

2.1.2 Building and Packaging Transistors

Although transistor

transistors can perform a variety of functions, for example amplification of signals, we will use them almost exclusively as switches. That is, a component that conducts, or allows charge to flow, between two terminals when we turn on the switch and does not conduct, or behaves like a resistor, when we turn off the switch.

There are several different types of transistor, but we will concentrate on one of the simplest, the Field Effect Transistor (FET), originally invented in 1952 by William Shockley at Bell Labs. The two transistor terminals are called the source and drain, and the switch that controls the flow of current is called the gate.

To create the switch, we combine materials with different properties. The most common design is the Metal Oxide Semiconductor Field-Effect Transistor (MOSFET), invented in 1960 by Martin Atalla. A MOSFET is built from N-type and P-type semiconductor materials with layers of metal. The two types are called N-MOSFET and P-MOSFET.

The basic idea is that applying a potential difference between gate and source creates an electric field that forms a channel between source and drain, allowing current to flow. Changing this potential difference changes the conductivity between source and drain. Thus, the gate acts like a switch controlling current.

For an N-MOSFET, a high potential difference between gate and source opens the channel and current flows (switch on). A low potential difference closes the channel (switch off). For a P-MOSFET, the behavior is reversed: high voltage turns it off and low voltage turns it on.

Even when the switch is off, a small current can still flow due to imperfections in the material. This is called leakage.

Transistors are usually not used alone but are packaged into larger components called logic styles. The most common is Complementary Metal-Oxide Semiconductor (CMOS), invented in 1963 by Frank Wanlass.

CMOS pairs an N-MOSFET with a P-MOSFET so that when one conducts, the other does not. This forms a circuit with a pull-up network (P-MOSFETs connected to V_{dd}) and a pull-down network (N-MOSFETs connected to GND). Only one network is active at a time.

CMOS circuits consume power mainly when switching states, except for small leakage currents. This reduces power consumption and heat, improving reliability and allowing smaller designs.

Compared to Transistor-Transistor Logic (TTL), which uses Bipolar Junction Transistors (BJTs), CMOS is lower power but often slower. A hybrid technology combining both is called BiCMOS.

PROOF:

Let (a_j) be a sequence of real numbers such that $\lim_{j \rightarrow \infty} a_j = L$.

Let $\varepsilon > 0$ be given.

By the definition

of limit, there exists N such that for all $j > N$:

$$|a_j - L| < \varepsilon/2.$$

Then for all $m, n > N$, we have:

$$|a_m - L| < \varepsilon/2 \text{ and } |a_n - L| < \varepsilon/2.$$

So:

$$\varepsilon = \varepsilon/2 + \varepsilon/2$$

$$|a_m - L| + |a_n - L|$$

Now,

$$|a_m - a_n|$$

$$= |(a_m - L) + (L - a_n)|$$

$$\leq |a_m - L| + |a_n - L|$$

$$< \varepsilon/2 + \varepsilon/2$$

$$= \varepsilon.$$

Thus, for all $m, n > N$:

$$|a_m - a_n| < \varepsilon.$$

Therefore, the sequence (a_j) is Cauchy. ✓

PROOF (All Cauchy sequences converge)

This proof is taken from:

Khan, Writing Proofs in Analysis, page 73.

Theorem:

All Cauchy sequences converge.

PROOF:

Let $\langle a_n \rangle$ be a Cauchy sequence. Let A be the set of its terms $\{a_n\}$.

CASE 1: A is finite

If A contains only one value, then the sequence is constant and converges to that value.

If A contains more than one value, then since the set of values is finite, the set of differences $a_n - a_m$ is also finite.

Let d be the smallest positive difference between any two distinct terms of the sequence, and let $\epsilon = d/2$.

Since the sequence is Cauchy, there exists N such that for all $m, n > N$:

$$|a_m - a_n| < \epsilon.$$

But the smallest positive distance between distinct terms is $d > \epsilon$, so this is only possible if:
 $a_m - a_n = 0$.

Thus, for all $n > N$, the sequence is constant.

Therefore, the sequence converges.

CASE 2: A is infinite

Since every Cauchy sequence is bounded, A is a bounded infinite set.

By the Bolzano–Weierstrass Theorem, A has an accumulation point p .

