Chapter 4
WAVES

IN THIS CHAPTER:

- ✔ *Transverse Waves*
- ✔ *Wave Terminology*
- ✔ *Standing Waves*
- ✔ *Resonance*
- ✔ *Longitudinal Waves*
- ✔ *Sound Waves*
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Transverse Waves

Propagating Waves

A **propagating wave** is a self-sustaining disturbance of a medium that travels from one point to another, carrying energy and momentum. Mechanical waves are aggregate phenomena arising from the motion of constituent particles. The wave advances, but the particles of the medium only oscillate in place. A wave has been generated on the string in Figure 4-1 by the sinusoidal vibration of the hand at its end. The wave furnishes a record of earlier vibrations of the source. Energy is carried by the wave from the source to the right, along the string. This direction, the direction of energy transport, is called the direction (or line) of propagation of the wave.

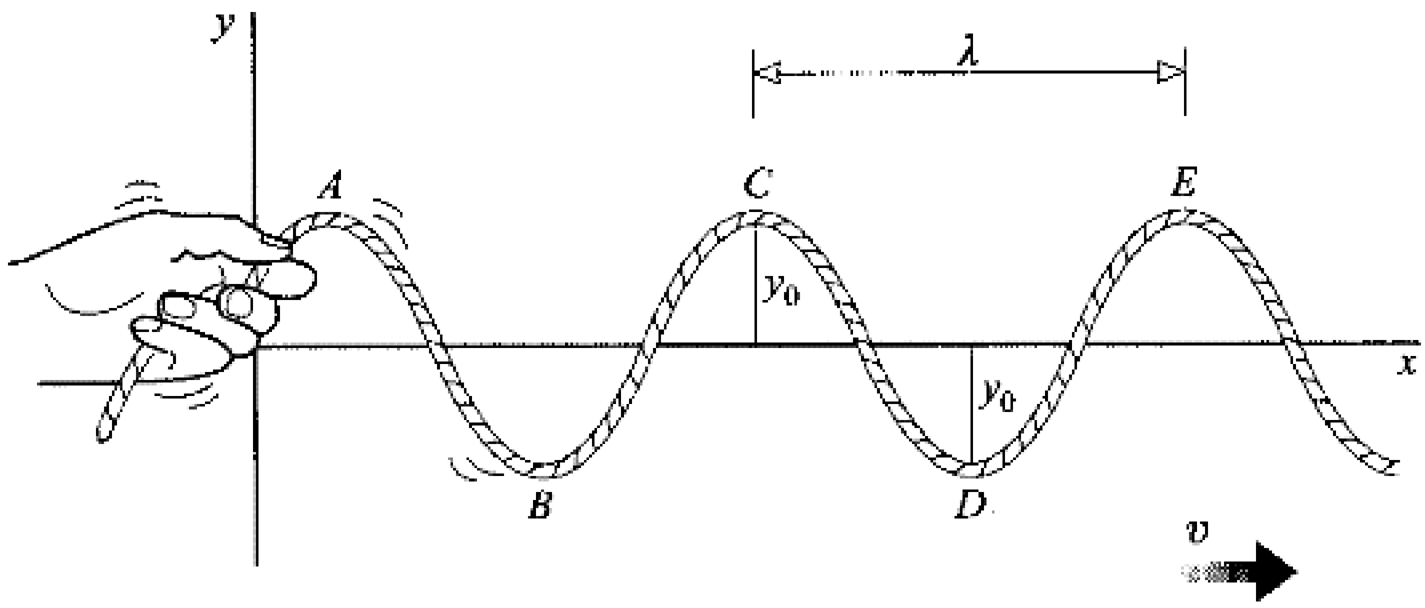


Figure 4-1

Each particle of the string (such as the one at point C) vibrates up and down, perpendicular to the line of propagation. Any wave in which the vibration direction is perpendicular to the direction of propagation is called a *transverse wave*. Typical transverse waves, besides those on a string, are electromagnetic waves—light and radio waves. By contrast, in sound waves, the vibration direction is parallel to the direction of propagation, as will be seen in a later section. Such a wave is called a *longitudinal* (or *compressional*) *wave*.

