].
		is adapted from [7] <appears in [7]>.
		has previously been used by Lax [7].

For		a recent account of the theory		we refer the reader to [7].
		a treatment of a more general case		
		a fuller <thorough> treatment		
		a deeper discussion of		
		direct constructions along more		
		classical lines		
		yet another method		

We introduce the notion of, following Kato [7].
 We follow [Ka] in assuming that

The main results of this paper were announced in [7].
 Similar results have been obtained independently by Lax and are to be published in [7].

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The author | wishes to express his thanks ⟨gratitude⟩ to
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HOW TO SHORTEN THE PAPER

General rules:

1. Remember: you are writing for an expert. Cross out all that is trivial or routine.
2. Avoid repetition: do not repeat the assumptions of a theorem at the beginning of its proof, or a complicated conclusion at the end of the proof. Do not repeat the assumptions of a previous theorem in the statement of a next one (instead, write e.g. "Under the hypotheses of Theorem 1 with f replaced by g ,"). Do not repeat the same formula—use a label instead.
3. Check all formulas: is each of them necessary?

Phrases you can cross out:

We denote by $\mathbb{R}$ the set of all real numbers.
 We have the following lemma.
 The following lemma will be useful.
 the following inequality is satisfied:

Phrases you can shorten (see also p. 38):

Let ε be an arbitrary but fixed positive number $\rightsquigarrow$ Fix $\varepsilon > 0$
 Let us fix arbitrarily $x \in X$ $\rightsquigarrow$ Fix $x \in X$
 Let us first observe that $\rightsquigarrow$ First observe that
 We will first compute $\rightsquigarrow$ We first compute
 Hence we have $x=1$ $\rightsquigarrow$ Hence $x=1$
 Hence it follows that $x=1$ $\rightsquigarrow$ Hence $x=1$

Taking into account (4) $\rightsquigarrow$ By (4)
 By virtue of (4) $\rightsquigarrow$ By (4)
 By relation (4) $\rightsquigarrow$ By (4)
 In the interval $[0, 1]$ $\rightsquigarrow$ In $[0, 1]$
 There exists a function $f \in C(X)$ $\rightsquigarrow$ There exists $f \in C(X)$
 For every point $p \in M$ $\rightsquigarrow$ For every $p \in M$
 It is defined by the formula $F(x) = \dots$ $\rightsquigarrow$ It is defined by $F(x) = \dots$
 Theorem 2 and Theorem 5 $\rightsquigarrow$ Theorems 2 and 5
 This follows from (4), (5), (6) and (7) $\rightsquigarrow$ This follows from (4)–(7)
 For details see [3], [4] and [5] $\rightsquigarrow$ For details see [3]–[5]
 The derivative with respect to t $\rightsquigarrow$ The t -derivative
 A function of class C^2 $\rightsquigarrow$ A C^2 function
 For arbitrary x $\rightsquigarrow$ For all x (For every x)
 In the case $n = 5$ $\rightsquigarrow$ For $n = 5$
 This leads to a contradiction with the maximality of f
 $\rightsquigarrow \dots$, contrary to the maximality of f
 Applying Lemma 1 we conclude that $\rightsquigarrow$ Lemma 1 shows that
 $\dots$, which completes the proof $\rightsquigarrow \dots$ ■

EDITORIAL CORRESPONDENCE

I would like to submit | the enclosed manuscript “.....”
 I am submitting | for publication in *Studia Mathematica*.

I have also included a reprint of my article for the convenience of the referee.

I wish to withdraw my paper as I intend to make a major revision of it.

I regret any inconvenience this may have caused you.

I am very pleased that the paper will appear in *Fundamenta*.

Thank you very much for accepting my paper for publication in

Please find enclosed two copies of the revised version.

As the referee suggested, I inserted a reference to the theorem of

We have followed the referee’s suggestions.

I have complied with almost all suggestions of the referee.

REFeree’S REPORT

The author proves the interesting result that

The proof is short and simple, and the article well written.

The results presented are original.

The paper is a good piece of work on a subject that attracts considerable attention.

I am pleased to		recommend it for publication in Studia Mathematica.
It is a pleasure to		
I strongly		

The only remark I wish to make is that condition B should be formulated more carefully.

A few minor typographical errors are listed below.

I have indicated various corrections on the manuscript.

The results obtained are not particularly surprising and will be of limited interest.

The results are		correct but only moderately interesting.
		rather easy modifications of known facts.

The example is worthwhile but not of sufficient interest for a research article.

The English of the paper needs a thorough revision.

The paper does not meet the standards of your journal.

Theorem 2 is false		as stated.
		in this generality.

Lemma 2 is known (see)

Accordingly, I recommend that the paper be rejected.

PART B: SELECTED PROBLEMS OF ENGLISH GRAMMAR

INDEFINITE ARTICLE (a, an, —)

Note: Use “a” or “an” depending on *pronunciation* and not spelling, e.g. a unit, an x .

1. Instead of the number “one”:

The four centres lie in **a** plane.

A chapter will be devoted to the study of expanding maps.

For this, we introduce **an** auxiliary variable z .

2. Meaning “member of a class of objects”, “some”, “one of”:

Then D becomes **a** locally convex space with dual space D' .

The right-hand side of (4) is then **a** bounded function.

This is easily seen to be **an** equivalence relation.

Theorem 7 has been extended to **a** class of boundary value problems.

This property is **a** consequence of the fact that

Let us now state **a** corollary of Lebesgue’s theorem for

After **a** change of variable in the integral we get

We thus obtain the estimate with **a** constant C .

in the plural:

The existence of **partitions** of unity may be proved by

The definition of **distributions** implies that

....., with suitable constants.

....., where G and F are **differential** operators.

3. In definitions of classes of objects

(i.e. when there are many objects with the given property):

A fundamental solution is a function satisfying

We call C **a** module of ellipticity.

A classical example of **a** constant C such that

We wish to find **a** solution of (6) which is of the form

in the plural:

The elements of D are often called **test** functions.

the set of $\left\{ \begin{array}{l} \text{points with distance 1 from } K \\ \text{all functions with compact support} \end{array} \right.$

The integral may be approximated by **sums** of the form

Taking in (4) **functions** v which vanish in U we obtain

Let f and g be **functions** such that

4. In the plural—when you are referring to each element of a class:

Direct sums exist in the category of abelian groups.

