Space, Time, Observers

We all have an intuitive sense of what space is and what time is. Space is something in which "bodies move" and time is something that sequences these movements. To make these notions quantitative we need to adopt a procedure to assign numbers to "locations" and put time stamps on events. It is in terms of these assignments or coordinates that we make the space-time explicit, and it is this explicit model that is used in physics. All the tourist maps we use and the scheduling we struggle to achieve are based on precisely such "made explicit" space and time.

There is no unique way to assign coordinates and time stamps. Herein enters an observer (= adopted procedure).

With such a procedure at hand, it is possible to formulate the phenomenon of motion of bodies in terms of kinematics - description of motion, and dynamics - laws of motion. The key point to note is that there is always an observer implicit directly in kinematics and indirectly in dynamics.

Einstein now observes several examples of relationships between classes of observers and the phenomena being described. Consider the problem of determining the distance between two points, say by laying down meter sticks. The answer will evidently depend on how the meter sticks are laid. Drawing on the experience of measuring distances along short straight lines and using the procedure of assigning Cartesian coordinates, an observer can determine the distance between two points with Cartesian coordinates (x1, y1, z1) and (x2, y2, z2) to be given by:

Distance^2 = (x2 - x1)^2 + (y2 - y1)^2 + (z2 - z1)^2

Now the interesting observation is that all observers assigning Cartesian coordinates will verify that the distance between two given points is numerically the same (assuming the same units are used!). Hence, as far as the problem of determining distance between points is concerned, any of this class of observers will do fine.

Observers are related by the transformation law:

(x0)i = Σ (j=1 to 3) Ai j xj + Bi, where Ai j is a 3-by-3 orthogonal matrix.

These leave the Cartesian nature of coordinates unchanged as well as the expression for distance invariant. For Bi = 0, Einstein calls this relativity of orientation.

The next example he considers is the phenomenon of motion of particles, governed by Newton's laws formulated in the so-called inertial frames.

The class of observers whose descriptions are equivalent are those who are in uniform relative motion, possibly differing in the orientation of the axes of the Cartesian frames and possibly with difference in the "zero" of their clocks. This is, of course, Galilean relativity. What is left invariant is the mass × acceleration.

When the phenomenon of motion is extended to include electromagnetic field and the motion of charges under their influence, a contradiction arises. Analysis of the famous moving magnet and conductor problem in the magnet's rest frame and the conductor's rest frame presents two alternatives. Either have Galilean transformations among the electric and magnetic fields so as to get the same force in both the frames, or allow a new transformation law for the force so as to be consistent with the Lorentz transformations which leave Maxwell's equations invariant. Which one of these is "correct"?

On the one hand, confirmation of constancy of speed of light puts Lorentz transformations on a firmer ground, and on the other hand Galilean transformations contain an unwarranted assumption of observer independence of simultaneity. Einstein chooses Lorentz transformations and we have the theory of special relativity. What two observers in uniform relative motion must agree on is the same value of the speed of light in vacuum.

This affects the kinematics in a profound manner. We will discuss the derivations a little later, but let us note at this stage that the length of a stick measured by a moving observer is a little less than that measured by an observer at rest with respect to the stick. Likewise, when an observer compares the successive ticks of a moving clock with a stationary clock, the moving clock always ticks slower. These consequences of the demand of invariance of the speed of light go by the names length contraction and time dilation respectively.

The new kinematics does not leave invariant the other Newtonian law, namely the law of gravitational force. Once again we face a similar dilemma as before: Do we limit the applicability of the new kinematics, or do we modify the law of gravitational force?

There is a peculiarity with the law of gravitation. The "charge" that enters in the force law, the gravitational mass, happens to be numerically equal to the measure of the inertia of a body, its inertial mass. This makes different bodies of varied compositions and weights fall to the ground with the same acceleration.

There is no "reason" for such conceptually widely different quantities to be numerically equal, except perhaps it is a clue to the nature of gravitational interaction.

All bodies fall at the same rate also means that an observer does so too and therefore, relative to the observer, the bodies continue to maintain their state of uniform motion. In the absence of any force of any other origin, this just means that the freely falling observer is the Newtonian inertial observer!

