

ASSIGNMENT 1

Translate the following text from English to Serbian.

Excerpt from page 57 of the book:
General Relativity: Basics and Beyond – Relativistic Cosmology

Relativistic Cosmology

Let us now leave the context of compact, isolated bodies and the spacetimes in their vicinity and turn our attention to the space-time appropriate to the whole universe. We can make no progress by piecing together space-times of individual compact objects such as stars, galaxies etc., since we will have to know all of them! Instead we want to look at the universe at the largest scale. Since our observations are necessarily finite (that there are other galaxies was discovered only about 90 years ago!), we have to make certain assumptions and explore their implications. These assumptions go under the lofty names of 'cosmological principles'.

One fact that we do know with reasonable assurance is that the universe is 'isotropic on a large scale'. What this means is the following. If we observe our solar system from any planet, then we do notice its structure, namely, other planets. If we observe the same from the nearest star (alpha centauri, about 4 light years), we will just notice the Sun. Likewise if we observe distant galaxies, they appear as structureless point sources (which is why it took so long to discover them). If we look still farther away then even clusters of galaxies appear as points. We can plot such sources at distances in excess of about a couple of hundred megaparsecs on the celestial sphere. What one observes is that the sources are to a great extent distributed uniformly in all directions. We summarize this by saying that the universe on the large scale is isotropic about us. We appear to occupy a special vantage point! One may accept this as a fact and ponder about what is special about our position and why we occupy it. An alternative is to reject the idea that there is anything special about our location in the universe and propose instead that the universe must be isotropic from all locations (clusters of galaxies). Since the universe appears isotropic to us at present, we assume that the same must be true for other observers elsewhere, i.e. there is a common 'present' at which isotropic picture holds for all observers. Denial of privileged position also amounts to assuming that the universe is spatially homogeneous, i.e. at each instant there is a spatial hypersurface (space at time t) on which all points are equivalent. Isotropy about each point means that there must be observers (timelike world line) who will not be able to detect any distinguished direction. The statement that on large scale the universe is spatially homogeneous and isotropic is called the cosmological principle. The so-called standard cosmology is based on spatial homogeneity and isotropy and this is what is discussed below.

In order to arrive at a suitable form of the metric, we need to characterize precisely what is meant by spatially homogeneous and isotropic in the context of geometry. The first task is to be able to identify a spatial slicing of the space-time. This is achieved by stipulating that there exists a one-parameter family of spacelike hypersurfaces, Σ_t , foliating the space-time. A space-time is said to be spatially homogeneous if there is a transitive action of a group of isometries on each of the spatial slices. Here, transitive action means that given any two points on a Σ_t , there is a diffeomorphism of Σ_t onto itself. This being an isometry

means that the metric remains the same. There can be more than one such isometries connecting two points.

Isotropy is a stipulation associated with observers. Let $x^\mu(t)$ be a timelike curve representing worldline of an observer. The observer is said to be an isotropic observer if at any point $p \in x^\mu(t)$ and for a pair of spacelike tangent vectors in the tangent space at p , there exists an isometry which leaves p and the tangent vector $u^\mu := dx^\mu/dt|_p$ unchanged but maps one direction to the other. A space-time is said to be isotropic at every point if there exists a space-time filling congruence of isotropic observers, i.e. a timelike vector field u^μ whose integral curves represent isotropic observers, variously called cosmic observers or fundamental observers.

Isotropy implies that the vector field must be orthogonal to surfaces of homogeneity. For if it were not, its projection on the tangent space to Σ_τ would give a distinguished direction which is disallowed by isotropy. If there are more than one family of hypersurfaces of homogeneity, then isotropy implies that at least one of these must be orthogonal to the vector field. Note that isotropy at each point does not imply or require spatial homogeneity, nor does spatial homogeneity imply isotropy. However, if we have both of these, then the isotropy vector field is orthogonal to the surfaces of homogeneity. We can choose the label τ of the family of hypersurfaces as a time coordinate and, given any choice of spatial coordinates x^i on a Σ_{τ_0} , carry them along the world lines of the isotropic observers.

Assignment 2: BINOMIAL THEOREM

Statement:

$$(a + b)^n = \sum_{k=0}^n C(n,k) a^{n-k} b^k$$

where:

$$C(n,k) = n! / (k!(n-k)!)$$

Meaning:

The coefficients in the expansion of $(a+b)^n$ are binomial coefficients (Pascal triangle row n).