In particular, closed sets are Borel sets.

Borel measurable functions are often called Borel mappings.

This makes it possible to apply H_2 -results to functions in any H_p .

If you are referring to all elements of a class, use “the”:

The real measures form a subclass of **the** complex ones.

5. In front of an adjective which is intended to mean “having this particular quality”:

This map extends to all of M in **an** obvious fashion.

A remarkable feature of the solution should be stressed.

Section 1 | gives **a** condensed exposition of
describes in **a** unified manner the recent results

A simple computation gives

Combining (2) and (3) we obtain, with **a** new constant C ,

A more general theory must be sought to account for these irregularities.

The equation (3) has **a** unique solution g for every f .

But: (3) has **the** unique solution $g = ABf$.

DEFINITE ARTICLE (the)

1. Meaning “mentioned earlier”, “that”:

Let $A \subset X$. If $aB = 0$ for every B intersecting **the** set A , then

Define $\exp x = \sum x^i/i!$. **The** series can easily be shown to converge.

2. In front of a noun (possibly preceded by an adjective) referring to a single, uniquely determined object (e.g. in definitions):

Let f be **the** linear form $\left| \begin{array}{l} g \mapsto (g, F). \\ \text{defined by (2).} \end{array} \right.$ [If there is only one.]

So $u = 1$ in **the** compact set K of all points at distance 1 from L .

We denote by $B(X)$ **the** Banach space of all linear operators in X .

....., under **the** usual boundary conditions.

....., with **the** natural definitions of addition and multiplication.

Using **the** standard inner product we may identify

3. In the construction: the + property (or another characteristic) + of + object:

The continuity of f follows from

The existence of test functions is not evident.

There is a fixed compact set containing **the supports of** all the f^j .

Then x is **the centre of** an open ball U .

The intersection of a decreasing family of such sets is convex.

But: Every nonempty open set in $\mathbb{R}^k$ is **a union of** disjoint boxes.
 [If you wish to stress that it is some union of not too well specified objects.]

4. In front of a cardinal number if it embraces all objects considered:

The two groups have been shown to have the same number of generators. [Two groups only were mentioned.]

Each of **the** three products on the right of (4) satisfies
 [There are exactly three products there.]

5. In front of an ordinal number:

The first Poisson integral in (4) converges to g .

The second statement follows immediately from **the first**.

6. In front of surnames used attributively:

the Dirichlet problem	<i>But:</i>	Taylor's formula
the Taylor expansion		[without "the"]
the Gauss theorem		a Banach space

7. In front of a noun in the plural if you are referring to a class of objects as a whole, and not to particular members of the class:

The real measures form a subclass of **the** complex ones.

This class includes **the** Helson sets.

ARTICLE OMISSION

1. In front of nouns referring to activities:

Application of Definition 5.9 gives (45).

Repeated application (use) of (4.8) shows that

The last formula can be derived by **direct** consideration of

Thus A is the smallest possible extension in which **differentiation** is always possible.

Using integration by parts we obtain

If we apply **induction** to (4), we get

Addition of (3) and (4) gives

This reduces the solution to **division** by Px .

Comparison of (5) and (6) shows that

2. In front of nouns referring to properties if you mention no particular object:

In questions of **uniqueness** one usually has to consider

By **continuity**, (2) also holds when $f = 1$.

By **duality** we easily obtain the following theorem.

Here we do not require **translation invariance**.

3. After certain expressions with “of”:

a type of convergence	the hypothesis of positivity
a problem of uniqueness	the method of proof
the condition of ellipticity	the point of increase

4. In front of numbered objects:

It follows from **Theorem 7** that

Section 4 gives a concise presentation of

Property (iii) is called the triangle inequality.

This has been proved in **part (a)** of the proof.

But: the set of solutions of **the** form (4.7)

To prove **the** estimate (5.3) we first extend

We thus obtain **the** inequality (3). [*Or:* inequality (3)]

The asymptotic formula (3.6) follows from

Since **the** region (2.9) is in U , we have

5. To avoid repetition:

the order and symbol of a distribution

the associativity and commutativity of A

the direct sum and direct product

the inner and outer factors of f [Note the plural.]

But: a deficit or an excess

6. In front of surnames in the possessive:

Minkowski's inequality, *but:* **the** Minkowski inequality

Fefferman and Stein's famous theorem,

more usual: the famous Fefferman–Stein theorem

7. In some expressions describing a noun, especially after “with” and “of”:

an algebra **with** unit e ; an operator **with** domain H^2 ; a solution **with** vanishing Cauchy data; a cube **with** sides parallel to the axes; a domain **with** smooth boundary; an equation **with** constant coefficients; a function **with** compact support; random variables **with** zero expectation

the equation **of** motion; the velocity **of** propagation;

an element **of** finite order; a solution **of** polynomial growth;

a ball **of** radius 1; a function **of** norm p

But: elements of **the** form $f = \dots$

a Banach space with a weak symplectic form w

two random variables with a common distribution

8. After forms of “have”:

It has	finite norm.	<i>But:</i> It has	a finite norm not exceeding 1.
	compact support.		a compact support contained in I .

It has	rank 2.
	cardinality c .
	absolute value 1.
	determinant zero.
<i>But:</i> It has	a zero of order at least 2 at the origin.
	a density g .
	[Unless g has appeared earlier; then: It has density g .]

9. In front of the name of a mathematical discipline:

This idea comes from game theory (homological algebra).

But: in **the theory of** distributions

10. Other examples:

We can assume that G is **in diagonal form**.

Then A is deformed into B by pushing it **at constant speed** along the integral curves of X .

G is now viewed as a set, **without group structure**.

INFINITIVE

1. Indicating aim or intention:

To prove the theorem, we first let

We now apply (5)	to study the group of
	to derive the following theorem.
	to obtain an x with norm not exceeding 1.

Here are some examples **to show** how

2. In constructions with “too” and “enough”:

This method is **too** complicated **to** be used here.

This case is important **enough to** be stated separately.

3. Indicating that one action leads to another:

We now apply Theorem 7 **to get** $Nf = 0$. [= and we get $Nf = 0$]

Insert (2) into (3) **to find** that

4. In constructions like “we may assume M to be”:

We may **assume M to be** compact.

