A technical text - astrophysics students

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Introduction to the Basic Concept of Modern Physics
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MAXWELL EQUATIONS, ETHER, MICHELSON–MORLEY EXPERIMENT, AND LORENTZ TRANSFORMATIONS

Maxwell equations in vacuum describe the propagation of electromagnetic signals with speed:

c = 1 / sqrt(mu0 * epsilon0)

According to the Galilean relativity principle, velocities must be added like vectors when going from one inertial frame to another.

Therefore, if a luminous signal has velocity c in one frame O, and O moves relative to another frame O', then the measured velocity of light in O' should be different.

This would imply:

- Maxwell equations are valid in O
- Maxwell equations are not valid in a generic inertial frame O'

This created a paradox.

ETHER HYPOTHESIS

In the nineteenth century, the natural solution seemed to be the existence of a medium called ether.

By analogy with elastic waves:

- sound needs air
- water waves need water
- light was assumed to need ether

Electromagnetic waves were interpreted as deformations of an extremely rigid and rare medium called ether.

This created a new problem:

Which reference frame is at rest with respect to ether

A technical text – computer science.

EARTH'S MOTION THROUGH ETHER

Earth moves in orbit with velocity approximately:

$v \approx 10^{-4} c$

To detect this motion experimentally, one would need precision of at least:

1 / 10000

Michelson and Morley reached this precision using light interference.

ELECTROSTATIC FORCE PROBLEM

Two charged particles at rest relative to each other obey Coulomb's law.

For equal charges:

Force is repulsive.

But in a moving frame, each charge also produces a magnetic field.

Then Lorentz force must be included.

If velocity is perpendicular to separation distance, total force is reduced by factor:

$(1 - v^2 / c^2)$

This would produce different accelerations in different inertial frames.

That contradicts Galilean relativity.

MICHELSON–MORLEY EXPERIMENT

A two-arm interferometer is used.

Light source L sends a beam.

Half-silvered mirror S splits beam into two parts.

The beams travel along arms 1 and 2, reflect back, recombine, and interfere.

Observed phase shift is proportional to time difference:

Delta T

between travel times of the two beams.

If arm lengths are equal:

l

and light speed is same in both directions:

c

then:

$\Delta T = 0$

and constructive interference occurs.

FIRST ARM (PERPENDICULAR MOTION)

If apparatus moves through ether with velocity:

v

parallel to second arm, then for first beam:

$$c^2 T^2 = v^2 T^2 + 4 l^2 \quad (1.1)$$

Hence:

$$T = \frac{2 l / c}{\sqrt{1 - v^2 / c^2}} \quad (1.2)$$

SECOND ARM (PARALLEL MOTION)

Forward time:

$$t_1 = l / (c - v)$$

Return time:

$$t_2 = l / (c + v) \quad (1.3)$$

Total time:

$$T' = t_1 + t_2$$

$$T' = \frac{2 l / c}{1 - v^2 / c^2}$$

$$T' = \frac{T}{\sqrt{1 - v^2 / c^2}} \quad (1.4)$$

TIME DIFFERENCE

For small v/c:

$\Delta T = T' - T$

$\Delta T \approx (T v^2) / (2 c^2)$

$\Delta T \approx (l v^2) / c^3$ (1.5)

Therefore apparatus should detect Earth's motion through ether.

NUMERICAL SENSITIVITY

Assume detectable time difference:

$\delta T \approx 5 * 10^{-17}$ s

Take:

$l = 2$ m

Then:

$l / c \approx 0.6 * 10^{-8}$ s

Sensitivity becomes:

$\delta v / c = \sqrt{\delta T * c / l}$

$\delta v / c \approx 10^{-4}$

Thus detectable velocity:

$v \approx 3 * 10^4$ m/s

This is approximately Earth's orbital speed.

But repeated experiments showed no ether wind.

Conclusion:

Ether does not exist.

FAILURE OF GALILEAN TRANSFORMATIONS

Galilean transformations:

$t' = t$

$x' = x - v t$ (1.6)

These are inadequate.

They must be replaced by new linear laws preserving speed of light.

A light signal emitted at origin satisfies:

$x = c\,t$

It must transform into:

$x' = c\,t'$

DERIVATION OF NEW TRANSFORMATIONS

Assume:

$x' = A(x - v\,t)$ (1.7)

By symmetry:

$x = A(x' + v\,t')$ (1.8)

Combining gives:

$t' = A\,[\,t - (x / v)(1 - 1/A^2)\,]$ (1.9)

A must be positive.

IMPOSE INVARIANCE OF LIGHT SPEED

Take:

$x = c\,t$

and require:

$x' = c\,t'$

Then:

$x' = c\,A\,t\,(1 - v/c)$

$t' = A\,t\,[1 - (c/v)(1 - 1/A^2)]$ (1.10)

Imposing:

$x' = c\,t'$

gives:

$1 - (c/v)(1 - 1/A^2) = 1 - v/c$ (1.11)

Hence:

$A = 1 / \sqrt{1 - v^2 / c^2}$ (1.12)

LORENTZ TRANSFORMATIONS

$x' = (x - v t) / \sqrt{1 - v^2 / c^2}$

$t' = (t - v x / c^2) / \sqrt{1 - v^2 / c^2}$

FINAL RESULT

The failure to detect ether and the invariance of light speed led Einstein to Special Relativity.

Space and time are not absolute.

