HOMEWORK – TRANSLATION OF TECHNICAL TEXT

Date: 13/5/2026
Topic: General relativity

Section 1.

HOMEWORK 13 – TECHNICAL TRANSLATION (NOTEPAD FORMAT)
SOURCE: General Relativity: Basics and Beyond
Authors: unknown in excerpt (standard reference text in GR literature)
Pages: 44–45

---
5. Elementary Properties and Peculiarities
---

(1)
The coefficient in front of T_ab is about 2 × 10^-48 cm^-1·gm^-1·sec^2.
From cosmology, the estimate of the possible cosmological constant Λ is about 10^-56 cm^-2.
So although the strict Newtonian limit would rule out Λ, Newtonian gravity itself is not tested to the extent of detecting the presence of Λ.
Thus, logically, the Λ term is admissible.

In fact, exactly the same logic can be applied to seek more general field equations.
Our second requirement was based on the form of the Newtonian limit and simplicity.
Simplicity is a matter of taste, and the level of accuracy of Newtonian gravity could permit higher derivatives of the metric and hence more general equations that could nonetheless show the same Newtonian limit.

In this sense, to propose the above equation as “the” equation governing determination of spacetime metric is a postulation and not a “derivation”.

---

(2)
The derivation above follows Weinberg [2].
There are other alternative heuristic derivations of the Einstein equations.

One is based on the comparison of “tidal forces” in the context of geometry.
In the Newtonian picture, tidal forces imply relative acceleration between two nearby bodies moving in the same inhomogeneous gravitational field.
This is given by gradients of the force, or double derivatives of the potential.

In the geometrical context, one represents free fall of nearby bodies by two neighboring geodesics and obtains an expression for their relative motion in terms of the Riemann tensor.
Identifying these expressions and referring to the Poisson equation leads to the attempt:
$R_{ab} = 4\pi G/c^4 T_{ab}$

This was in fact the first equation considered by Einstein.
However, the contracted Bianchi identity implies that the trace of T_{ab} must be constant, which is an unphysical restriction on matter.
The correction is replacing the Ricci tensor by the Einstein tensor.

This still retains the identification of tidal accelerations with geodesic deviation for non-relativistic sources of Newtonian gravity.
Details may be found in Wald [17].
Weinberg [2] also provides another derivation.
We now accept the Einstein equations as a law of nature and proceed to their implications.

---

(3)
Mathematically, the Einstein tensor involves second derivatives of the metric.
The equations therefore form a system of 10 nonlinear partial differential equations for 10 unknown functions g_ab(x^c).

However, the equations are not independent.
They satisfy 4 differential identities implied by the contracted Bianchi identities.
There is also freedom to perform arbitrary coordinate transformations.

To specify a solution, one must fix coordinates either explicitly or via coordinate conditions.
In this respect, the equations are similar to Maxwell equations for the gauge potential.

Being partial differential equations, they are local.
Solutions therefore allow extension and matching across different regions.
Examples appear in the Schwarzschild solution.

---

(4)
On the gravitational side, the equations involve only the Ricci tensor and Ricci scalar, not the full Riemann tensor.
On the matter side, only T_ab is used, not full microscopic structure of matter.

For example, a perfect fluid can be composed of different microscopic constituents, but its stress-energy tensor has the same form.
Different fluids are distinguished only by equations of state.

For a photon gas, one uses T_ab in terms of pressure and density without reference to Maxwell fields.

Even if T_ab = 0 in a region, the geometry is only Ricci-flat, not necessarily Riemann-flat.

Vacuum spacetime is not necessarily Minkowski spacetime.

This allows non-flat geometries near bodies even outside matter distributions.

Remark:
In n dimensions, the Riemann tensor has (1/12)n^2(n^2 − 1) independent components.
For n = 2 this is 1 (Ricci scalar).
For n = 3 it is 6 (Ricci tensor fully determines curvature).
For n ≥ 4, Ricci-flat does not imply Riemann-flat.