We **define K to be** the section of H over S .

If we **take** the contour G **to lie** in U , then

We **extend f to be** homogeneous of degree 1.

The class A is defined by **requiring** all the functions f **to satisfy**

Partially order P by **declaring $X < Y$ to mean** that

5. In constructions like “ M is assumed to be”:

The map M	is assumed ⟨expected/found/considered/taken/ claimed⟩ to be open. will be chosen to satisfy (2). can be taken to be constant. can easily be shown to have is also found to be of class S .
-------------	---

This investigation is **likely to produce** good results.

[= It is very probable it will]

The close agreement of the six elements is **unlikely to be**
a coincidence. [= is probably not]

6. In the structure “for this to happen”:

For this to happen, F must be compact.

[= In order that this happens]

For the last estimate to hold, it is enough to assume

Then **for such a map to exist**, we must have

7. As the subject of a sentence:

To see that this is not a symbol **is** fairly easy.

[Or: It is fairly easy to see that]

To choose a point at random in the interval $[0, 1]$ **is** a conceptual
experiment with an obvious intuitive meaning.

To say that u is maximal **means** simply that

After expressions with “it”:

It is necessary ⟨useful/very important⟩ **to consider**

It makes sense to speak of

It is therefore **of interest to look** at

8. After forms of “be”:

Our goal ⟨method/approach/procedure/objective/aim⟩ **is to find**

The problem ⟨difficulty⟩ here **is to construct**

9. With nouns and with superlatives, in the place of a relative clause:

The theorem **to be proved** is the following. [= which will be proved]

This will be proved by the method **to be described** in Section 6.

For other reasons, **to be discussed** in Chapter 4, we have to

He was **the first to propose** a complete theory of

They appear to be **the first to have suggested** the now accepted
interpretation of

10. After certain verbs:

These properties **led him to suggest** that

Lax **claims to have obtained** a formula for

This map **turns out to satisfy**

At first glance M **appears to differ** from N in two major ways:

A more sophisticated argument **enables** one **to prove** that

[Note: “enable” requires “one”, “us” etc.]

He **proposed to study** that problem. [Or: He proposed studying

We **make** G **act** trivially on V .

Let f **satisfy** (2). [Not: “Let f satisfies”]

We **need to consider** the following three cases.

We **need not consider** this case separately.

[“need to” in affirmative clauses, without “to” in negative clauses; also note: “we only need to consider”, but: “we need only consider”]

ING-FORM

1. As the subject of a sentence (note the absence of “the”):

Repeating the previous argument and **using** (3) **leads** to

Since **taking** symbols **commutes** with lifting, A is

Combining Proposition 5 and Theorem 7 **gives**

2. After prepositions:

After making a linear transformation, we may assume that

In passing from (2) to (3) we have ignored the factor n .

In deriving (4) we have made use of

On substituting (2) into (3) we obtain

Before making some other estimates, we prove

The trajectory Z enters X **without meeting** $x = 0$.

Instead of using the Fourier method we can multiply

In addition to illustrating how our formulas work, it provides

Besides being very involved, this proof gives no information on

This set is obtained **by letting** $n \rightarrow \infty$.

It is important to pay attention to domains of definition

when trying to

The following theorem is the key **to constructing**

The reason **for preferring** (1) to (2) is simply that

3. In certain expressions with “of”:

The **idea of combining** (2) and (3) came from

The **problem** considered there was that **of determining** $WF(u)$ for

We use the **technique of extending**

This method has the disadvantage of		being very involved.
		requiring that f be positive.
		[Note the infinitive.]

Actually, S has the much stronger **property of being** convex.

4. After certain verbs, especially with prepositions:

We **begin by analyzing** (3).

We **succeeded** (were successful) **in proving** (4).

[*Not*: “succeeded to prove”]

We next **turn to estimating**

They **persisted in investigating** the case

We are **interested in finding** a solution of

We were **surprised at finding out** that

[*Or*: surprised to find out]

Their study **resulted in proving** the conjecture for

The success of our method will **depend on proving** that

To compute the norm of **amounts to finding**

We should **avoid using** (2) here, since

[*Not*: “avoid to use”]

We **put off discussing** this problem to Section 5.

It is **worth noting** that [*Not*: “worth to note”]

It is worth while discussing here this phenomenon.

[*Or*: worth while to discuss; “worth while” with ing-forms is best avoided as it often leads to errors.]

It is an idea **worth carrying out**.

[*Not*: “worth while carrying out”, *nor*: “worth to carry out”]

After **having finished proving** (2), we will turn to

[*Not*: “finished to prove”]

However, (2) **needs handling** with greater care.

One more case **merits mentioning** here.

In [7] he **mentions having proved** this for f not in S .

5. Present Participle in a separate clause (note that the subjects of the main clause and the subordinate clause must be the same):

We show that f satisfies (2), thus **completing** the analogy with

Restricting this to R , we can define

[*Not*: “Restricting, the lemma follows”. The lemma does not restrict!]

The set A , **being** the union of two intersecting continua, is connected.

6. Present Participle describing a noun:

We need only consider paths **starting** at 0.

We interpret f as a function with image **having** support in

We regard f as **being** defined on

7. In expressions which can be rephrased using “where” or “since”:

Now J is defined to equal Af , the function f **being** as in (3).

[= where f is

This is a special case of (4), the space X here **being** $B(K)$.

We construct three maps of the form (5), each of them **satisfying** (8).

Then $\lim_t a(x, t) < 1$, the limit **being assumed** to exist for every x .

The ideal is defined by $m = \dots$, it **being understood** that
 Now, F **being** convex, we can assume that [= since F is]
 Hence $F = \emptyset$ (it **being** impossible to make A and B intersect).
 [= since it is impossible]

[Do not write “a function being an element of X ” if you mean
 “a function which is an element of X ”.]

8. In expressions which can be rephrased as “the fact that X is”:

Note that M **being** cyclic implies F is cyclic.

The probability of X **being** rational equals $1/2$.

In addition to f **being** convex, we require that

PASSIVE VOICE

1. Usual passive voice:

This theorem was proved by Milnor in 1976.

In items 2–6, passive voice structures replace sentences with subject “we” or impersonal constructions of other languages.

2. Replacing the structure “we do something”:

This identity **is established** by observing that

This difficulty **is avoided** above.

