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## **A Beginner's Guide to Motion and Mechanical Force**

In mechanics, objects experience different forms of resistance. When a body slides, the opposing force is sliding friction. When it rolls, the resistance is rolling friction. To keep a vehicle moving forward, a tractive force must overcome resistances and the pull of gravity, which is a force of attraction acting between masses. During deceleration, the vehicle applies a braking effort that removes kinetic energy from the system. In the context of fields and fluids, vector flux describes the rate at which a field passes through a given area. When analyzing motion, velocity is treated as a vector, while speed is the value (magnitude) of that vector. In hydraulics, velocity head expresses the kinetic energy of fluid flow per unit weight. Many forms of velocity appear in physics. Translational velocity describes the movement of the center of mass of a body, while in electromagnetism, electrons primarily move with a slow drift velocity. Changes in velocity arise from accelerative force, causing acceleration. For charged particles, this leads to charged particle acceleration inside electric or magnetic fields. Rotating systems experience inertial force, stemming from resistance to acceleration. When velocities become extreme, an object may achieve acceleration through the speed of sound, transitioning between subsonic and supersonic regimes. Stability in motion is associated with an equilibrant force, which balances all other forces and contributes to the stability of the system. Rotational dynamics include important concepts such as the moment of momentum (angular momentum) and the moment of force with respect to a specific axis. A noncentral force (or off-central force) generates torque, resulting in motion around a fixed point.

### **Non-formal technical text**

Autoverse physics—the internal logic of the self-contained computer model—then how could she, outside the model, interact with it at all? By constructing little surrogate hands

in the Autoverse, to act as remote manipulators? Construct them out of what? There were no molecules small enough to build anything finely structured, at that scale; the smallest rigid polymers which could act as "fingers" would be half as thick as the entire nutrose ring. In any case,

although the target molecule would be free to interact with these surrogate hands according to pure Autoverse physics, there'd be nothing authentic about the way the hands themselves magically followed the movements of her gloves. Maria could see no joy in simply shifting the point where the rules were broken—and the rules had to be broken, somewhere. Manipulating the contents of the Autoverse meant violating its laws. That was obvious ... but it was still frustrating. She gazed at the array of Petri dishes floating in the workspace, their contents portrayed in colors which coded for the health of the bacteria. "False colors" . . . but that phrase was tautological. Any view of the Autoverse was necessarily stylized: a color-coded map, displaying selected attributes of the region in question. Some views were more abstract, more heavily processed than others—in the sense that a map of the Earth, color-coded to show the health of its people, would be arguably more abstract than one displaying altitude or rainfall—but the real-world ideal of an unadulterated, naked-eye view was simply untranslatable.

A few of the cultures were already looking decidedly sick, fading from electric blue to dull brown. Maria summoned up a three-dimensional graph, showing population versus time for the full range of nutrient mixtures. The cultures with only a trace of the new stuff were, predictably, growing at almost the pace of the control; with increasing mutose substitution the ascent gradually

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slowed, until, around the eighty-five percent line, the population was static. Beyond that were ever steeper trajectories into extinction. In small doses, mutose was simply irrelevant, but at high enough concentrations it was insidious:

similar enough to nutrose—*A. lamberti*'s usual food—to be taken part-way through the metabolic process, competing for the same enzymes, tying up valuable biochemical resources . . . but eventually reaching a step where that one stray blue-red spike formed an insurmountable barrier to the reaction geometry, leaving the bacterium with nothing but a useless byproduct and a net energy loss. A culture with ninety percent mutose was a world where ninety per cent of the food supply had no nutritional value whatsoever—but had to be ingested indiscriminately along with the worthwhile ten percent. Consuming ten times as much for the same return wasn't a viable solution;

to survive in the long term, *A. lamberti* would have to chance upon some means of rejecting mutose before wasting energy on it—or, better still, find a way to turn it back into nutrose, transforming it from a virtual poison into a source of food.

Maria displayed a histogram of mutations occurring in the bacteria's three nutrose epimerase genes; the enzymes these genes coded for were the closest things *A. lamberti* had to a tool to render mutose digestible—although none, in their original form, would do the job. No mutants had yet persisted for more than a couple of generations; all the changes so far had evidently done more harm than good. Partial sequences of the mu

She saved the modified sugar, optimistically dubbing it mutose. Then, changing the length scale by a factor of a million, she started up twenty-one tiny cultures of *Autobacterium lamberti*, in solutions ranging from pure nutrose, to a fifty-fifty mixture, to one hundred percent mutose.