The clue of equality of the two masses provides us with a definition of inertial frames as precisely those in which gravitational field cannot be detected. Furthermore, an observer who detects gravitational field is accelerated relative to an inertial frame. Thus we can trade off a gravitational field, for an observer accelerated relative to an inertial observer. Since relatively accelerated observers are involved, Lorentzian kinematics is not immediately applicable.

Rotating platforms provide a convenient "laboratory" for a thought experiment. Imagine determining the circumference and the radius of a rotating platform. The measuring sticks tangential to the circumference will undergo Lorentz contraction while those along the radial direction will not be contracted. Thus the ratio of the circumference to radius of the rotating platform, obtained by taking the ratio of the number of measuring sticks along the circumference and the number along the radius, will be greater than $2\pi$ while that of a non-rotating platform will be $2\pi$. Hence, the geometry on a rotating platform will be non-Euclidean. But by equivalence principle, acceleration is equivalent to a gravitational field (locally) and therefore one must infer that gravity affects the geometry. This gravitational field is of course inferred by the observer who is co-rotating with the platform. We will return to the rotating platform later again.

Thus the response (motion) of bodies to a gravitational field is independent of their masses and the gravitational field also changes the geometry of space. Since a gravitational field is produced by masses, the spatial geometry is also influenced by the masses. Thus, geometry of space is changeable. This is quite a novel inference! Does space-time geometry also change with distribution of masses?

This could be so if clocks tick at different rates in a gravitational field. Consider an observer stationed at a height of h from the ground and another observer freely falling. The freely falling observer will have a speed $v = g t$ relative to the stationary observer after a time t and will have fallen through a distance of $s = 1/2 g t^2$. As per Lorentzian kinematics, the rate of freely falling clock will be:

$$\tau_{falling} = \tau_{fixed} \sqrt{(1 - g^2 t^2)} = \tau_{fixed} \sqrt{(1 - 2 g s)} = \tau_{fixed} \sqrt{(1 - 2 \phi_{grav})}$$

The final expression depends only on the gravitational potential difference between the stationary observer and instantaneous.

It is clear from this argument that the gravitational potential affects the rates of clocks and since gravitational potential changes with the distribution of masses, so do the clock rates and hence the space-time geometry too is affected by distribution of masses.

Thus, replacing gravitational field by an accelerated observer and the Lorentzian kinematics leads us to a space-time geometry which is affected by presence of gravitational field which in turn depends on distribution of masses. One puzzle still remains. If gravitational field can be "gotten rid of" as in a freely falling lift, is gravity "fictitious"? It can't be. After all Earth is freely falling in the gravitational field of the Sun and real tides – which are effects of Newtonian gravity – do exist! So, while metrical property within a freely falling lift will be that in the absence of gravitational field, something else must remain encoded in the geometry that will account for the tides.

From the examples of two-dimensional surfaces, we know that the non-Euclidean geometries have non-zero curvature. This is most easily seen on the surface of a sphere. Consider a triangle made up of sides which are portions of great circles on the sphere. If a triangle is "large", with two points on the equator and the third one the north pole (say), then the sum of angles is greater than 180 degrees. Now bring the two equatorial points closer to the pole. Note that the generic latitude is not a great circle (the longitudes always are). So the small triangle will look more and more "distorted", but the sum of its angles will get closer and closer to 180 degrees. In short, non-zero curvature is detectable as deviation from Euclidean geometry, only for larger triangles.

The same is true for tidal forces in Newtonian gravity. The differential forces on two extremes of a body are larger when the separation of the two extremes is larger. Thus we see a parallel between the effects of curvature in geometry and the tidal forces of gravity.

At a qualitative level then, we see that effects of gravitational field can be mimicked by a space-time geometry which has curvature which in turn must depend on the distribution of masses since Newtonian gravitational potential does. The observed equality of gravitational mass and inertial mass, combined with Lorentzian kinematics leads to replacing gravitational interaction as revealing a space-time geometry which is curved in general and is changeable.

Space-time is a dynamical entity. In the process, the principle of relativity also gets extended to all observers regardless of their state of motion. As Einstein says: "Theory of relativity is intimately connected with a theory of space and time..." In the subsequent chapters we will formalize and make these arguments precise and quantitative.

Ako želiš, mogu da napra