THE FIRST ASSIGNMENT: Translate from English to Serbian an excerpt from the book *General Relativity: Basics and Beyond*, page 68, authored by Valeri P. Frolov and Andrei Zelnikov).

5.2.3 Cosmic Microwave Background Radiation

In building up our knowledge of the universe, we used several different kinds of observations in conjunction with certain theoretical models. One class of observations is the observations of structures in the universe i.e. (super) clusters of galaxies, voids, _laments etc. and their statistics. From the

summary of thermal history discussed above, all these correspond to matter

dominated era and our current observations go back to about z _ 10 (about

billion years after the big bang). Clearly, there is a huge range (essentially

in_nite) of red shift values that are still to be subjected to observations. One

of the crucial tool for these observations is the Cosmic Microwave Background

Radiation (CMBR) alluded to earlier.

According to the Big Bang model, the universe would have gone through an

epoch where protons, electrons, photons and neutral atoms (hydrogen) would

have been in equilibrium. After a drop of temperature to about 40000K, the

photons would decouple and stream freely carrying with them the information

at the decoupling epoch. These photons constitute the CMBR. Observe

that we cannot get a direct snapshot of period prior to decoupling by electromagnetic

observations since during the plasma epoch all prior information

would have been washed out. If we could observe the analogously predicted

neutrino background, then we could have a similar snapshot of a much earlier

epoch. But this is beyond our means. It turns out that the angular distribution

of CMBR photons contains a wealth of information allowing us to constrain

models of much earlier era. This is what we will discuss brie_y.

The CMBR was _rst predicted by George Gamow and his collaborators

in the late 40's when they were trying to obtain the abundance of chemical

elements via the hot big bang. Their prediction remained unnoticed since their

main goal of chemical abundances did not work out. It could not have worked out since we now know that except the very light nuclei, all others are produced in the interiors of stars where not only are the temperatures high but also the densities. The prediction of CMBR was e_ectively forgotten until it was discovered accidentally by Wilson and Penzias in 1965 [24]. Penzias and Wilson in fact were testing an antenna built to observe echo satellite and they observe a background `hiss' not attributable to any particular direction in the sky. They reported an equivalent temperature (at wave length of 7.35 cm) of 3:5_ 1_K. Its theoretical signi_cance (identi_cation with CMBR) was provided by Dicke, Peebles, Roll and Wilkinson [25]. This was of course observation at one frequency. Since then CMBR has been observed at wavelengths ranging from about 100 cm down to about 0.05 cm. The lower wavelengths are observed from balloon, rocket borne instruments and _nally from the COsmic Background Experiment satellite. These ranges cover both sides of the Planck distribution curve and the current value of the photon temperature is 2:725 _ 0:0010K

THE SECOND ASSIGNMENT:

THE SECOND ASSIGNMENT

IMPORTANT NOTE:

A) This assignment is based on an excerpt taken from the following source: https://en.wikipedia.org/wiki/Bounded_function

Students are required to **translate the text from English into Serbian**.

The task is purely a translation exercise. No rewriting, summarizing, or interpretation is required.

A function f defined on a set X with real or complex values is called bounded if the set of its values (its image) is bounded.

In other words, there exists a real number M such that:

$|f(x)| \leq M$

for all x in X.

A function that is not bounded is called unbounded.

If f is real-valued and:

$f(x) \leq A$ for all x in X,

then f is bounded above by A.

If:

$f(x) \geq B$ for all x in X,

then f is bounded below by B.

A real-valued function is bounded if and only if it is both bounded above and bounded below.

An important special case is a bounded sequence, where X is the set of natural numbers N.

A sequence $f = (a0, a1, a2, \ldots)$ is bounded if there exists a real number M such that:

$|an| \leq M$

for every natural number n.

The set of all bounded sequences forms the sequence space $l\infty$.

The definition of boundedness can be generalized to functions $f: X \to Y$ taking values in a more general space Y by requiring that the image f(X) is a bounded set in Y.

Related notions:

Weaker than boundedness is local boundedness.

A family of bounded functions may be uniformly bounded.

A bounded operator $T: X \to Y$ is not a bounded function in the usual sense (unless $T = 0$), but it preserves bounded sets:

If $M \subseteq X$ is bounded, then $T(M) \subseteq Y$ is also bounded.

This notion extends to general functions when X and Y have a notion of bounded sets.

Examples:

The sine function $\sin: R \to R$ is bounded since:
$|\sin(x)| \leq 1$ for all x in R.

The function $f(x) = 1 / (x^2 - 1)$, defined for all real x except $x = -1$ and $x = 1$, is unbounded because it grows without bound near those points.

The function $f(x) = 1 / (x^2 + 1)$, defined for all real x, is bounded since:
$|f(x)| \leq 1$ for all x.

The arctangent function $y = \arctan(x)$ is bounded with:
$-\pi/2 < y < \pi/2$.

Every continuous function on a closed interval [a, b] is bounded.

