CHAPTER 2

STATICS

Basic Attacks and Strategies for Solving Problems in this Chapter

Statics is the study of the condition of objects (usually at rest) with especial regard to the forces involved. Fundamental to physics is the concept of a force. Intuitively, a force is a push or a pull acting on some object. More precisely, Newton's laws help us to define a force. Newton's first law *states that an object at rest remains at rest and an object in motion remains in motion with constant velocity in the absence of external forces.* Newton's second law *is the basis of dynamics, but one consequence of it is that the* weight *force of any mass is* $W = mg$, *where* g *is the gravitational acceleration* $= 9.8$ m/s^2 *near the surface of the Earth.* Newton's third law *says that for every action force there is an equal and opposite reaction force.*

Other than weight, several important forces are tension (the force in a string or cable), the normal force N *acting perpendicular to a surface, the force of static friction* ($F_s \le \mu_s N$), *the force of kinetic friction* ($F_k = \mu_k N$), *and a pivot or reaction force* R *acting at an angle* θ *with respect to the surface. For example, in standing on the floor, you exert a force of magnitude* W *on the floor; the floor responds by exerting a force* $N = R$ *on you. The reaction force of the floor prevents you from falling through the floor.*

In order to solve a statics (or any) physics problem, first write down the information in terms of numbers and symbols. Then draw a figure showing the relevant objects and angles. Next, choose points in the system

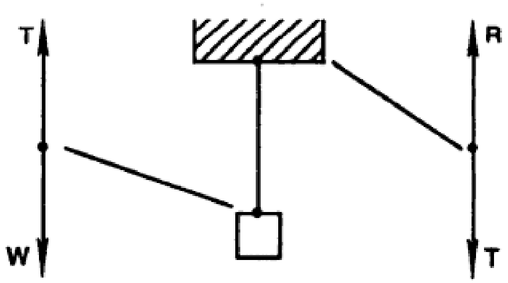


Figure 1

and draw free body diagrams for those points. For example, in Figure 1, two important free body diagrams are shown for the case of a mass suspended by a cord from a ceiling.

In statics, we now apply the two conditions of equilibrium. The first condition is that the sum of the forces in each direction is zero: $\Sigma \vec{F} = 0$. *The equilibrium is said to be static if also the velocity* $\vec{v} = 0$. *For example, in Figure 1, choosing the positive direction as down, we get*

$$\Sigma F_y = W - T = 0 \textit{ and } T - R = 0.$$

Hence, $T = W$ *and* $R = T$; *the weight determines both the tension in the string and the reaction force of the ceiling. In Figure 2, a force* F *pulls an object of mass* m *on a flat but rough surface with coefficient of static friction* μ_s *and coefficient of kinetic friction* μ_k. *Resolving* F *into its* x *and* y *components, we find* $F_x = F \cos\theta$ *and* $F_y = F \sin\theta$. *Static equilibrium in the y–direction gives*

$$\Sigma F_y = F_y + N - W = 0 \quad \textit{or} \quad N = mg - F \sin\theta$$

to find the normal force. Note that the normal force is not always equal to mg! *If the object starts out at rest, then it will begin to move when*

$$\Sigma F_x = F_x - F_s = 0 \quad \text{or} \quad F \cos\theta = \mu_s N.$$

If the object is moving at constant velocity, then

$$\Sigma F_x = F_x - F_k = 0 \quad \textit{or} \quad \mu_k N = F \cos\theta.$$

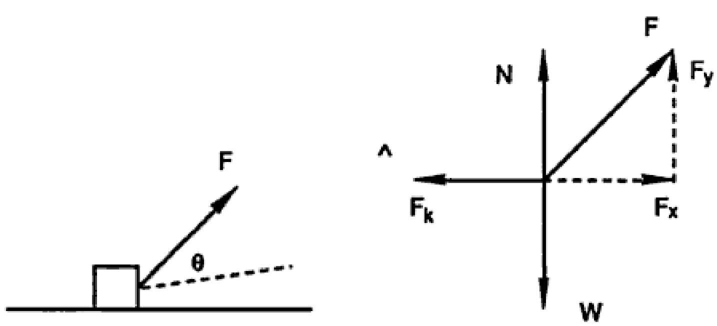


Figure 2

The second condition, that of rotational equilibrium, *is that the sum of all torques is zero:* $\Sigma \vec{\tau} = 0$ *where the torque* $\vec{\tau} = \vec{r} \times \vec{F}$ *is a cross product. Note that position vector* $\vec{r}$ *where the force acts and the force* $\vec{F}$ *must be drawn with a common origin of find the angle* θ *between them; then the right hand rule is used to find the direction of the torque. Figure 3 shows a standard boom problem, where the boom has weight* $B = m_b g$ *and the person has weight* $W = mg$. *The first equilibrium condition gives*