Since the sequence is Cauchy, for every $\epsilon > 0$ there exists N such that for all $m, n > N$:

$$|a_m - a_n| < \epsilon/2.$$

Since p is an accumulation point of A, there exists $k > N$ such that:

$$|a_k - p| < \epsilon/2.$$

Then for all $n > N$:

$$\begin{aligned} & |a_n - p| \\ &= |(a_n - a_k) + (a_k - p)| \\ &\leq |a_n - a_k| + |a_k - p| \\ &< \epsilon/2 + \epsilon/2 \\ &= \epsilon. \end{aligned}$$

Thus, $a_n \rightarrow p$.

PROOF 2 (product of limits theorem)

This proof is taken from:

Khan, Writing Proofs in Analysis, page 84.

Theorem:

If f and g are functions defined on a set with accumulation point a , and

$\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = H$,

then

$\lim_{x \rightarrow a} f(x)g(x) = LH$.

PROOF:

Let f and g be functions defined on a set with accumulation point a such that

$\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = H$.

Let $\epsilon > 0$ be given.

By the definition of limit, there exists $\delta_1 > 0$ such that if $0 < |x - a| < \delta_1$, then

$|f(x) - L| < \epsilon / (2(|H| + 1))$.

By the definition of limit, there exists $\delta_2 > 0$ such that if $0 < |x - a| < \delta_2$, then

$|f(x) - L| < 1$.

Then $|f(x)| \leq |L| + 1$.

By the definition of limit, there exists $\delta_3 > 0$ such that if $0 < |x - a| < \delta_3$, then

$|g(x) - H| < \epsilon / (2(|L| + 1))$.

Let $\delta = \min(\delta_1, \delta_2, \delta_3) > 0$.

Then if $0 < |x - a| < \delta$, we have:

$|f(x)g(x) - LH|$

$= |f(x)g(x) - f(x)H + f(x)H - LH|$

$= |f(x)(g(x) - H) + H(f(x) - L)|$

$\leq |f(x)| |g(x) - H| + |H| |f(x) - L|$

$< (|L| + 1) * \epsilon / (2(|L| + 1)) + |H| * \epsilon / (2(|H| + 1))$

$< \epsilon/2 + \epsilon/2 = \epsilon$.

Therefore,

$\lim_{x \rightarrow a} f(x)g(x) = LH$.

StORY: Benjamin Parzybok – “Reparador Pico”

Reparador Pico sat on a dirty child’s dollhouse and drank sweetwine made by Marzo. The wine tasted of plastic, car oil, and the smell of the dump. The tin shed around him rattled. Nails popped from the roof. This meant the trucks had arrived. The nackers were coming for new trash.

Pico finished the wine and hid the bottle in the dollhouse. He took his tool belt and tharpoon from the wall. Outside, he hit his chest with the tharpoon and said: “Aha!”

The pepenadores (dump workers) gathered at the edge of the settlement. Pico joined them, keeping distance from his parents. They used to run for new trash, but now the nackers took everything first. So they searched only old garbage.

At one end, Pico saw Mouse. Mouse wore a cowboy hat and an orange jumpsuit with one arm missing. She liked to stand out. Pico liked her. He followed her at a distance, as he always did. He kept about fifteen steps behind her. Pico sharpened his tharpoon with a stone in his pocket. The wine made him feel bold, like he could fight someone. Around him, other pepenadores waited—old people buried in layers of clothes, living like part of the dump itself.

Some dump workers were young, like Pico. Most ignored him, their faces marked with sweat and dirt. All of them watched the trucks on the horizon. They waited for the main dump area to clear of nackers. Then they slowly moved in.

Mouse went first, toward the great wall of trash. Pico followed at a distance. He was not sure what she was looking for, but he kept following anyway. He told himself she would probably send him away if she noticed.

After a while, a nacker picked up Mouse's trail. It followed her quietly, hiding among the trash. This one was different—newer, silent, with smooth joints. Pico crouched and moved behind it, staying hidden in the garbage hills.

Mouse stopped and began digging with her tharpoon. She had found something valuable. The nacker waited, watching her. Pico also waited, afraid. He thought about attacking the machine, but he was not ready.

Then Mouse pulled a car door out of the ground. It was valuable—metal, electronics, worth money. The nacker would want it. Pico tightened his grip on his tharpoon and ran toward them. He was slow, sweating, dizzy from heat and wine.

The nacker was faster. It moved on six legs across the trash, reaching forward with a mechanical arm. Pico knew he was too late—but he kept running anyway.