ASSIGNMENT 1: Translate the following text from English to Serbian. Internals Class Association

vo:
Student je dužan da neizmenjen tekst ispod prevede sa engleskog na srpski jezik.

Izvor:
Pasus preuzet iz knjige *General Relativity: Basics and Beyond*, Chapter 12, str. 185–186.

ZADATAK:
Prevesti sledeći engleski tekst na srpski jezik.

TEKST ZA PREVOĐENJE (NEIZMENJEN)

Numerical Relativity

Einstein equation is a complicated system of partial differential equations and even though it can be cast as an initial value problem, obtaining solutions for physically realistic situations is a very tall order and Numerical Simulations are crucially needed. Perhaps the strongest need is felt for extracting wave forms of gravitational waves from compact sources in a variety of possible motions. The numerical simulations however have also unexpectedly revealed the critical phenomena in collapse situation and are a tool to explore issues such as cosmic censorship. In this chapter we will describe the basic ingredients of numerical relativity and highlight some of the recent developments. The primary reference is the beautiful review by Luis Lehner [96].

Problems of interest: In the weak field regimes - metric close to Minkowski metric or observation length scales are large compared to the Schwarzschild radius of compact source - systematic perturbative analytical methods such as post-Newtonian expansion exists which are quite reliable. In these regimes too numerical simulations are used e.g. N-body simulations for galaxies, but these are not general relativistic simulations. It is the strong field regimes where numerical methods play a crucial role. This class of problems involve compact stars such neutron stars or black holes, individually or in binaries, formation of horizons (or otherwise) in a gravitational collapse, oscillatory or otherwise approach to cosmological singularities (the Belinskii-Kalatnikov-Lifshitz conjecture) etc. Some of these are primarily of theoretical interest such as exploration of critical collapse, cosmic censorship, BKL singularities or the general two body problem. Some, however, have practical applications in the astrophysical context, especially the two body problem is strongly motivated by the efforts towards direct detection of gravitational waves. Following [96] we will first describe the numerical approach in somewhat general terms.

The basic problem: The basic problem is of course to solve the Einstein equation numerically. The covariant form of the equation implies that a physical solution can be obtained as many different metric coefficients as functions of corresponding (local) coordinates. In arbitrary coordinates, the equation is a local, partial differential equation with no particular type - hyperbolic or parabolic or elliptic. Whichever way a solution, $g\mu\nu(x)$, is obtained, it is a local solution and one attempts to extend it in some ‘maximal way’. Already in the discussion of causality and determinism, we noted that not every solution of the equation is physically admissible and the globally hyperbolic solutions are the physically relevant ones.

These space-times already have a $R \times \Sigma^3$ topology and can be viewed (sliced) as evolution of spatial hyper-surfaces in the space-time.

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In the previous chapter we saw that performing a ‘space + time’ decomposition and further imposition of coordinate conditions, it is possible to split the equations into elliptic (constraint) and hyperbolic ones for which local existence, uniqueness and well-posedness properties hold. This means that we input (i) a 3-manifold Σ, (ii) two symmetric tensor fields of rank 2 on it namely g_ij and K_ij one of which is a Riemannian 3-metric, (iii) a lapse function N, (iv) a shift vector N_i and construct a space-time from the evolution equations satisfied by the two tensor fields.

There are other methods of viewing the Einstein equation as an evolution of some data specified on 3-manifolds which will be null hypersurfaces in the evolved space-time e.g. the characteristic value formulation. We will focus on the Cauchy framework and refer the reader to the references for other approaches [96, 97].

In the previous section, we have already given the 3+1 decomposition as well as obtained the extrinsic curvature K_ij in terms of ∂_t g_ij (eqn. 11.15). We also had the Hamilton’s form of evolution equations. In the numerical approach, it is more customary to present the evolution equations in terms of L_n; n^μ = N^{-1} (t^μ − N^μ) or dt := ∂*t* − *L*{N^μ} = N L_n. Thus the equations are presented as[1]

ASSIGNMENT 2

The aim of the second assignment is to translate a mathematical (theoretical) text from English into Serbian, while preserving technical terminology and logical structure of proofs.

SOURCE

Terence Tao – Analysis
pp. 94–97

TEXT FOR TRANSLATION

Lemma 4.2.3.

The sum, product, and negation operations on rational numbers are well-defined, in the sense that if one replaces a/b with another rational number a'/b' which is equal to a/b, then the output of the above operations remains unchanged, and similarly for c/d.

Proof. We just verify this for addition; we leave the remaining claims to Exercise 4.2.2.

Suppose a/b = a'/b', so that b and b' are non-zero and ab' = a'b.

We now show that:

a/b + c/d = a'/b' + c/d

By definition, the left-hand side is (ad + bc)/bd and the right-hand side is (a'd + b'c)/b'd, so we have to show that:

(ad + bc)b'd = (a'd + b'c)bd

which expands to:

ab'd^2 + bb'cd = a'bd^2 + bb'cd

But since ab' = a'b, the claim follows. Similarly if one replaces c/d by c'/d'.

We note that the rational numbers a/b behave in a manner identical to the integers a:

(a/1) + (b/1) = (a + b)/1
(a/1) × (b/1) = (ab)/1
-(a/1) = (-a)/1

Also, a/1 and b/1 are only equal when a and b are equal. Because of this, we will identify a with a/1 for each integer a:

a = a/1

The above identities then guarantee that the arithmetic of the integers is consistent with the arithmetic of the rationals.