Shegenes scrolled by in a small window; Maria gazed at the blur of codons, and mentally urged the process on—if not straight toward the target (since she had no idea what that was), then at least. . . outward, blindly, into the space of all possible mistakes.

It was a nice thought. The only trouble was, certain portions of the genes were especially prone to particular copying errors, so most of the mutants were "exploring" the same dead ends again and again.

Arranging for *A. lamberti* to mutate was easy; like a real-world bacterium, it made frequent errors every time it duplicated its analogue of DNA. Persuading it to mutate "usefully"

was something else. Max Lambert himself—inventor of the Autoverse, creator of *A. lamberti*, hero to a generation of cellular-automaton and artificial-life freaks—had spent much of the last fifteen years of his life trying to discover why the subtle differences between real-world and Autoverse biochemistry made natural selection so common in one system, and so elusive in the other. Exposed to the kind of stressful opportunities which *E. coli* would have exploited within a few dozen generations, strain after strain of *A. lamberti* had simply died out.

Only a few die-hard enthusiasts still continued Lambert's work. Maria knew of just seventy-two people who'd have the slightest idea what it meant if she ever succeeded. The artificial life scene, now, was dominated by the study of Copies—patchwork creatures, mosaics of ten thousand different ad hoc rules . . . the antithesis of everything the Autoverse stood for.

Real-world biochemistry was far too complex to simulate in every last detail for a creature the size of a gnat, let alone a human being. Computers could model all the processes of life—but not on every scale, from atom to organism, all at the same time. So the field had split three ways. In one camp, traditional molecular biochemists continued to extend their painstaking calculations, solving Schrodinger's equation more or less exactly for ever larger systems, working their way up to entire replicating strands of DNA, whole mitochondrial sub-assemblies, significant patches of the giant carbohydrate chain-link fence of a cell wall. . . but spending ever more on computing power for ever diminishing returns.

At the other end of the scale were Copies: elaborate refinements of whole-body medical simulations, originally designed to help train surgeons with virtual operations, and to take the place of animals in drug tests. A Copy was like a high-resolution CAT scan come to life, linked to a medical encyclopedia to spell out how its every tissue and organ should behave . . . walking around inside a state-of-the-art architectural simulation. A Copy possessed no individual atoms or molecules; every organ in its virtual body came in the guise of specialized

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sub-programs which knew (in encyclopedic, but not atomic, detail) how a real liver or brain or thyroid gland functioned . . . but which couldn't have solved Schrodinger's equation for so much as a single protein molecule. All physiology, no physics.

Lambert and his followers had staked out the middle ground. They'd invented a new physics, simple enough to allow several thousand bacteria to fit into a modest computer simulation, with a

consistent, unbroken hierarchy of details existing right down to the subatomic scale ( anm excerpt from Greg Egan's novel Permutation city).

## Mathematical text

### Functions, fields

A finite or countably infinite selection of elements of a set  $A$  is a list.

Members of a list are assumed to be in a definite order, given by their indices or by the implied order of reading from left to right. Lists are usually written without brackets:  $a_1, \dots, a_n$ , though sometimes written as ordered  $n$ -tuples  $(a_1, \dots, a_n)$ . Elements of a list need not be distinct.

A countably infinite list of elements of a set  $A$  is also often called a sequence of elements of  $A$ . The set of all distinct members of a list is called the underlying subset of the list.

If  $A$  and  $B$  are sets:

A union  $A \cup B$  is the set of all elements that belong to  $A$  or  $B$ .

A intersect  $A \cap B$  is the set of all elements belonging to both  $A$  and  $B$ .

A difference  $A \setminus B$  is the set of all elements of  $A$  which do not belong to  $B$ .

More generally, if  $\{A_i \mid i \in W\}$  is a possibly infinite collection of sets:

Union over  $i$  in  $W$ :  $\bigcup A_i =$  all elements belonging to at least one  $A_i$ .

Intersection over  $i$  in  $W$ :  $\bigcap A_i =$  all elements belonging to all  $A_i$ .

A function  $f$  from a nonempty set  $A$  to a nonempty set  $B$  assigns each  $a$  in  $A$  a unique element  $f(a)$  in  $B$ .  $A$  is the domain,  $B$  is the range. We write:

$f: A \rightarrow B$

and

$f(a) = b$

We also write  $f: a \rightarrow a^3$  for a function assigning the cube of an integer.

The set of all functions from  $A$  to  $B$  is written  $B^A$ .

If  $A'$  is a nonempty subset of  $A$ , the restriction of  $f$  to  $A'$  is the function  $f': A' \rightarrow B$  defined by  $f'(a) = f(a)$ .