When this **is substituted** in (3), an analogous description of K
 is obtained.

Nothing **is assumed** concerning the expectation of X .

3. Replacing the structure “we prove that X is”:

The function M **is easily shown to have**
may be said to be regular if

This equation **is known to hold** for

4. Replacing the construction “we give an object X a structure Y ”:

Note that E **can be given** a complex **structure** by

The letter A **is here given a bar** to indicate that

5. Replacing the structure “we act on something”:

This order behaves well when g **is acted upon** by an operator.

Hence F **can be thought of** as

So all the terms of (5) **are accounted for**.

The preceding observation, when **looked at** from a more general
 point of view, leads to

In the physical context already **referred to**, K is

6. Meaning “which will be (proved etc.)”:

Before stating the result **to be proved**, we give

This is a special case of convolutions **to be introduced** in Chapter 8.

We conclude with two simple lemmas **to be used** mainly in

QUANTIFIERS

This implies that A contains $\left| \begin{array}{l} \text{all open subsets of } U. \\ \text{all } y \text{ with } Gy = 1. \end{array} \right.$

Let B be the collection of $\left| \begin{array}{l} \text{all transforms } F \text{ of the form} \\ \text{all } A \text{ such that} \end{array} \right.$

In this way F is defined at **all** points of X .

This holds for **all** $n \neq 0$ (for **all** m which have/for **all** other m /for **all but** a finite number of indices i)

The domain X contains **all the** boundary except the origin.

The integral is taken over **all of** X .

Hence E, F and G $\left| \begin{array}{l} \text{all extend to a neighbourhood of } U. \\ \text{all have their supports in } U. \\ \text{are all zero at } x. \\ \text{are all equal.} \end{array} \right.$

There exist functions R , **all of whose** poles are in U , with

Each of the following nine conditions implies **all the others**.

Such an x exists iff **all the** intervals A_x have

For **every** g in X (not in X) there exists an N

[But: for all f and g , for any two maps f and g ;

“every” is followed by a *singular* noun.]

To **every** f there corresponds a unique g such that

Every invariant subspace of X is of the form

[Do not write: “Every subspace is not of the form”

if you mean: “No subspace is of the form”;

“every” must be followed by an *affirmative* statement.]

Thus $f \neq 0$ at **almost every** point of X .

Since $A_n = 0$ for **each** n , [Each = every, considered separately]

Each term in this series is either 0 or 1.

Consequently, F is bounded on **each** bounded set.

Each of these four integrals is finite.

These curves arise from, and **each** consists of

There remain four intervals of length $1/16$ **each**.

Thus X assumes values $0, 1, \dots, 9$, **each** with probability $1/10$.

The functions $F_1, \dots, F_n$ are **each** defined in the interval $[0, 1]$.

Those n disjoint boxes are translates **of each other**.

If K is now **any** compact subset of H , there exists

[Any = whatever you like; write “for all x ”, “for every x ” if you just mean a quantifier.]

Every measure can be completed, so whenever it is convenient, we may assume that **any** given measure is complete.

There is a subsequence such that

There exists an x with

[Or: there exists x , *but*: there is an x]

There are sets satisfying (2) but not (3).

There is only one such f .

There is a unique function f such that

Each f lies in zA for **some** A (at least one A /
exactly one A /at most one A).

Note that **some of** the X_n may be repeated.

Thus F has **no** pole in U (hence **none** in K). [Or: no poles]

Call a set dense if its complement contains **no** nonempty open subset.

If **no two** members of A have an element in common, then

No two of the spaces X , Y , and Z are isomorphic.

It can be seen that **no** x has more than one inverse.

In other words, for **no** real x does $\lim F_n(x)$ exist.

[Note the inversion after the negative clause.]

If there is **no** bounded functional such that

..... provided **none of** the sums is of the form

Let A_n be a sequence of positive integers **none of which** is 1 less than a power of two.

If there is an f such that, set

If **there are** $\langle \text{is} \rangle$ **none**, define

None of these are $\langle \text{is} \rangle$ possible.

Both f and g are obtained by

[Or: f and g are both obtained]

For **both** C^∞ and analytical categories,

It behaves covariantly with respect to maps of **both** X and G .

We now apply (3) to **both** sides of (4).

Both $\langle \text{these/the} \rangle$ conditions are restrictions only on

[Note: “the” and “these” after “both”]

It lies on no segment **both of whose** endpoints are in K .

Two consecutive elements do not belong **both to** A
or **both to** B .

Both its sides are convex. [Or: Its sides are both convex.]

Let B and C be nonnegative numbers, not **both** 0.

Choose points x in M and y in N , **both** close to z , and

We show how this method works in two cases.

In **both** $\langle \text{In each} \rangle$, C is

In **either** case, it is clear that [= In both cases]
 Each f can be expressed in **either of** the forms (1) and (2).
 [= in any of the two forms]
 The density of $X + Y$ is given by **either of** the two integrals.
 The two classes coincide if X is compact. In that case we write $C(X)$ for **either of** them.
Either f or g must be bounded.

Let u and v be two distributions **neither of** which is
 [Use “neither” when there are *two* alternatives.]
 This is true for **neither of** the two functions.
Neither statement is true.
 In **neither** case can f be smooth.
 [Note the inversion after the negative clause.]
 He proposes two conditions, but **neither** is satisfactory.

NUMBER, QUANTITY, SIZE

1. Cardinal numbers:

Hence A and B are also F -functions, any **two** of A , B , and C being independent.

the multi-index with $\left\{ \begin{array}{l} \text{all entries } \mathbf{zero} \text{ except the } k\text{th which is } \mathbf{one} \\ \text{the last } k \text{ entries } \mathbf{zero} \end{array} \right.$

This shows that there are no **two** points a and b such that
 There are **three** that the reader must remember. [= three of them]
 We have defined A , B , and C , and **the three** sets satisfy
 For **the two** maps defined in Section 3,
 [“The” if only two maps are defined there.]
 Clearly, R is concentrated at **the** n points $x_1, \dots, x_n$ defined above.

for **at least** $\langle$ **at most** $\rangle$ one k ; with norm **at least equal to** 2

There are **at most 2 such** r in $(0, 1)$.
 There is **a unique** map satisfying (4).
 Equation (4) has **a unique** solution g for each f .
But: it has **the unique** solution $g = ABf$.
 Problem (4) has **one and only one** solution.
Precisely r **of** the intervals are closed.
 In Example 3 only **one of** the x_j is positive.
 If $p = 0$ then there are **an additional** m arcs.