More generally, every continuous function from a compact space into a metric space is bounded.

Every entire complex function $f: C \to C$ is either unbounded or constant (Liouville's theorem).

In particular, sin(x) on complex numbers is unbounded.

The Dirichlet function (0 for rationals, 1 for irrationals) is bounded.

Continuous functions need not be bounded on non-compact domains, for example:

$g(x, y) = x + y$
$h(x, y) = 1 / (x + y)$

B)

assignmen 2: https://en.wikipedia.org/wiki/Periodic_function

A periodic function is a function that repeats its values at regular intervals. For example, the trigonometric functions, which are used to describe waves and other repeating phenomena, are periodic. Many aspects of the natural world have periodic behavior, such as the phases of the Moon, the swinging of a pendulum, and the beating of a heart.

The length of the interval over which a periodic function repeats is called its period. Any function that is not periodic is called aperiodic.

Definition

A graph of the sine function. It is periodic with a fundamental period of 2π.

A function is defined as periodic if its values repeat at regular intervals. For example, the positions of the hands on a clock display periodic behavior as they cycle through the same positions every 12 hours. This repeating interval is known as the period.

More formally, a function f is periodic if there exists a constant P such that

$f(x + P) = f(x)$

for all values of x in the domain. A nonzero constant P for which this condition holds is called a period of the function.

If a period P exists, any integer multiple nP (for a positive integer n) is also a period. If there is a least positive period, it is called the fundamental period (also primitive period or basic period). Often, "the" period of a function is used to refer to its fundamental period.

Geometrically, a periodic function's graph exhibits translational symmetry. Its graph is invariant under translation in the x-direction by a distance of P. This implies that the entire graph can be formed from copies of one particular portion, repeated at regular intervals.

Examples

Periodic behavior can be illustrated through both common, everyday examples and more formal mathematical functions.

Real-valued functions

Functions that map real numbers to real numbers can display periodicity, which is often visualized on a graph.

Sawtooth wave

An example is the function f that represents the "fractional part" of its argument. Its period is 1. For instance,

$$f(0.5) = f(1.5) = f(2.5) = \ldots = 0.5$$

The graph of the function f is a sawtooth wave.

Trigonometric functions

A plot of $f(x) = \sin(x)$ and $g(x) = \cos(x)$; both functions are periodic with period 2π.

The trigonometric functions are common examples of periodic functions. The sine function and cosine function are periodic with a fundamental period of 2π, as illustrated in the figure to the right. For the sine function, this is expressed as:

$$\sin(x + 2\pi) = \sin x$$

for all values of x.

The field of Fourier series investigates the concept that an arbitrary periodic function can be expressed as a sum of trigonometric functions with matching periods.

Exotic functions

Some functions are periodic but possess properties that make them less intuitive. The Dirichlet function, for example, is periodic, with any nonzero rational number serving as a period. However, it does not possess a fundamental period.

Complex-valued functions

Functions with a domain in the complex numbers can exhibit more complex periodic properties.

Complex exponential

The complex exponential function is a periodic function with a purely imaginary period:

$$e^{ikx} = \cos(kx) + i \sin(kx)$$

Given that the cosine and sine functions are both periodic with period 2π, Euler's formula demonstrates that the complex exponential function has a period L such that

$$L = 2\pi / k$$

Double-periodic functions

A function on the complex plane can have two distinct, incommensurate periods without being a constant function. The elliptic functions are a primary example of such functions. ("Incommensurate" in this context refers to periods that are not real multiples of each other.)

Properties

Periodic functions can take on values many times. More specifically, if a function f is periodic with period P, then for all x in the domain of f and all positive integers n,

$$f(x + nP) = f(x)$$

A significant property related to integration is that if $f(x)$ is an integrable periodic function with period P, then its definite integral over any interval of length P is the same. That is, for any real number a:

$$\int_{a}^{a+P} f(x)\, dx = \int_{0}^{P} f(x)\, dx$$

If $f(x)$ is a function with period P, then $f(ax)$, where a is a non-zero real number such that ax is within the domain of f, is periodic with period $P / |a|$. For example, $f(x) = \sin(x)$ has period 2π and therefore $\sin(5x)$ will have period $2\pi / 5$.

A key property of many periodic functions is that they can be described by a Fourier series. This series represents a periodic function as a sum of simpler periodic functions, namely sines and cosines. For example, a sound wave from a musical instrument can be broken down into the fundamental note and various overtones. This decomposition is a powerful tool in fields like physics and signal processing. While most "well-behaved" periodic functions can be represented this way, Fourier series can only be used for periodic functions or for functions defined on a finite length.

Any function that is a combination of periodic functions with the same period is also periodic (though its fundamental period may be smaller). This includes:

addition, subtraction, multiplication and division of periodic functions,
taking a power or a root of a periodic function (provided it is defined for all x)

Generalizations

The concept of periodicity can be generalized beyond functions on the real number line. For example, the idea of a repeating pattern can be applied to shapes in multiple dimensions, such as a periodic tessellation of the plane. A sequence can also be viewed as a function defined on the natural numbers, and the concept of a periodic sequence is defined accordingly.