Wave Terminology

The **period** (T) of a wave is the time it takes the wave to go through one complete cycle. It is the time taken for a particle such as the one at A to move through one complete vibration or cycle, down from point A and then back to A. The period is the number of seconds per cycle. The **frequency** (f) of a wave is the number of cycles per second. Thus,

$$f = \frac{1}{T}$$

If T is in seconds, then f is in hertz (Hz), where 1 Hz = 1 s^{-1}. The period and frequency of the wave are the same as the period and frequency of the vibration

The top points on the wave, such as A and C, are called *wave crests*. The bottom points, such as B and D, are called *troughs*. As time goes on, the crests and troughs move to the right with speed v, the speed of the wave. The **amplitude** of a wave is the maximum disturbance undergone during a vibration cycle, distance y_0 in Figure 4-1. The **wavelength** (λ) is the distance along the direction of propagation between corresponding points on the wave, distance AC, for example. In a time T, a crest moving with speed v will move a distance λ to the right. Therefore, s = vt gives

$$\lambda = vT = \frac{v}{f} \quad \text{and} \quad v = f\lambda$$

This relation holds for all waves, not just for waves on a string.

In-phase vibrations exist at two points on a wave if those points undergo vibrations that are in the same direction, in step. For example, the particles of the string at points A and C in Figure 4-1 vibrate in-phase, since they move up together and down together. Vibrations are in-phase if the points are a whole number of wavelengths apart. The pieces of the string at A and B vibrate opposite to each other; the vibrations there are said to be 180°, or half a cycle, *out-of-phase*.

The **speed of a transverse wave** on a stretched string or wire is

$$v = \sqrt{\frac{\text{tension in string}}{\text{mass per unit length of string}}}$$

Standing Waves

At certain vibrational frequencies, a system can undergo resonance. That is to say, it can efficiently absorb energy from a driving source in its environment which is oscillating at that frequency (see Figure 4-2).

These and similar vibration patterns are called **standing waves**, as compared to the propagating waves considered above. These might better not be called waves at all since they do not transport energy and momentum. The stationary points (such as B and D) are called **nodes**; the points of greatest motion (such as A, C, and E) are called **antinodes**.

The distance between adjacent nodes (or antinodes) is $\frac{1}{2}\lambda$.

We term the portion of the string between adjacent nodes a **segment**, and the length of a segment is also $\frac{1}{2}\lambda$.