2. Ordinal numbers:

The first two are simpler than **the third**.
 Let S_i be **the first of** the remaining S_j .
The n th trial is the last.
 It follows that X_1 appears at **the** $(k + 1)$ **th** place.

The gain **up to and including the n th** trial is

The elements of **the third and fourth** rows are in I .
 [Note the plural.]

Therefore F has a zero of **at least third** order at x .

3. Fractions:

Two-thirds of its diameter is covered by

But: **Two-thirds** of the gamblers are ruined.

Obviously, G is **half** the sum of the positive roots.

[*Note:* Only “half” can be used with or without “of”.]

On the average, about **half** the list will be tested.

But J contains an interval of **half** its length in which

Note that F is greater by **a half** (a third).

The other player is half (one third) as fast.

We divide J **in half**.

All sides were increased by the same **proportion**.

About **40 percent** of the energy is dissipated.

A positive **percentage** of summands occurs in all k partitions.

4. Smaller (greater) than:

Observe that n is	greater (less) than k .
	much (substantially) greater than k .
	no greater (smaller) than k .
	greater (less) than or equal to k .
	[<i>Not:</i> “greater or equal to”] strictly less than k .

All points at a distance **less than** K from A satisfy (2).

We thus obtain a graph of **no more than** k edges.

This set has	fewer elements than K has.
	no fewer than twenty elements.

Therefore F can have no jumps **exceeding** $1/4$.

The degree of P **exceeds** that of Q .

Find the density of **the smaller of** X and Y .

The smaller of the two satisfies

It is dominated (bounded/estimated/majorized) by

5. How much smaller (greater):

25 is **3 greater** than 22; 22 is **3 less** than 25.

Let a_n be a sequence of positive integers none of which is **1 less** than a power of two.

The degree of P **exceeds** that of Q **by at least** 2.

Consequently, f is **greater by a half** (a third).

It follows that C is **less than a third** of the distance between

Within I , the function f **varies** (oscillates)
by **less than** l .

The upper and lower limits of f **differ by at most** 1.

We thus have in A **one** element **too many**.

On applying this argument k **more times**, we obtain

This method is recently **less and less** used.

A succession of **more and more** refined discrete models.

6. How many times as great:

twice (ten times/one third) **as long as**; half as big as

The longest edge is at most 10 times as long as the shortest one.

Now A has **twice as many** elements as B has.

Clearly, J contains a subinterval **of half its length** in which

Observe that A has four times the radius of B .

The diameter of L is $1/k$ times (twice) **that of** M .

7. Multiples:

The k -fold integration by parts shows that

We have shown that F covers M **twofold**.

It is bounded by **a multiple of** t (a constant times t).

This distance is less than **a constant multiple of** d .

Note that G acts on H as **a multiple**, say n , of V .

8. Most, least, greatest, smallest:

Evidently, F has **the most** (the fewest) points when

In **most** cases it turns out that

Most of the theorems presented here are original.

The proofs are, **for the most part**, only sketched.

Most probably, his method will prove useful in

What **most** interests us is whether

The **least** such constant is called the norm of f .

This is **the least** useful of the four theorems.

The method described above seems to be **the least** complex.

That is **the least** one can expect.

The elements of A are comparatively big, but **least** in number.

None of those proofs is easy, and John's **least of all**.

The best estimator is a linear combination U such that
 $\text{var } U$ is (the) **smallest possible**.

The expected waiting time is **smallest** if

Let L be **the smallest number** such that

Now, F has **the smallest** norm among all f such that

It is **the largest of** the functions which occur in (3).

There exists **a smallest** algebra with this property.

Find the **second largest** element in the list L .

9. Many, few, a number of:

There are [Note the plural.]	a large number of illustrations. only a finite number of f with $Lf = 1$. a small number of exceptions. an infinite number of sets a negligible number of points with
------------------------------------	--

Ind c is **the number of times** that c winds around 0.

We give **a number of** results concerning [= some]

This may happen in **a number of** cases.

They correspond to the values of **a countable number of** invariants.

..... for **all n except a finite number** (for all but finitely many n).

Thus Q contains **all but a countable number** of the f^i .

There are only **countably many** elements q of Q with $\text{dom } q = S$.

The theorem is fairly general. There are, however, **numerous** exceptions.

A variety of other characteristic functions can be constructed in this way.

There are **few** exceptions to this rule. [= not many]

Few of various existing proofs are constructive.

He accounts for all the major achievements in topology over **the last few** years.

The generally accepted point of view in this domain of science seems to be changing **every few** years.

There are **a few** exceptions to this rule. [= some]

Many interesting examples are known. We now describe **a few of** these.

Only **a few of** those results have been published before.

Quite a few of them are now widely used.

[= A considerable number]

10. Equality, difference:

A equals B or A is equal to B [Not: " A is equal B "]

The Laplacian of g is $4r > 0$.

Then r is about kn .

The inverse of FG is GF .

The norms of f and g coincide.

Therefore F has the same number of zeros and poles in U .

They **differ by** a linear term (by a scale factor).

The differential of f is **different from** 0.

Each member of G **other than** g is

Lemma 2 shows that F is not identically 0.

Let a , b and c be **distinct** complex numbers.

Each w is Pz for precisely m distinct values of z .

Functions which are equal a.e. are indistinguishable as far as integration is concerned.

11. Numbering:

Exercises 2 to 5 furnish other applications of this technique.

[*Amer.*: Exercises 2 through 5]

in the third and fourth rows

the derivatives up to order k

from row k onwards

the odd-numbered terms

in lines 16–19

the next-to-last column

the last paragraph but one of the previous proof

The matrix with $\left| \begin{array}{l} 1 \text{ in the } (i, j) \text{ entry and zero elsewhere} \\ \text{all entries zero except for } N - j \text{ at } (N, j) \end{array} \right.$

This is $\left| \begin{array}{l} \text{hinted at in Sections 1 and 2.} \\ \text{quoted on page 36 of [4].} \end{array} \right.$

HOW TO AVOID REPETITION

1. Repetition of nouns:

Note that the continuity of f implies **that** of g .