Thus just as we embedded the natural numbers inside the integers, we embed the integers inside the rational numbers. In particular, all natural numbers are rational numbers; for instance 0 is equal to 0/1 and 1 is equal to 1/1.

Observe that a rational number a/b is equal to 0 = 0/1 if and only if a × 1 = b × 0, i.e. if the numerator a is equal to 0.

Thus if a and b are non-zero then so is a/b.

We now define a new operation on the rationals: reciprocal.

If x = a/b is a non-zero rational (so that a, b ≠ 0), then we define the reciprocal x^{-1} of x to be:

x^{-1} := b/a

It is easy to check that this operation is consistent with equality: if two rational numbers a/b and a'/b' are equal, then their reciprocals are also equal.

We however leave the reciprocal of 0 undefined.

Proposition 4.2.4 (Laws of algebra for rationals).

Let x, y, z be rationals. Then the following laws of algebra hold:

$x + y = y + x$
$(x + y) + z = x + (y + z)$
$x + 0 = 0 + x = x$
$x + (-x) = (-x) + x = 0$

$xy = yx$
$(xy)z = x(yz)$
$x \cdot 1 = 1 \cdot x = x$

$x(y + z) = xy + xz$
$(y + z)x = yx + zx$

If x is non-zero, we also have:

$x \cdot x^{-1} = x^{-1} \cdot x = 1$

Remark 4.2.5.

The above identities assert that the rationals Q form a field.

Proof (sketch).

One writes $x = a/b$, $y = c/d$, $z = e/f$ for integers a, c, e and non-zero integers b, d, f, and verifies each identity using algebra of integers.

We only prove associativity of addition:

$(x + y) + z = x + (y + z)$

Left-hand side:

$(x + y) + z$
$= ((ad + bc)/bd) + (e/f)$
$= (adf + bcf + bde)/bdf$

Right-hand side:

$x + (y + z)$
$= (a/b) + ((cf + de)/df)$
$= (adf + bcf + bde)/bdf$

Thus they are equal.

Other identities are similar.

We can now define the quotient x/y of two rational numbers ($y \neq 0$) by:

$x/y := x \times y^{-1}$

ASSIGNMENT 3: : Please translate the following text into Serbian.

The excerpt is taken from Teppo's story *Lost Technique of Blackmail*.

Using the stopdrops as a way to send anonymous messages had been my idea. It had labeled me with a Director tag, and until the Systemic Introspect & Reorganization, I had been in charge of security for InterCore Express. After that, well, I fared better than a lot of people at ICE in that I still had a job, but with the i3Cee's kinder, gentler approach to corporate intrigue (read: none), the ROI of a fully staffed Security Directorate didn't pass budget audit. SecD got broken up—most went to SysAdmD, the knuckle-draggers given new uniforms and new offices (EnforD), and me and a few others were downgraded to desk jobs. I went from "Director" to "Theorist," and had a few turns to really sink into a never-ending depression, a hole where I could theorize all I liked.

I had a SysAdmD Section Manager, who really didn't know what to do with me, and I was pretty sure he was hoping that I would EOE voluntarily, saving him the headache of doing my PIPe every turn. I wasn't about to give him the satisfaction. He got back at me by never bothering to R & U any of my GPARs. It's a very unfulfilling relationship.

Which explains why I found myself leaving the office and heading out into the field to investigate the mysterious package. I should have walked it over to EnforD and let them go hit people, but that would have taken the matter out of my hands. Plus there was the issue of the stopdrops. Eventually, a doc audit would bring up the whole history of their use, and my Section Monkey would be thrilled to find my tag all over the documentation. It'd be all the excuse he'd need to WTF me.

I went Out of Office. As much as I hated that three square, it was mine, and I had been there a long time. It's funny what you'll fight to keep.

Depot 12-B4 was still inRing, next to a Baskin-Robbins Emporium 31 on the Malachite Layer. I took an express 'tubebus, and walked the few clicks from the depot. It was still ante-meridiem and the reflected sunlight wasn't too bad.

The Ring circled the planet like a lopsided halo, cleaving to the ecliptic. The outer edge was bubbled with a couple thousand climatologies where brain trusts kept trying to replicate moss and lichens in an artificial environment. InRing was home to humanity and we sprawled across every meter of space. By design, of course, regardless of the GoogleTube PR claim to the contrary.

I wasn't quite sure why they still maintained the conceit that the Ring was meant as a data structure and not as a habitat. Old corporate habits, I suppose, but after the GoogleTube Infrastructure Accords, it was hard to believe they hadn't planned for this possibility. Especially after the white paper by the pair of GoogleTube Extrapolationists was leaked. Sure, they had been ostracized from campus for writing the

document, but when your corporate mandate says you never delete anything, it gets hard for the rest of the world to believe you wouldn't actually use your own data. Even the theoretical kind.

Anyway, the GTI Accords opened up the Ring to the rest of the CorCongloms and over the next couple of clocks, the Ring went from a pristine packet landscape to a population density of a thousand per. The Retail Interregnum cleaned house, so to speak, and in the resulting economic vacuum, the SIX moved in.

Basing their dispersal theory on the New Modality of the Chicago School Theory of Economic Rapture, the SIX remodeled the Ring into an economic web that took advantage of the population density by maximizing isolation variables while pushing separation anxiety to nearly zero. It was all high throughput packet flow—1PB/f optimization to each node cluster, delivering every sort of digital signal that a body could desire (for everything that was still meatspace based, there was InterCore Express, the official package delivery service of the Ring).