They transform according to Lorentz, not Galileo

COMPACTNESS AND THE HEINE–BOREL THEOREM
Strana 115

4.4.3 UNIFORM CONTINUITY ON CLOSED BOUNDED INTERVALS

Using the Heine–Borel Theorem, it can be shown that every continuous function on a closed bounded interval is uniformly continuous on that interval.

The basic idea is the following:

If f is continuous on the closed bounded interval [a, b], then for every $\varepsilon > 0$ and for every point x in [a, b], there exists a $\delta > 0$ such that whenever y belongs to the interval:

$(x - \delta, x + \delta)$

it follows that:

$|f(x) - f(y)| < \varepsilon$

Thus, around every point x in [a, b], there exists an open interval having the desired continuity property.

The Heine–Borel Theorem states that the closed bounded interval [a, b] is compact.

Therefore, every open cover of [a, b] has a finite subcover.

This means that the infinitely many open intervals centered at each point x can be reduced to finitely many intervals covering [a, b].

Since each of these finitely many intervals has an associated positive δ, one may try to choose the smallest δ and use it in proving uniform continuity.

FIRST SUBTLETY

There is a complication.

From continuity at x, one only knows that:

$|f(y) - f(x)| < \varepsilon$

for y near x.

However, uniform continuity requires showing:

$|f(y) - f(z)| < \varepsilon$

for any two nearby points y and z.

So the estimate must compare arbitrary nearby points, not only points with x.

STANDARD ANALYSIS TRICK

To solve this, instead of using ε, use:

$\varepsilon / 2$

Because continuity works for every positive number, it also works for ε/2.

So choose $\delta > 0$ such that:

$|f(y) - f(x)| < \varepsilon/2$

whenever y lies in:

$(x - \delta, x + \delta)$

Then if both y and z are in that interval, the triangle inequality gives:

$|f(y) - f(z)| \leq |f(y) - f(x)| + |f(z) - f(x)|$

Hence:

$|f(y) - f(z)| < \varepsilon/2 + \varepsilon/2 = \varepsilon$

So the first difficulty is resolved.

SECOND SUBTLETY

Another problem remains.

If y and z are close together, how do we know they both lie in the same interval of the finite open cover?

Even though [a, b] is covered by finitely many open intervals, two nearby numbers do not automatically belong to the same one.

SOLUTION USING ENDPOINTS

Consider all endpoints of the finitely many open intervals in the finite cover.

Because the cover is finite, there are only finitely many such endpoints.

Choose δ not as the minimum of the interval radii, but as the smallest distance between any two distinct endpoints.

Then if:

$|y - z| < \delta$

there can be at most one endpoint between y and z.

This guarantees that y and z must both lie inside one of the covering intervals.

WHY THIS WORKS

Intervals in an open cover must overlap sufficiently to cover the whole closed interval [a, b].

Therefore, an endpoint of one interval must lie inside another interval.

So if y and z are very close and straddle at most one endpoint, they still remain together inside some open interval of the finite cover.

CONCLUSION

Using compactness from the Heine–Borel Theorem:

- every point has a local continuity neighborhood
- finitely many such neighborhoods cover [a, b]
- one common δ can be chosen
- therefore continuity becomes uniform continuity

FINAL RESULT

Every continuous function on a closed bounded interval [a, b] is uniformly continuous on that interval.

The material is taken from Jonathan M. Kane, Writing Proofs in Analysis, pages 115–116.

EXCERPT FROM THE STORY
DOWN AND OUT IN THE YEAR 2000
Page 98

By Kim Stanley Robinson

From the book Cyberpunk
Edited by Victoria Blake

DIALOGUE IN A COLLAPSING URBAN LANDSCAPE

“What you doing here?”

“Sitting!”

The man was startled and nervous.

“Just sitting in a park!”

“This ain’t no park, man.

This is our front yard.

You see any front yard to these apartment buildings here?

No.

This here is our front yard, and we don’t like people just coming into it and sitting down anywhere!”

The man stood and walked away.

He looked back once.

His expression was angry and frightened.

The other man sitting on the park benches looked at Lee curiously.

TWO DAYS LATER

Lee was nearly out of money.

He walked over to Connecticut Avenue, where his old friend Victor played harmonica for coins whenever he could not find other work.

Today Victor was there, performing:

Amazing Grace

He stopped when he saw Lee.

“Robbie! What’s happening?”

“Not much. You?”

Victor gestured toward his empty hat lying on the sidewalk before him.

“You see it.

Don’t even have seed coin for the cap, man.”

“So you ain’t been getting any gardening work lately?”

“No, no. Not lately.

I do all right here, though.

People still pay for music, man, some of them.

Music’s the angle.”

MEMORY OF FORMER WORK

Victor looked at Lee, his face tightened against the sun.

They had once worked together for the park service.

During summer mornings they drove a truck through city streets.

They stopped at every tree.

One of them would hoist the other upward in slings.

The man raised into the branches had to balance like an acrobat.

He moved carefully while cutting every branch below twelve feet.

The chainsaw had to be handled precisely to avoid cutting legs or causing injury.

Those had been good times.

But now the park service was gone.

Victor sat behind an empty hat and looked at Lee with stoic resignation.

NATURE RETURNS

“Do you ever look up at the trees anymore, Robbie?”

“Not much.”

“I do.

They’re growing wild, man.

Every summer they go like crazy.

Pretty soon people are gonna have to drive their cars through the branches.

The streets’ll be tunnels.

And with half the buildings in this area falling down...

I like the idea that the forest is taking this city back again.

Running over it like kudzu.

Till maybe it just be forest again at last