The passage from Riemann's theory to **that** of Lebesgue is

The diameter of F is about twice **that** of G .

His method is similar to **that** used in our previous paper.

The nature of this singularity is the same as **that**

which f has at $x = 0$.

Our results do not follow from **those** obtained by Lax.

One can check that the metric on T is **the one** we have just described.

It follows that S is the union of two disks. Let D be **the one** that contains

The cases $p = 1$ and $p = 2$ will be **the ones** of interest to us.

We prove a uniqueness result, similar to **those** of the preceding section.

Each of the functions on the right of (2) is **one**
to which

Now, F has many points of continuity. Suppose x is **one**.

In addition to a contribution to W_1 , there may be **one**
to W_2 .

We now prove that the constant pq cannot be replaced by
a smaller **one**.

Consider the differences between these integrals and

the corresponding ones with f in place of g .

The geodesics (4) are **the only ones** that realize the distance between
their endpoints.

On account of the estimate (2) and similar **ones** which can be

We may replace A and B by whichever is the larger of **the two**.
[Not: “the two ones”]

This inequality applies to conditional expectations as well as to ordinary **ones**.

One has to examine the equations (4). If **these** have no solutions, then

Thus D yields operators D^+ and D^- . **These** are formal adjoints of each other.

This gives rise to the maps F_i . All the other maps are suspensions of **these**.

So F is the sum of A , B , C and D . The last two of **these** are zero.

Both f and g are connected, but **the latter** is in addition compact.
[The latter = the second of *two* objects]

Both AF and BF were first considered by Banach, but only **the former** is referred to as the Banach map, **the latter** being called the Hausdorff map.

We have thus proved Theorems 1 and 2, **the latter** without using

Since the vectors G_i are orthogonal to **this last** space,

As a consequence of **this last** result,

Let us consider sets of the type (1), (2), (3) and (4).

These last two are called

We shall now describe a general situation in which **the last-mentioned** functionals occur naturally.

2. Repetition of adjectives, adverbs or phrases like “ x is”:

If f and g are measurable functions, then **so are** $f + g$ and $f \cdot g$.

The union of measurable sets is a measurable set; **so is** the complement of every measurable set.

The group G is compact and **so is** its image under f .

It is of the same fundamental importance in analysis **as is** the construction of

Note that F is bounded but **is not** necessarily **so** after division by G .

Show that there are many **such** Y .

There is only one **such** series for each y .

Such an h is obtained by

3. Repetition of verbs:

A geodesic which meets bM **does so** either transversally or

This will hold for $x > 0$ if it **does** for $x = 0$.

Note that we have not required that, and we shall not **do so** except when explicitly stated.

The integral might not converge, but it **does so** after

We will show below that the wave equation can be put in this form,
as can many other systems of equations.

The elements of L are not in S , **as** they **are** in the proof of

4. Repetition of whole sentences:

The same is true for f in place of g .

The same being true for f , we can [= Since the same]

The same holds for $\langle$ applies to $\rangle$ the adjoint map.

We shall assume that **this is the case**.

Such was the case in (2).

The L^2 theory has more symmetry **than is the case**
in L^1 .

Then either or **In the latter** $\langle$ former $\rangle$ **case**,

For k **this is no longer true**.

This is not true of (2).

This is not so in other queuing processes.

If **this is so**, we may add

If $f_i \in L$ and if $F = f_1 + \dots + f_n$ then $F \in H$, and every
 F is **so** obtained.

We would like to If U is open, **this** can be done.

On S , **this** gives the ordinary topology of the plane.

Note that **this** is not equivalent to

[Note the difference between “this” and “it”: you say “*it* is not
equivalent to” if you are referring to some object explicitly
mentioned in the preceding sentence.]

Consequently, F **has the stated** $\langle$ desired/claimed $\rangle$ **properties**.

WORD ORDER

General remarks: The normal order is: subject + verb + direct object + adverbs in
the order manner-place-time.

Adverbial clauses can also be placed at the beginning of a sentence, and some adverbs
always come between subject and verb. Subject almost always precedes verb,
except in questions and some negative clauses.

1. ADVERBS

1a. Between subject and verb, but after forms of “be”; in compound tenses
after first auxiliary

• *Frequency adverbs:*

This has **already** been proved in Section 8.

This result will **now** be derived computationally.

Every measurable subset of X is **again** a measure space.

We **first** prove a reduced form of the theorem.

There has **since** been little systematic work on
 It has **recently** been pointed out by Fix that
 It is **sometimes** difficult to
 This **usually** implies further conclusions about f .
 It **often** does not matter whether

- *Adverbs like “also”, “therefore”, “thus”:*

Our presentation is **therefore** organized in such a way that
 The sum in (2), though formally infinite, is **therefore** actually finite.
 One must **therefore** also introduce the class of
 But C is connected and is **therefore** not the union of

These properties, with the exception of (1), **also** hold
 for t .

We will **also** leave to the reader the verification that
 It will **thus** be sufficient to prove that
 So (2) implies (3), since one would **otherwise** obtain

The order of several topics has **accordingly** been changed.

- *Emphatic adverbs (clearly, obviously, etc.):*

It would **clearly** have been sufficient to assume that
 But F is **clearly** not an I -set.
 Its restriction to N is **obviously** just f .
 This case must **of course** be excluded.
 The theorem **evidently** also holds if $x = 0$.

The crucial assumption is that the past history
in no way influences

We did not **really** have to use the existence of T .
 The problem is to decide whether (2) **really** follows
 from (1).

The proof is now **easily** completed.

The maximum is **actually** attained at some point of M .

We then **actually** have [= We have even more]

At present we will **merely** show that

A stronger result is **in fact** true.

Throughout integration theory, one **inevitably** encounters ∞ .

But H itself can **equally well** be a member of S .

1b. After verb—most adverbs of manner:

We conclude **similarly** that

One sees **immediately** that

Much relevant information can be obtained **directly** from (3).

This difficulty disappears **entirely** if

This method was used **implicitly** in random walks.

- 1c. After an object if it is short:

We will prove the theorem **directly** without using the lemma.

But: We will prove **directly** a theorem stating that

This is true for every sequence that shrinks to x **nicely**.

Define Fg **analogously** as the limit of

Formula (2) defines g **unambiguously** for every g' .