$$\Sigma F_x = R_x - T_x = 0; \textit{ hence } R_x = T\cos\theta.$$

Also,

$$\Sigma F_y = R_y + T_y - W - B = 0 \textit{ or } R_y = W + B - T\sin\theta.$$

If R *and* T *are unknown, one cannot find them just from these two equations. Hence, choose the point where the boom contacts the wall as the origin for calculating torques. Rotational equilibrium then implies*

$$\begin{aligned}\Sigma\tau &= \Sigma rF\sin\theta \\ &= (0)(R) - xW\sin 90 - d/2\,B\sin 90 + dT\sin(180-\theta) \\ &= 0\end{aligned}$$

or solving for the tension $T = (xW + Bd/2)/(d\sin\theta)$. *The positive and negative directions come from the right hand rule. The angles come from moving the position vector such that it and the force have a common origin (Figure 4).*

The concept of rotational equilibrium can also be used to locate the center of gravity or gravitational center of a system of objects. This is just the pivot point where the system balances as in the childhood seesaw. More importantly, the center of gravity often coincides with the center of mass of an object where $\vec{r}_{cm} = \Sigma m\vec{r} / \Sigma m$. *In the beam problem, Figure 3, we assumed the weight of the beam acted at the center of mass of the beam* d/2.

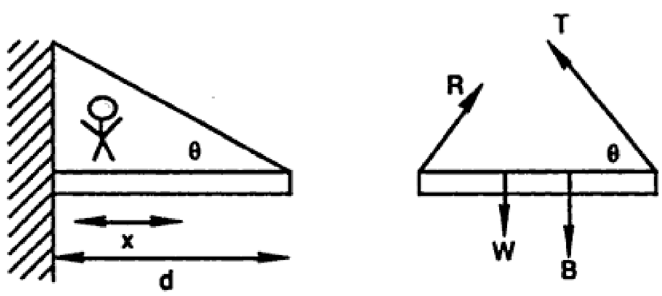


Figure 3

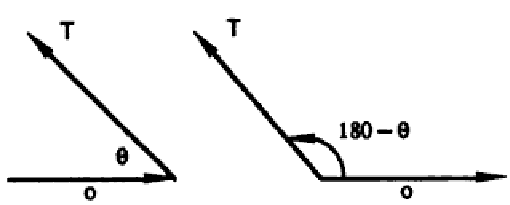


Figure 4

In order that both forces have the same line of action, the center of gravity of the body must lie vertically below the point of attachment of the cord.

Let us emphasize again that the forces $\vec{w}_1$ and $\vec{T}_1$ are not an action-reaction pair, although they are equal in magnitude, opposite in direction, and have the same line of action. The weight $\vec{w}_1$ is a force of attraction exerted on the body by the earth. Its reaction is an equal and opposite force of attraction exerted on the earth by the body. The reaction is one of the set of forces acting on the earth, and therefore it does not appear in the free-body diagram of the suspended block.

The reaction to the force $\vec{T}_1$ is an equal downward force, $\vec{T}_1'$, exerted on the cord by the suspended body.

$T_1 = T_1'$ (from Newton's third law).

The force $\vec{T}_1'$ is shown in part (c), which is the free-body diagram of the cord. The other forces on the cord are its own weight $\vec{w}_2$ and the upward force $\vec{T}_2$ exerted on its upper end by the ceiling. Since the cord is also in equilibrium,

$$\Sigma F_y = T_2 - w_2 - T_1' = 0$$

$$T_2 = w_2 + T_1' . \qquad \text{(1st law)}$$

The reaction to $\vec{T}_2$ is the downward force $\vec{T}_2'$ in part (d), exerted on the ceiling by the cord.

$$T_2 = T_2' \qquad \text{(3rd law)}$$

As a numerical example, let the body weight 20 lb and the cord weigh 1 lb. Then

$$T_1 = w_1 = 20 \text{ lb},$$

$$T_1' = T_1 = 20 \text{ lb},$$

$$T_2 = w_2 + T_1' = 1 \text{ lb} + 20 \text{ lb} = 21 \text{ lb},$$

$$T_2' = T_2 = 21 \text{ lb}.$$

• PROBLEM 2-2

Three forces acting on a particle and keeping it in equilibrium must be coplanar and concurrent. Show that the vectors representing the forces, when added in order, form a closed triangle; and further show that the magnitude of any force divided by the sine of the angle between the lines of action of the other two is a constant quantity.