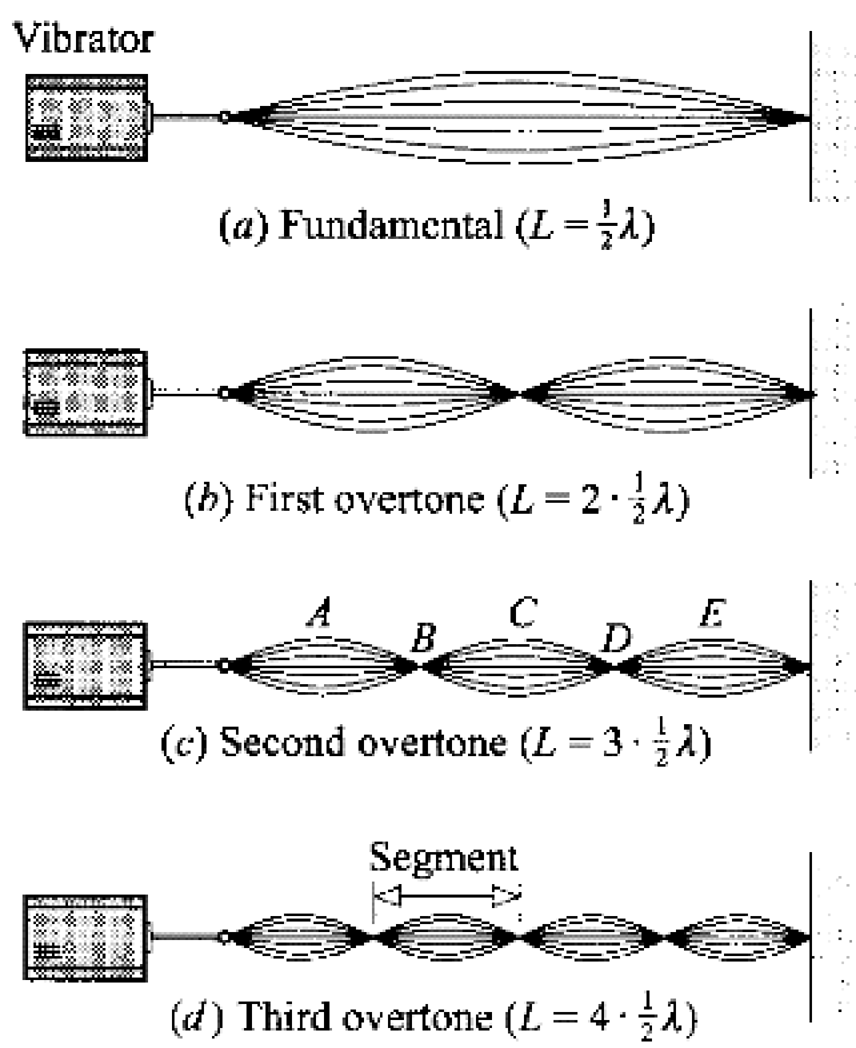


Figure 4-2

Resonance

A string will resonate only if the vibration wavelength has certain special values: the wavelength must be such that a whole number of wave segments (each $\frac{1}{2}\lambda$ long) exactly fit on the string. A proper fit occurs when nodes and antinodes exist at positions demanded by the constraints on the string. In particular, the fixed ends of the string must be nodes. Thus, as shown in Figure 4-2, the relation between the wavelength λ and the length L of the resonating string is $L = n\left(\frac{1}{2}\lambda\right)$, where n is any integer. Because $\lambda = vT = v/f$, the shorter the wave segments at resonance, the higher will be the resonance frequency. If we call the fundamental resonance frequency f_1, then Figure 4-2 shows that the higher resonance frequencies are given by $f_n = nf_1$.

Longitudinal Waves

Longitudinal (compressional) waves occur as lengthwise vibrations of air columns, solid bars, and the like. At resonance, nodes exist at fixed points, such as the closed end of an air column in a tube, or the location of a clamp on a bar. Diagrams such as Figure 4-2 are used to display the resonance of longitudinal waves as well as transverse waves. However, for longitudinal waves, the diagrams are mainly schematic and are used simply to indicate the locations of nodes and antinodes. In analyzing such diagrams, we use the fact that the distance between node and adjacent antinode is $\frac{1}{4}\lambda$.

Sound Waves

Sound waves are compression waves in a material medium such as air, water, or steel. When the compressions and rarefactions of the waves strike the eardrum, they result in the sensation of sound, provided the frequency of the waves is between about 20 Hz and 20 000 Hz. Waves with frequencies above

20 kHz are called *ultrasonic* waves. Those with frequencies below 20 Hz are called *infrasonic* waves.

Equations for Sound Speed

In an ideal gas of molecular mass M and absolute temperature T, the speed of sound v is given by

$$v = \sqrt{\frac{\gamma RT}{M}} \qquad \text{(ideal gas)}$$

where R is the gas constant, and γ is the ratio of specific heats c_p/c_v. γ is about 1.67 for monatomic gases (He, Ne, Ar), and about 1.40 for diatomic gases (N_2, O_2, H_2).

The speed of compression waves in other materials is given by

$$v = \sqrt{\frac{\text{modulus}}{\text{density}}}$$

If the material is in the form of a bar, Young's modulus Y is used. For liquids, one must use the bulk modulus.

The **speed of sound in air** at 0°C is 331 m/s. The speed increases with temperature by about 0.61 m/s for each 1°C rise. More precisely, sound speeds v_1 and v_2 at absolute temperatures T_1 and T_2 are related by

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

The speed of sound is essentially independent of pressure, frequency, and wavelength.

Intensity

The **intensity** (I) of any wave is the energy per unit area, per unit time; in practice, it is the average power carried by the wave through a unit area erected perpendicular to the direction of propagation of the wave. Suppose that in a time Δt an amount of energy ΔE is carried through an area ΔA that is perpendicular to the propagation direction of the wave. Then

$$I = \frac{\Delta E}{\Delta A\, \Delta t} = \frac{P_{av}}{\Delta A}$$

It may be shown that, for a sound wave with amplitude A_0 and frequency f, traveling with speed v in a material of density ρ,

$$I = 2\pi^2 f^2 \rho v A_O^2$$

If f is in Hz, ρ is in kg/m^3, v is in m/s, and A_o (the maximum displacement of the atoms or molecules of the medium) is in m, then I is in W/m^2.

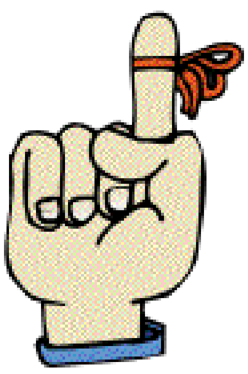
Loudness

Loudness is a measure of the human perception of sound. Although a sound wave of high intensity is perceived as louder than a wave of lower intensity, the relation is far from linear. The sensation of sound is roughly proportional to the logarithm of the sound intensity. But the exact relation between loudness and intensity is complicated and not the same for all individuals.

