

WHY IS THE KNOWLEDGE OF LINEAR ALGEBRA KEY TO UNDERSTANDING OF QUANTUM MECHANICS

An excerpt from the book (Quantum mechanics, by R. Shankar, page V)

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Introduction:

This chapter is followed by one on classical mechanics, where the Lagrangian and Hamiltonian formalisms are developed in some depth. It is for the instructor to decide how much of this to cover; the more students know of these matters, the better they will understand the connection between classical and quantum mechanics.

Chapter 3 is devoted to a brief study of idealized experiments that betray the inadequacy of classical mechanics and give a glimpse of quantum mechanics. Having trained and motivated the students I now give them the postulates of quantum mechanics of a single particle in one dimension. I use the word "postulate" here to mean "that which cannot be deduced from pure mathematical or logical reasoning, and given which one can formulate and solve quantum mechanical problems and interpret the results." This is not the sense in which the true axiomatist would use the word. For instance, where the true axiomatist would just postulate that the dynamical variables are given by Hilbert space operators, I would add the operator identifications, i.e., specify the operators that represent coordinate and momentum (from which others can be built). Likewise, I would not stop with the statement that there is a Hamiltonian operator that governs the time evolution through the equation $i\hbar \frac{d}{dt} \psi = H \psi$; I would say the H obtained from the classical Hamiltonian by substituting for x and p the corresponding operators. While the more general axioms have the virtue of surviving as we progress to systems of more degrees of freedom, with or without classical counterparts, students given just these will not know how to calculate anything such as the spectrum of the oscillator. Now one can, of course, try to "derive" these operator assignments, but to do so one would have to appeal to ideas of a postulatory nature themselves. (The same goes for "deriving" the Schrodinger equation.) As we go along, these postulates are generalized to more degrees of freedom and it is for pedagogical reasons that these generalizations are postponed. Perhaps when students are finished with this book, they can free themselves from the specific operator assignments and think of quantum mechanics as a general mathematical formalism obeying certain postulates (in the strict sense of the term). The postulates in Chapter 4 are followed by a lengthy discussion of the same, with many examples from fictitious Hilbert spaces of three dimensions. Nonetheless, students will find it hard. It is only as they go along and see these postulates used over and over again in the rest of the book, in the setting up of problems and the interpretation of the results, that they will catch on to how the game is played. It is hoped they will be able to do it on their own when they graduate. I think that any attempt to soften this initial blow will be counterproductive in the long run. Chapter 5 deals with standard problems in one dimension. It is worth mentioning that the scattering off a

step potential is treated using a wave packet approach. If the subject seems too hard at this stage, the instructor may decide to return to it after Chapter 7 (oscillator), when students have gained more experience. But I think

that sooner or later students must get acquainted with this treatment of scattering.

The classical limit is the subject of the next chapter. The harmonic oscillator is discussed in detail in the next. It is the first realistic problem and the instructor may be eager to get to it as soon as possible. If the instructor wants, he or she can discuss the classical limit after discussing the oscillator.

2.

We will assume that the theorem proved for finite dimensions, namely, that the eigenfunctions of a Hermitian operator form a complete basis, holds in Hilbert space.

(The trouble with infinite-dimensional spaces is that even if you have an infinite number of orthonormal eigenvectors, you can never be sure you have them all, since adding or subtracting a few still leaves you with an infinite number of them.)

Since K is a Hermitian operator, functions that were expanded in the X basis with components

$$f(x) = \langle x | f \rangle$$

must also have an expansion in the K basis. To find the components, we start with a ket $|f\rangle$ and do the following:

$$\begin{aligned} f(k) &= \int \langle k | x \rangle \langle x | f \rangle dx \\ &= (1/\sqrt{2\pi}) \int e^{-ikx} f(x) dx \quad (1.10.34) \end{aligned}$$

The passage back to the X basis is done as follows:

$$\begin{aligned} f(x) &= \int \langle x | k \rangle \langle k | f \rangle dk \\ &= (1/\sqrt{2\pi}) \int e^{ikx} f(k) dk \quad (1.10.35) \end{aligned}$$

Thus the familiar Fourier transform is just the passage from one complete basis to another, $|k\rangle$. Either basis may be used to expand functions that belong to the Hilbert space.