- 1d. At the beginning—adverbs referring to the whole sentence:

Incidentally, we have now constructed

Actually, Theorem 3 gives more, namely

Finally, (2) shows that $f = g$. [*Not*: “At last”]

Nevertheless, it turns out that

Next, let V be the vector space of

More precisely, Q consists of

Explicitly (**Intuitively**), this means that

Needless to say, the boundedness of f was assumed only for simplicity.

Accordingly, either f is asymptotically dense or

- 1e. In front of adjectives—adverbs describing them:

a **slowly varying** function

probabilistically significant problems

a method **better suited** for dealing with

The maps F and G are **similarly obtained** from H .

The function F has a **rectangularly shaped** graph.

Three-quarters of this area is covered by **subsequently chosen** cubes. [Note the singular.]

- 1f. “only”

We need the openness **only** to prove the following.

It reduces to the statement that **only** for the distribution F do the maps F_i satisfy (2). [Note the inversion.]

In this chapter we will be concerned **only** with

In (3) the X_j assume the values 0 and 1 **only**.

If (iii) is required for finite unions **only**, then

We need **only** require (5) to hold for bounded sets.

The proof of (2) is similar, and will **only** be indicated briefly.

To prove (3), it **only** remains to verify

2. ADVERBIAL CLAUSES

- 2a. At the beginning:

In testing the character of, it is sometimes difficult to

For $n = 1, 2, \dots$, consider a family of

2b. At the end (normal position):

The averages of F_n become small **in small neighbourhoods** of x .

2c. Between subject and verb, but after first auxiliary—only short clauses:

The observed values of X will **on average** cluster around

This could **in principle** imply an advantage.

For simplicity, we will **for the time being** accept as F only C^2 maps.

Accordingly we are **in effect** dealing with

The knowledge of f is **at best** equivalent to

The stronger result is **in fact** true.

It is **in all respects** similar to matrix multiplication.

2d. Between verb and object if the latter is long:

It suffices **for our purposes** to assume

To a given density on the line there corresponds **on the circle**
the density given by

3. INVERSION AND OTHER PECULIARITIES

3a. Adjective or past participle after a noun:

Let Y be the complex X with the origin **removed**.

Theorems 1 and 2 **combined** give a theorem

We now show that G is in the symbol class **indicated**.

We conclude by the part of the theorem **already proved** that

The bilinear form **so defined** extends to

Then for A **sufficiently small** we have

By queue length we mean the number of customers **present**
including the customer **being served**.

The description is the same with the roles of A and B **reversed**.

3b. Direct object or adjectival clause placed farther than usual—when they are long:

We must **add** to the right-hand side of (3) **the probability** that

This is equivalent to **defining** in the z -plane **a density** with

Let F be the **restriction** to D **of** the unique linear map

The **probability** at birth **of** a lifetime exceeding t is at most

3c. Inversion in some negative clauses:

We do not assume that, **nor do** we assume a priori that

Neither is the problem simplified by assuming $f = g$.

The “if” part now follows from (3), since **at no point can** S exceed
the larger of X and Y .

The fact that **for no** x **does** Fx contain y implies that

In no case does the absence of a reference imply any claim to
originality on my part.

3d. Inversion—other examples:

But F is compact and **so is** G .

If f, g are measurable, then **so are** $f + g$ and $f \cdot g$.

Only for $f = 1$ | **can** one expect to obtain
| **does** that limit exist.

3e. Adjective in front of forms of “be”—for emphasis:

By far **the most important** is the case where

Much more subtle are the following results of John.

Essential to the proof are certain topological properties of M .

3f. Subject coming sooner than in some other languages:

Equality occurs in (1) iff f is constant.

The natural **question** arises whether it is possible to

In the following applications **use** will be made of

Recently **proofs** have been constructed which use

3g. Incomplete clause at the beginning or end of a sentence:

Put differently, the moments of arrival of the lucky customers constitute a renewal process.

Rather than discuss this in full generality, let us look at

It is important that the tails of F and G are of comparable magnitude,
a statement made more precise by the following inequalities.

WHERE TO INSERT A COMMA

General rules: Do not over-use commas—English usage requires them less often than in many other languages. Do not use commas around a clause that defines (limits, makes more precise) some part of a sentence. Put commas before and after non-defining clauses (i.e. ones which can be left out without damage to the sense). Put a comma where its lack may lead to ambiguity, e.g. between two symbols.

1. Comma not required:

We shall now prove that f is proper.

The fact that f has radial limits **was** proved in [4].

It is reasonable to ask **whether** this holds for $g = 1$.

Let M denote the set of all paths that satisfy (2).

There is a polynomial P such that $Pf = g$.

The element **given** by (3) is of the form (5).

Let M be the manifold to whose boundary f maps K .

Take an element **all** of whose powers are in S .

We call F proper if G is dense.

There exists a D such that $D \sim H$ **whenever** $H \sim G$.

Therefore $F(x) = G(x)$ for all $x \in X$.

Let F be a nontrivial continuous linear operator in V .

2. Comma required:

The proof of (3) depends on the notion of M -space, which has already been used in [4].

We will use the map H , which has all the properties required.

There is only one such f , and (4) defines a map from

In fact, we can do even better.

In this section, however, we will not use it explicitly.

Moreover, F is countably additive.

Finally, (d) and (e) are consequences of (4).

Nevertheless, he succeeded in proving that

Conversely, suppose that

Consequently, (2) takes the form

In particular, f also satisfies (1).

Guidance is also given, whenever necessary or helpful, on further reading.

This observation, when looked at from a more general point of view, leads to

It follows that f , being convex, cannot satisfy (3).

If $e = 1$, which we may assume, then

We can assume, by decreasing k if necessary, that

Then (5) shows, by Fubini's theorem, that

Put this way, the question is not precise enough.

Being open, V is a union of disjoint boxes.

This is a special case of (4), the space X here being $B(K)$.

In [2], X is assumed to be compact.

For all x , $G(x)$ is convex.

[Comma between two symbols.]

In the context already referred to, K is the complex field.

[Comma to avoid ambiguity.]

3. Comma optional:

By Theorem 2, there exists an h such that

For z near 0, we have

If h is smooth, then M is compact.

Since h is smooth, M is compact.