---

(5)
Newtonian gravity is described by a single potential satisfying a time-independent Poisson equation.
Time-dependent gravitational fields arise only through time-varying matter density.

In Einstein's theory, gravity is dynamic.
Even in vacuum, propagating gravitational waves exist.
These have been indirectly observed via binary pulsars, while direct detection was achieved later in experiments.

SECTION 2
TOPIC: SQUEEZING THEOREM (SANDWICH THEOREM)
DATE: _____

PROOF (SQUEEZING THEOREM)

Let f, g, and h be three functions with the same domain, and let a be an accumulation point of that domain.

Assume that for all x in that domain:
g(x) <= f(x) <= h(x)

and that:
lim (x -> a) g(x) = lim (x -> a) h(x) = L

Then:
lim (x -> a) f(x) = L

PROOF STEPS:

• Assume that a is an accumulation point of the common domain of f, g, and h.

• Assume:
lim (x -> a) g(x) = L
lim (x -> a) h(x) = L

• Assume:
g(x) <= f(x) <= h(x) for all x in the common domain.

• Let epsilon > 0 be given.

• Then there exists delta1 > 0 such that:
for all x with 0 < |x - a| < delta1,
|g(x) - L| < epsilon

• There exists delta2 > 0 such that:
for all x with 0 < |x - a| < delta2,
|h(x) - L| < epsilon

• Let delta = min(delta1, delta2).

• Then for all x with 0 < |x - a| < delta:
|g(x) - L| < epsilon
|h(x) - L| < epsilon

• Since:
g(x) <= f(x) <= h(x)

it follows that:
-L - epsilon < g(x) - L <= f(x) - L <= h(x) - L < epsilon

• Therefore:
|f(x) - L| < epsilon

• Hence:
lim (x -> a) f(x) = L

This proof is taken from Jonathan Kane, *Writing Proofs in Analysis* (2010).

Translation: English → Serbian
Source: Kim Stanley Robinson – *Down and Out in the Year 2000*

TEXT FOR TRANSLATION

Lee shook his head distastefully at the idea.

“Delmont, you know any weed I can buy? I need a finger baggie for twenty.”

“Not so easy to do, Robbie.” Delmont hemmed and hawed and they dickered for a while, then he sent Lee over to Jim Johnson, who made the sale under a cheery exchange of the day’s news, over by the chess tables.

After that Lee bought a pack of cigarettes in a liquor store and went up to the little triangular park between 17th, S, and New Hampshire, where no police or strangers ever came. They called it Fish Park for the incongruous cement whale sitting by one of the trash cans.

He sat down on the long broken bench, among his acquaintances who were hanging out there, and fended them off while he carefully emptied the Marlboros, cut some tobacco into the weed, and refilled the cigarette papers with the new mix. With their ends twisted he had a dozen more joints.

They smoked one and he sold two more for a dollar each before he got out of the park.

But he was still anxious, and since it was the hottest part of the day and few people were about, he decided to visit his plants. He knew it would be at least a week till harvest, but he wanted to see them. Anyway, it was about watering day.

East between 16th and 15th he hit no-man's-land. The mixed neighborhood of fortress apartments and burned-out hulks gave way to a block or two of entirely abandoned buildings.

Here the police had been at work, and looters had finished the job. The buildings were battered and burnt out, their ground floors blasted wide open, some of them collapsed entirely into heaps of rubble.

No one walked the broken sidewalk; sirens a few blocks off and the distant hum of traffic were the only signs that the whole city wasn't just like this.

Little jumps in the corner of his eye were no more than that; nothing there when he looked directly.

The first time, Lee had found walking down the abandoned street nerve-racking; now he was reassured by the silence, the stillness, the no-man's-land smell of torn asphalt and wet charcoal, the wavering streetscape empty under a sour-milk sky.

INSTRUCTIONS FOR STUDENTS

- Translate the text into Serbian.
- Maintain narrative tone and informal spoken dialogue where appropriate.
- Preserve names and street references as given.
- Pay attention to colloquial expressions and urban slang.

- Do not omit any part of the text.
- Ensure consistency in tense and narrative voice.