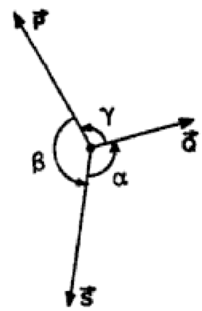


Figure A

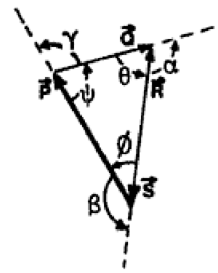


Figure B

Solution: Let the three forces be $\vec{P}$, $\vec{Q}$, and $\vec{S}$, at angles α, β, and γ to one another as shown in figure (a). In order that the three forces shall be in equilibrium, the resultant $\vec{R}$ of $\vec{P}$ and $\vec{Q}$ must be equal and opposite to $\vec{S}$. The vectors $\vec{P}$, $\vec{Q}$, and $\vec{S}$ are concurrent and, since the vector $\vec{R}$ is in the same plane as $\vec{P}$ and $\vec{Q}$, they are coplanar.

But the resultant of $\vec{P}$ and $\vec{Q}$ is obtained by vector addition, as in figure (b). That is, $\vec{R}$ is the third side of the triangle formed by placing the tail of $\vec{Q}$ at the head of $\vec{P}$. The force $\vec{S}$ is equal and opposite to $\vec{R}$ and thus will occupy the same space as $\vec{R}$, the third side of the triangle, but will be opposite in direction to $\vec{R}$. Thus $\vec{P} + \vec{Q} + \vec{S}$ taken in order, form a closed triangle and their sum is of necessity zero. Applying the law of sines to the triangle of figure (b)

$$\frac{P}{\sin\theta} = \frac{Q}{\sin\phi} = \frac{S}{\sin\psi}$$

$$\therefore \frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{S}{\sin(180-\gamma)}$$

$$\therefore \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{S}{\sin\gamma} = \text{const.}$$

• PROBLEM 2-3

A 200 lb man hangs from the middle of a tightly stretched rope so that the angle between the rope and the horizontal direction is 5°, as shown in Figure A. Calculate the tension in the rope. (Figure B).

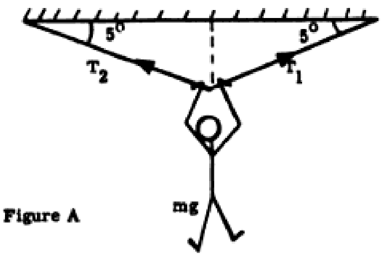


Figure A

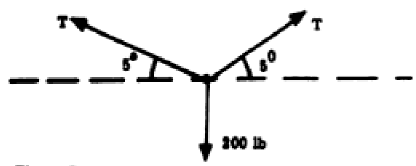


Figure B

Solution: Since the two sections of the rope are symmetrical with respect to the man, the tensions in them must have the same magnitude, (Fig. B.) This can be arrived at by summing the forces in the horizontal direction and setting them equal to zero since the system is in equilibrium. Then

$$\Sigma F_x = T_1 \cos 5^\circ - T_2 \cos 5^\circ = 0$$

and

$$T_1 = T_2 = T$$

Considering the forces in the vertical direction,

$$\Sigma F_y = T \sin 5^\circ + T \sin 5^\circ - 200 \text{ lb} = 0$$

$$200 \text{ lb} = 2T \sin 5^\circ = 2T(0.0871)$$

$$T = \frac{(200)}{(2)(0.0871)} = 1150 \text{ lbs.}$$

Note the significant force that can be exerted on objects at either end of the rope by this arrangement. The tension in the rope is over five times the weight of the man. Had the angle been as small as 1°, the tension would have been

$$T = \frac{200}{2 \sin 1^\circ} = \frac{200}{(2)(0.0174)} = 5730 \text{ lbs.}$$

This technique for exerting a large force would only be useful to move something a very small distance, since any motion of one end of the rope would change the small angle considerably and the tension would decrease accordingly.

• PROBLEM 2-4

Find the tension in the cable shown in Figure A. Neglect the weight of the wooden boom.

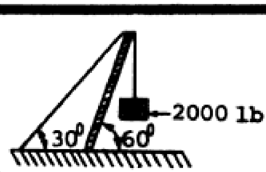


Fig. A

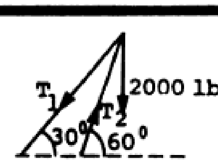


Fig. B: Force Diagram

Solution: Take the directions of the tensions in the cable and the boom to be as shown in the force diagram(fig.B). We assume at this point, that the given directions are correct. However, the forces may turn out to point in the opposite direction. If this is the case, our solutions for the tensions will be negative.