It is possible to use (4) here, but it seems preferable to

This gives (3), because $\langle$ since $\rangle$ we may assume

Integrating by parts, we obtain

The maps X , Y , and Z are all compact.

We have $X = FG$, where F is defined by

Thus $\langle$ Hence/Therefore $\rangle$, we have

HYPHENATION

1. Non(-):

Write consistently either

nontrivial, nonempty, nondecreasing, nonnegative, or
non-trivial, non-empty, non-decreasing, non-negative.

[*But*: non-locally convex, non-Euclidean]

2. Hyphen required:

one-parameter group

two-stage computation

n -fold integration

out-degree

global-in-time solution [But: solution global in time]

3. Hyphen optional:

right hand side or right-hand side

second order equation or second-order equation

selfadjoint or self-adjoint

halfplane or half-plane

seminorm or semi-norm

a blow-up, a blow up, or a blowup [But: to blow up]

the n th element or the n -th element

SOME TYPICAL ERRORS

1. Spelling errors:

Spelling should be either British or American throughout:

Br.: colour, neighbourhood, centre, fibre, labelled, modelling

Amer.: color, neighborhood, center, fiber, labeled, modeling

“an unified approach” $\rightsquigarrow$ a unified approach

“a M such that” $\rightsquigarrow$ an M such that

[Use a or an according to pronunciation.]

“preceeding” $\rightsquigarrow$ preceding

“occurring” $\rightsquigarrow$ occurring

“developped” $\rightsquigarrow$ developed

“loosing” $\rightsquigarrow$ losing

“it’s norm” $\rightsquigarrow$ its norm

2. Grammatical errors:

“Let f denotes” $\rightsquigarrow$ Let f denote

“Most of them is” $\rightsquigarrow$ Most of them are

“There is a finite number of” $\rightsquigarrow$ There are a finite number of

“In 1964 Lax has shown” $\rightsquigarrow$ In 1964 Lax showed
 [Use the past tense if a date is given.]
 “the Taylor’s formula” $\rightsquigarrow$ Taylor’s formula [Or: the Taylor formula]
 “the section 1” $\rightsquigarrow$ Section 1
 “Such map exists” $\rightsquigarrow$ Such a map exists [But: for every such map]
 “in case of smooth norms” $\rightsquigarrow$ in the case of smooth norms
 “We are in the position to prove” $\rightsquigarrow$ We are in a position to prove
 “We now give few examples” [= not many]
 $\rightsquigarrow$ We now give a few examples [= some]
 “ F is equal G ” $\rightsquigarrow$ F is equal to G [Or: F equals G]
 “ F is greater or equal to G ” $\rightsquigarrow$ F is greater than or equal to G
 “This is precised by” $\rightsquigarrow$ This is made more precise by
 “This allows to prove” $\rightsquigarrow$ This allows us to prove
 “This makes clear that” $\rightsquigarrow$ This makes it clear that
 “The first two ones are” $\rightsquigarrow$ The first two are
 “a not dense set” $\rightsquigarrow$ a non-dense set
 [But: This set is not dense]
 “Since $f = 0$, then M is closed”
 $\rightsquigarrow$ Since $f = 0$, it follows that M is closed
 “....., as it is shown in Sec. 2” $\rightsquigarrow$, as is shown in Sec. 2
 “Every function being an element of X is convex”
 $\rightsquigarrow$ Every function which is an element of X is convex
 “Every f is not convex” $\rightsquigarrow$ No f is convex
 “Setting $n = p$, the equation can be solved by”
 $\rightsquigarrow$ Setting $n = p$, we can solve the equation by”
 [Because we set.]
 “We have $\langle$ get/obtain $\rangle$ that B is empty”
 $\rightsquigarrow$ We see $\langle$ know/conclude/deduce/find/infer $\rangle$ that B is empty

3. Wrong word used:

“Summing (2) and (3) by sides” $\rightsquigarrow$ Summing (2) and (3)
 “In the first paragraph” $\rightsquigarrow$ In the first section
 “which proves our thesis”
 $\rightsquigarrow$ which proves our assertion $\langle$ conclusion/statement $\rangle$
 [thesis = dissertation]
 “to this aim” $\rightsquigarrow$ to this end
 “At first, note that” $\rightsquigarrow$ First, note that
 “At last, C is dense because” $\rightsquigarrow$ Finally, C is dense because
 “for every two elements” $\rightsquigarrow$ for any two elements
 “....., what completes the proof” $\rightsquigarrow$, which completes the proof
 “....., what is impossible” $\rightsquigarrow$, which is impossible

“We denote it shortly by c ” $\rightsquigarrow$ We denote it briefly by c

“This map verifies (2)” $\rightsquigarrow$ This map satisfies (2)

“continuous in the point x ” $\rightsquigarrow$ continuous at x

“disjoint with B ” $\rightsquigarrow$ disjoint from B

“equivalent with B ” $\rightsquigarrow$ equivalent to B

“independent on B ” $\rightsquigarrow$ independent of B
[*But*: depending on B ,
independence from B]

“similar as B ” $\rightsquigarrow$ similar to B

similarly as in Sec. 2	$\rightsquigarrow$	similarly to Sec. 2
		as (just as) in Sec. 2
		as is the case in Sec. 2
		in much the same way as
		in Sec. 2

“on Fig. 3” $\rightsquigarrow$ in Fig. 3

“in the end of Sec. 2” $\rightsquigarrow$ at the end of Sec. 2

4. Wrong word order:

“a bounded by 1 function” $\rightsquigarrow$ a function bounded by 1

“the described above condition” $\rightsquigarrow$ the condition described above

“the obtained solution” $\rightsquigarrow$ the solution obtained

“the mentioned map” $\rightsquigarrow$ the map mentioned

[*But*: the above-mentioned map]

“the both conditions” $\rightsquigarrow$ both conditions, both the conditions

“its both sides” $\rightsquigarrow$ both its sides

“the three first rows” $\rightsquigarrow$ the first three rows

“the two following sets” $\rightsquigarrow$ the following two sets

“This map we denote by f ” $\rightsquigarrow$ We denote this map by f

“Only for $x = 1$ the limit exists” $\rightsquigarrow$ Only for $x = 1$ does the limit exist

“For no x the limit exists” $\rightsquigarrow$ For no x does the limit exist

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