Week 10, Homework, 6.2.2026.

Assignment 1: Translate: THEOREM (Cesàro type result)

Let (a_n) be a sequence.

Let (b_n) be a sequence of strictly positive real numbers such that:

$\sum_{i=0}^{\infty} b_i = \infty$

If:

$\lim_{n\to\infty} (a_n / b_n) = L \in R$

then also:

$\lim_{n\to\infty} ((a_1 + a_2 + ... + a_n) / (b_1 + b_2 + ... + b_n)) = L$

COROLLARY

Let (a_n) and (b_n) be sequences in R such that (b_n) is strictly increasing and:

$\lim_{n\to\infty} b_n = \infty$

If:

$\lim_{n\to\infty} ((a_n - a_{n-1}) / (b_n - b_{n-1})) = L \in R$

then:

$\lim_{n\to\infty} (a_n / b_n) = L$

PROOF

Define:

$A_n = \sum_{i=1}^{n} a_i$
$B_n = \sum_{i=1}^{n} b_i$

Let $\varepsilon > 0$ and $\mu = \varepsilon / 2$.

By assumption, there exists $k \in N$ such that for all $n > k$:

$(L - \mu)b_n < a_n < (L + \mu)b_n$

Now split the sum:

$A_n = a_1 + a_2 + ... + a_k + a_{k+1} + ... + a_n$

So:

$A_k + (L - \mu)(b_{k+1} + ... + b_n)$
$<$
A_n
$<$
$A_k + (L + \mu)(b_{k+1} + ... + b_n)$

Hence:

$A_k + (L - \mu)(B_n - B_k)$
$<$
A_n
$<$
$A_k + (L + \mu)(B_n - B_k)$

Divide by B_n:

$A_k/B_n - (L - \mu)(B_k/B_n) + (L - \mu)$
$<$
A_n/B_n
$<$
$(L + \mu) + A_k/B_n - (L + \mu)(B_k/B_n)$

Fix k.

Since $B_n \to \infty$, we have:

$A_k/B_n \to 0$
and
$B_k/B_n \to 0$

Therefore:

$A_k/B_n - (L \pm \mu)(B_k/B_n) \to 0$

So for large n:

$|A_k/B_n - (L \pm \mu)(B_k/B_n)| < \mu$

Hence:

$L - 2\mu < A_n/B_n < L + 2\mu$

So:

$|A_n/B_n - L| < \varepsilon$

Therefore:

$\lim_{n\to\infty} (A_n / B_n) = L$

b) PROOF (EXISTENCE OF SQUARE ROOTS)

Let $r \in R$ and $r \geq 0$.

We want to show: there exists $s \in R$ such that $s^2 = r$.

Case 1: $r = 0$
Then $s = 0$ satisfies $s^2 = 0$.

Case 2: $r > 0$

Define the set:
$S = \{ x \in R \mid x^2 \leq r \}$

Step 1: S is nonempty
$0 \in S$ since $0^2 = 0 \leq r$.

Step 2: S is bounded above
If $x > r + 1$, then $x^2 > r$, so $x \notin S$.
Hence $r + 1$ is an upper bound of S.

Step 3: By completeness of R
S has a least upper bound.
Let $s = \sup S$.

Step 4: Show that $s^2 = r$

We consider three cases:

Case A: $s^2 > r$
Then we can find a smaller number than s which is still an upper bound of S.
This contradicts that s is the least upper bound.

So $s^2 > r$ is impossible.

Case B: $s^2 < r$
Then we can construct a number slightly bigger than s that is still in S.
This contradicts that s is an upper bound of S.

So $s^2 < r$ is impossible.

Step 5: Conclusion
Only possibility is:
$s^2 = r$

Therefore:
for every $r \geq 0$, there exists $s \in R$ such that $s^2 = r$.

QED

Assignment 2:

Hard real-time systems (for example, a nuclear power plant control system) have deadlines that the system must meet to avoid catastrophic failures such as loss of equipment or life.

Soft real-time systems (for example, a car's fuel-economy optimization system) have deadlines that the system can miss, but timeliness is still a desirable trait.

In real-time systems, computers have sensor input devices and control output devices.

The designer of a real-time computer system must know worst-case delays between the time an input device generates an interrupt and the time the device's driver can control the output device

to respond.

This worst-case analysis must take into account the delays the operating system introduces as well as the delays the application and device drivers impose.

Because Windows doesn't prioritize device IRQs in any controllable way and user-level applications execute only when a processor's IRQL is at passive level, Windows isn't always suitable as a real-time operating system.

The system's devices and device drivers—not Windows—ultimately determine the worst-case delay.

This factor becomes a problem when the real-time system's designer uses off-the-shelf hardware.

The designer can have difficulty determining how long every off-the-shelf device's ISR or DPC might take in the worst case.

Even after testing, the designer can't guarantee that a special case in a live system won't cause the system to miss an important deadline.

Furthermore, the sum of all the delays a system's DPCs and ISRs can introduce usually far exceeds the tolerance of a time-sensitive system.

Although many types of embedded systems (for example, printers and automotive computers) have real-time requirements, Windows Embedded Standard doesn't have real-time characteristics.It is simply a version of Windows XP that makes it possible, using system-designer technology that Microsoft licensed from VenturCom (formerly Ardence and now part of IntervalZero), to produce small-footprint versions of Windows XP suitable for running on devices with limited resources.

Source: Windows Internals by Mark Russinovich, page 104

Assignment 3 (Translate into Serbian)

:

MEMORIES OF MOMENTS, BRIGHT AS FALLING STARS
by Car Rambo

After eating, we helped wash dishes before heading to the library. We had to

wait a half hour for a terminal to free. Finally a man gathered his tablet and stood, stretching his shoulders.

“I’ll wait,” I said, and gestured Grizz forward.
She nodded and went forward to slide her hand into the log-in gloves. Within a moment, her eyes had the glassy stare that means the meat’s occupant is elsewhere.

I looked around. Chairs and desks dotted the place, all of them occupied. I went outside to the parking garage for a smoke.

Daylight had fled. At the structure’s edge, where the street was dimly visible, I panhandled a dozen people before I found one willing to admit to smoking. I lit the cigarette, a Marlboro Brute, and leaned back against the wall, which was patchworked with graffiti layers. Maybe by the time I was done, a booth would have opened up. It was getting late, after all.

I closed my eyes as the nicotine rush hit me. Footsteps came across the cement floor toward me. I opened my eyes.

It was Lorelei. She wore a slick, bright red jacket and lipstick to match her short skirt and chunky boots. Silver hoops all along each ear’s edge graduated to match her narrowing cartilage. She looked good. Very good.

“Nice night, ain’t it?” she said as she moved to lean on the wall beside me. “Gimme.”

I passed the smoke over and she took a drag.

“Want to try something to make the nice night even nicer?” she asked, smiling as she leaned back to return the cigarette.

“Meaning?”

“It’s good stuff.” She fished in the jacket before holding out the lighter and one-hitter. The end was packed with gray lintish dust. “Never had better.”

I took the pipe and sparked it. The blue smoke rushed into my lungs like a fist, like a physical jolt, and the world dropped half an inch beneath my feet. Everything was tinged with colors, an iridescence like gasoline on a rain puddle. I was standing there with Lorelei and at the same time I was on a vast dark plain, feeling the world teeter and slip.