We can thus correct ourselves at the end of the problem. The first condition of equilibrium yields

$$\Sigma F_x = T_2 \cos 60^\circ - T_1 \cos 30^\circ = 0 \qquad (1)$$

$$\Sigma F_y = T_2 \sin 60^\circ - T_1 \sin 30^\circ - 2000 = 0 \qquad (2)$$

We wish to find T_1, the tension in the cable. Solving for T_2 in terms of T_1 in equation (1) gives

$$T_2 = \frac{T_1 \cos 30^\circ}{\cos 60^\circ}$$

Substituting this in equation (2),

$$\left(\frac{T_1 \cos 30^\circ}{\cos 60^\circ}\right)\sin 60^\circ - T_1 \sin 30^\circ = 2000$$

Solving for T_1:

$$T_1(\cos 30^\circ \tan 60^\circ - \sin 30^\circ) = 2000$$

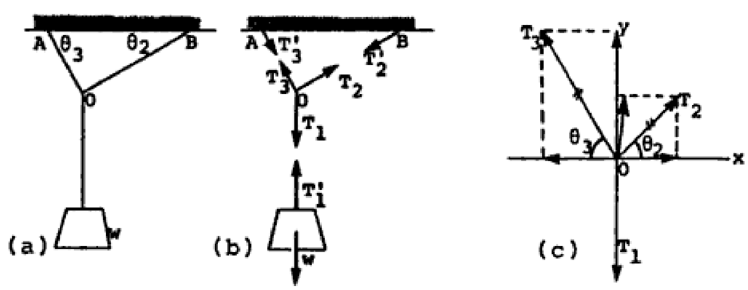
$$T_1 = \frac{2000}{\cos 30^\circ \tan 60^\circ - \sin 30^\circ} = \frac{2000}{(0.8660)(1.7321) - (0.5000)}$$

$$= \frac{2000}{1.5 - 0.5} = 2000 \text{ lb}$$

Since our answer is positive, the force acts in the direction assumed in the beginning.

• PROBLEM 2-5

In figure A, a block of weight w hangs from a cord which is knotted at O to two other cords fastened to the ceiling. Find the tensions in these three cords. Let w = 50 lb, $\theta_2 = 30°$, and $\theta_3 = 60°$. The weights of the cords are negligible.



(a) A block hanging in equilibrium. (b) Forces acting on the block, on the knot, and and the ceiling. (c) Forces on the knot O resolved into x- and y- components.

Solution: In order to use the conditions of equilibrium to compute an unknown force, we must consider some body which is in equilibrium and on which the desired force acts. The hanging block is one such body and the tension in the vertical cord supporting the block is equal to the weight of the block. The inclined cords do not exert forces on the block, but they do act on the knot at O. Hence, we consider the knot as a small body in equilibrium, whose own weight is negligible.

The free body diagrams for the block and the knot are shown in figure B, where $\vec{T}_1$, $\vec{T}_2$, and $\vec{T}_3$ represent the forces exerted on the knot by the three cords and $\vec{T}'_1$, $\vec{T}'_2$, and $\vec{T}'_3$ are the reactions to these forces.

Consider first the hanging block. Since it is in equilibrium,

$$T'_1 = w = 50 \text{ lb}$$

Since $\vec{T}_1$ and $\vec{T}'_1$ form an action-reaction pair,

$$T'_1 = T_1$$

Hence $\quad T_1 = 50$ lb.

To find the forces $\vec{T}_2$ and $\vec{T}_3$, we resolve these forces (see fig. C) into rectangular components. Then, from Newton's second law,

$$\Sigma F_x = T_2 \cos\theta_2 - T_3 \cos\theta_3 = 0,$$

$$\Sigma F_y = T_2 \sin\theta_2 + T_3 \sin\theta_3 - T_1 = 0$$

We have $\quad T_2 \cos 30° - T_3 \cos 60° = 0$

$$T_2 \sin 30° + T_3 \sin 60° = 50$$

or $\quad 0.866\, T_2 - 0.500\, T_3 = 0$

$$0.500\, T_2 + 0.866\, T_3 = 0$$

Solving these equations simultaneously, we find the tensions to be

$$T_2 = 25 \text{ lb}, \qquad T_3 = 43.3 \text{ lb}.$$

Finally, we know from Newton's third law that the inclined cords exert on the ceiling the forces $\vec{T}'_2$ and $\vec{T}'_3$, equal and opposite to $\vec{T}_2$ and $\vec{T}_3$, respectively.