Functions  $f$  and  $g$  in  $B^A$  are equal iff  $f(a) = g(a)$  for all  $a$  in  $A$ .

A function  $f$  is:

- monic (injective):  $f(a_1) \neq f(a_2)$  whenever  $a_1 \neq a_2$
- epic (surjective): every element of  $B = f(a)$  for some  $a$  in  $A$
- bijective: both monic and epic

If  $f: A \rightarrow B$  is bijective, it has an inverse  $f^{-1}: B \rightarrow A$  defined by:  
 $f^{-1}(b) = a$  iff  $f(a) = b$

A bijective function from  $A$  to itself is a permutation of  $A$ .  
The identity permutation is  $a \rightarrow a$ .

Cartesian product:

$A_1 \times A_2 =$  all ordered pairs  $(a_1, a_2)$  with  $a_i$  in  $A_i$ .

More generally:

$A_1 \times \dots \times A_n =$  all ordered  $n$ -tuples  $(a_1, \dots, a_n)$

This can be formulated as functions  $f: \{1, \dots, n\} \rightarrow$  union of  $A_i$  satisfying  $f(i)$  in  $A_i$ .

For an arbitrary index set  $W$ , the product  $\text{product}(A_i) =$  all functions  $f: W \rightarrow$  union  $A_i$  with  $f(i)$  in  $A_i$ . This uses the Axiom of Choice.

If  $A_i = A$  for all  $i$  in  $W$  then the product  $= A^W$ .

If  $W = \{1, \dots, n\}$  we write  $A^n$ .

Standard notation for number sets:

$N =$  nonnegative integers

$Z =$  integers

$Q =$  rational numbers

$R =$  real numbers

$C =$  complex numbers

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## FIELDS

A field  $F$  is a nonempty set with addition  $+$  and multiplication  $*$  satisfying:

(1) Associativity:

$$a + (b + c) = (a + b) + c$$

$$a * (b * c) = (a * b) * c$$

(2) Commutativity:

$$a + b = b + a$$

$$a * b = b * a$$

(3) Distributivity:

$$a * (b + c) = ab + ac$$

(4) Identity elements:

There exist  $0$  and  $1$  ( $0 \neq 1$ ) such that:

$$a + 0 = a$$

$$a * 1 = a$$

(5) Additive inverses:

For each  $a$ , there exists  $-a$  such that:

$$a + (-a) = 0$$

(6) Multiplicative inverses:

For each  $a \neq 0$ , there exists  $a^{-1}$  such that:

$$a^{-1} * a = 1$$

We use:

$$a - b = a + (-b)$$

$$a / b = a * b^{-1}$$

Exponent notation:

For integer  $n > 0$ :

$$na = a + \dots + a \text{ (n times)}$$

$$a^n = a * \dots * a \text{ (n times)}$$

For  $n = 0$ :

$$0a = 0; a^0 = 1$$

For  $n < 0$ :

$$a^n = (a^{-1})^{-n}$$

Multiplication distributes over sums:

$$a * (\sum_{i=1..n} b_i) = \sum_{i=1..n} a * b_i$$

Examples:

$\mathbb{Q}$  and  $\mathbb{R}$  with usual  $+$  and  $*$  are fields.

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### Example: Complex Numbers

Let  $C = \mathbb{R}^2$  with:

$$(a,b) + (c,d) = (a+c, b+d)$$

$$(a,b) * (c,d) = (ac - bd, ad + bc)$$

Additive identity:  $(0,0)$

Multiplicative identity:  $(1,0)$

Multiplicative inverse:

$$(a,b)^{-1} = (a/(a^2+b^2), -b/(a^2+b^2)) \text{ for } (a,b) \neq (0,0)$$

Identify:

$$\text{Real number } a \equiv (a,0)$$

$$i = (0,1)$$

$$i^2 = (-1,0)$$

So write  $z = a + bi$

$$\text{Real part: } \operatorname{Re}(z) = a$$

$$\text{Imag part: } \operatorname{Im}(z) = b$$

Complex conjugate:

$$z = a - bi$$

Useful identities:

$$z + z' = \text{conjugate}(z + z')$$

- $z = \text{conjugate}(-z)$   
 $zz' = \text{conjugate}(zz')$   
 $z^{-1} = \text{conjugate}(z)^{-1}$   
 $\text{abs}(z) = \text{sqrt}(a^2 + b^2)$

Triangle inequality:

$$\text{abs}(y + z) \leq \text{abs}(y) + \text{abs}(z)$$