Antiperiodic functions

One subset of periodic functions is that of antiperiodic functions. This is a function f such that

$$f(x + P) = -f(x)$$

for all x. For example, the sine and cosine functions are π-antiperiodic and 2π-periodic. While a P-antiperiodic function is a $2P$-periodic function, the converse is not necessarily true.

Bloch-periodic functions

A further generalization appears in the context of Bloch's theorems and Floquet theory, which govern the solution of various periodic differential equations. In this context, the solution (in one dimension) is typically a function of the form

$$f(x + P) = e^{ikP} f(x),$$

where k is a real or complex number (the Bloch wavevector or Floquet exponent). Functions of this form are sometimes called Bloch-periodic in this context. A periodic function is the special case $k = 0$, and an antiperiodic function is the special case $k = \pi / P$. Whenever kP / π is rational, the function is also periodic.

Quotient spaces as domain

In signal processing you encounter the problem that Fourier series represent periodic functions and that Fourier series satisfy convolution theorems (i.e. convolution of Fourier series corresponds to multiplication of represented periodic function and vice versa), but periodic functions cannot be convolved with the usual definition, since the involved integrals diverge. A possible way out is to define a periodic function on a bounded but periodic domain. To this end you can use the notion of a quotient space:

$$R / Z = \{x + Z : x \in R\} = \{\{y : y \in R \wedge y - x \in Z\} : x \in R\}.$$

That is, each element in R / Z is an equivalence class of real numbers that share the same fractional part. Thus a function like $f : R / Z \to R$ is a representation of a 1-periodic function.

Calculating period

Consider a real waveform consisting of superimposed frequencies, expressed in a set as ratios to a fundamental frequency, f: $F = 1/f\ [f1\ f2\ f3\ \ldots\ fN]$ where all non-zero elements ≥ 1 and at least one of the elements of the set is 1. To find the period, T, first find the least common denominator of all the elements in the set. Period can be found as $T = LCD / f$. Consider that for a simple sinusoid, $T = 1/f$. Therefore, the LCD can be seen as a periodicity multiplier.

For set representing all notes of Western major scale: [1 9/8 5/4 4/3 3/2 5/3 15/8] the LCD is 24 therefore $T = 24/f$.

For set representing all notes of a major triad: [1 5/4 3/2] the LCD is 4 therefore $T = 4/f$.

For set representing all notes of a minor triad: [1 6/5 3/2] the LCD is 10 therefore $T = 10/f$.

If no least common denominator exists, for instance if one of the above elements were irrational, then the wave would not be periodic.

THE THIRD ASSIGNMENT: Translate from English to Serbian an excerpt from the story *"Memories of Moments, Bright as Falling Stars"* by Cat Rambo

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IMPORTANT NOTE:
This is an excerpt from a literary story. The task is to translate the text from English to Serbian while preserving tone, narrative voice, and informal conversational style.

MEMORIES OF MOMENTS, BRIGHT AS FALLING STARS

by Cat Rambo

By Cat Rambo I was surprised; I'd never heard Ajah make anyone an offer like that.

"The Exams are your big chance. Get a good night's sleep and make the most of them. Face them fully charged."

I rolled my eyes. "For what? Like there's a chance." But he and Grizz ignored me.

"We need to make a library run still," she said.

"Yeah, yeah, that's fine. I'm up till midnight, maybe later," Ajah told her.

Despite my doubts, relief seeped into my bones. We'd been given a night's respite, and who knew what would happen after the Exams? "Thanks, Ajah," I said, and he grunted acknowledgement as he slid a plate before me.

The portabella bits had been browned in curry powder and oil, and the eggs were fresh and good. Grizz ate methodically, scraping her plate free, but she looked up to catch my eye and gave me a heartfelt smile, rare on her square-set face.

As her gaze swung back to her plate, my glance tangled with Lorelei's. I could not read her expression.

Lorelei and I used to pal around before Grizz and I met up. She and I grew up next to each other, and it's hard not to know someone intimately when you've shared hour after hour channel surfing while one mother or the other went out on work or errands. We suffered through the same street bullies and uninterested teachers. She was the first girl I ever kissed. You don't forget that.

But I knew I wanted Grizz for keeps the first moment I saw her. She came swaggering into the shelter wearing a rabbit-fur jacket and pseudo-leather pants. She'd been tricking in a swank bar, but then someone snatched all her hard-earned cash. So there she was, with a bruise on her face and a cracked wrist, but still holding herself hard and arrogant, and the only person in the world who could glimpse the softness underneath was me, it seemed like. So I sauntered up, invited her outside for a smoke, and then within a half hour, we were pressed against the wall together, my hands up her shirt like I'd never touched her before, feeling her firm little nipples against the skin of my palms.