Intensity (or loudness) level (β) is defined by an arbitrary scale that corresponds roughly to the sensation of loudness. The zero on this scale is taken at the sound-wave intensity $I_0 = 1.00 \times 10^{-12}$ W/m^2, which corresponds roughly to the weakest audible sound. The intensity level, in decibels, is then defined by

$$\beta = 10 \log\left(\frac{I}{I_o}\right)$$

The *decibel* (dB) is a dimensionless unit. The normal ear can distinguish between intensities that differ by an amount down to about 1 dB.



Remember!

Beats

The alternations of maximum and minimum intesity produced by the superposition of two waves of slightly different frequencies are called *beats*. The number of beats per second is equal to the difference between the frequencies of the two waves that are combined.

Doppler Effect

Suppose that a moving sound source emits a sound of frequency f_s. Let v be the speed of sound, and let the source approach the listener or observer at speed v_s, measured relative to the medium conducting the sound. Suppose further that the observer is moving toward the source at speed v_o, also measured relative to the medium. Then the observer will hear a sound of frequency f_o given by

$$f_o = f_s\left(\frac{v + v_o}{v - v_s}\right)$$

If either the source or the observer is moving away from the other, the sign on its speed in the equation must be changed.

When the source and the observer are approaching each other, more wave crests strike the ear each second than when both are at rest. This causes the ear to perceive a higher frequency than that emitted by the source. When the two are receding, the opposite effect occurs; the frequency appears to be lowered.

Because $v + v_o$ is the speed of a wave crest relative to the observer, and because $v - v_s$ is the speed of a wave crest relative to the source, an alternative form is

$$f_o = f_s\left(\frac{\text{crest speed relative to observer}}{\text{crest speed relative to source}}\right)$$

Interference Effects

Two sound waves of the same frequency and amplitude may give rise to easily observed interference effects at a point through which they both pass. If the crests of one wave fall on the crests of the other, the two waves are said to be *in-phase*. In that case, they reinforce each other and give rise to a high intensity at that point.

However, if the crests of one wave fall on the troughs of the other, the two waves will exactly cancel each other. No sound will then be heard at the point. We say that the two waves are then 180° (or a half wavelength) *out-of-phase*.

Intermediate effects are observed if the two waves are neither in-phase nor 180° out-of-phase, but have a fixed phase relationship somewhere in between.

Chapter 5

ELECTRICITY AND MAGNETISM

IN THIS CHAPTER:

- ✔ *Coulomb's Law and Electric Fields*
- ✔ *Potential and Capacitance*
- ✔ *Current, Resistance, and Ohm's Law*
- ✔ *Electrical Power*
- ✔ *Equivalent Resistance, Simple Circuits, and Kirchhoff's Laws*
- ✔ *Magnetic Fields*
- ✔ *Induced EMF and Magnetic Flux*
- ✔ *Electric Generators and Motors*
- ✔ *Inductance; R-C and R-L Time Constants*
- ✔ *Alternating Current*
- ✔ *Solved Problems*

Coulomb's Law and Electric Fields

Coulomb's Law

Suppose that two point charges, q and q´, are a distance r apart in vacuum. If q and q´ have the same sign, the two charges repel each other; if they have opposite signs, they attract each other. The force experienced by either charge due to the other is called a **Coulomb** or **electric force,** and it is given by **Coulomb's law**,

$$F_E = k\frac{qq'}{r^2} \quad (\text{in vacuum})$$

As always in the SI, distances are measured in meters and forces in newtons. The SI unit for charge q is the *coulomb* (C). The constant k in Coulomb's Law has the value

$k = 8.988 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$

which we shall usually approximate as $9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$. Often, k is replaced by $1/4\pi\varepsilon_0$, where $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2$ is called the *permittivity of free space.* Then Coulomb's Law becomes

$$F_E = \frac{1}{4\pi\varepsilon_o}\frac{qq'}{r^2} \quad (\text{in vacuum})$$

When the surrounding medium is not a vacuum, forces caused by induced charges in the material reduce the force between point charges. If the material has a *dielectric constant* K, then ε_0 in Coulomb's Law must be replaced by $K\varepsilon_0 = \varepsilon$, where ε is called the *permittivity of the material*. Then,