Example:

$$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

PROOF IDEA 1 (combinatorial)
 $(a+b)^n = (a+b)(a+b)\dots(a+b)$ [n times]

Each term $a^m b^{(n-m)}$:

- choose m factors where we pick "a"

- remaining $n-m$ factors give “b”

Number of ways:
 $C(n,m)$

So coefficient = $C(n,m)$

Thus:

$$(a+b)^n = \sum_{m=0}^n C(n,m) a^m b^{(n-m)}$$

PROOF IDEA 2 (induction)

Base:

$$(a+b)^1 = a + b$$

Assume true for n :

$$(a+b)^n = \sum_{k=0}^n C(n,k) a^{(n-k)} b^k$$

Multiply by $(a+b)$:

$$(a+b)^{(n+1)} = (a+b)^n (a+b)$$

Expand and group terms \rightarrow use identity:

$$C(n,k) + C(n,k-1) = C(n+1,k)$$

Result:

$$(a+b)^{(n+1)} = \sum_{k=0}^{n+1} C(n+1,k) a^{(n+1-k)} b^k$$

PROOF IDEA 3 (via exponentials)

$$e^{(a+b)} = e^a * e^b$$

Use series:

$$e^x = \sum_{n=0}^{\infty} x^n / n!$$

Compare coefficients:

$$(a+b)^n / \sum_{k=0}^n (a^k/k!) (b^{(n-k)}/(n-k)!) =$$

Multiply by $n!$:

$$(a+b)^n = \sum_{k=0}^n C(n,k) a^k b^{(n-k)}$$

GENERALIZATION (Newton binomial)

$$(a+b)^r = \sum_{k=0}^{\infty} C(r,k) a^{(r-k)} b^k$$

where:

$$C(r,k) = r(r-1)...(r-k+1) / k!$$

COMMON

USE

Example:

$$\begin{aligned} & x^5 + 4x^4 + 6x^3 + 4x^2 + x \\ = & x(x^4 + 4x^3 + 6x^2 + 4x + 1) \\ = & x(x+1)^4 \end{aligned}$$

(the

source:

https://artofproblemsolving.com/wiki/index.php/Binomial_Theorem?srltid=AfmBOoqP6ti6jY5XLmQrT-XXEIX0gCPY4_InHdoP74H41dyxcnUCMW3h .

CERPT — Bruce Sterling

(From the storu „User-centric: Design Team”)
Bruce Sterling

ORIGINAL (ENGLISH)

User-centric: “Design Team”
[Engineer, Graphic Designer, Legal Expert, Marketer, Programmer, Social Anthropologist & Team Coordinator]

Subject: New Product Brainstorm

Another new product launch. Well, we all know what that means. Nobody ever said that they’re easy. But I do believe the seven of us—given our unique backgrounds and our proven skills—are just the people to turn things around for this company.

Things aren’t as bad as the last quarterly report makes them look. Despite what the shareholders may think, we’ve definitely bottomed out from that ultrasonic cleanser debacle. Sales in muscle-gel apps remain strong. Plus, the buzz on our new product category just couldn’t be hotter.

People across our industry agree that locator tag microtechnology is a killer app in the intelligent-environment market. MEMS tech is finally out of the lab and bursting into the marketplace, and our cross-licenses and patents look very solid.

As for the development budget—well, this is the biggest new product budget I’ve seen in eight years with this company.

My point is, we’ve got to get away from our old-fashioned emphasis on “technology for tech’s sake.” That approach is killing us in the modern marketplace. Yes, of course MEMS locator chips are a “hot, sweet” technology—and yes, “If you build it, they will come.” Our problem is, we do build it, and they do come, but they *give all the money to somebody else*.

We can’t live on our reputation as a cutting-edge engineering outfit. Design awards just don’t pay the bills. That’s not what our shareholders want, and it’s not what the new management wants. No matter how we may grumble, this company has got to be competitive in the real world

From: Design Engineer
To: Design Team
Subject: Re: New Product Brainstorm

FYI, User specs: Classic early adapter type. Male. Technically proficient. 18–35 age demographic. NAFTA/Europe. Owns lots of trackable, high-value-added, mobile hardware products: sporting goods, laptops, bicycles, luggage, possibly several cars.

From: Marketer
To: Design Team
Subject: User Specs

I just read the Engineer's e-mail, and gee whiz, people. That is dullsville. That is marketing poison. Do you have any idea how burned out the Male- Early-Adapter thing is in today's competitive environment? These guys have digital toothbrushes now. They're nerd-burned, they've been consumer-carpet-bombed! There's nothing left of their demographic! They're hiding in blacked-out closets hoping their shoes will stop paging their belt buckles. Nerds can't push this product into the high-volume category that we need for a breakeven. We need a housekeeping technology. I mean ultra-high volume, in the realm of soaps, mops, brooms, scrubbing brushes, latex gloves, light bulbs. An impulse buy, but high-margin and everywhere.