(e) Converting the vectors into coordinate form and computing the dot product (scalar product):

$$(3\hat{x} + \hat{y} + 2\hat{z}) \cdot (2\hat{x} + 0\hat{y} + 0\hat{z}) =$$

$$6 + 0 + 0 = 6$$

(f) Find the form of $\vec{A}$ and $\vec{C}$ in a reference frame obtained from the old reference frame by a rotation of $\pi/2$ clockwise looking along the positive z axis. The new unit vectors $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ are related to the old $\hat{x}$, $\hat{y}$, $\hat{z}$ by (see fig. 3)

$$\hat{x}' = \hat{y}; \quad \hat{y}' = -\hat{x}; \quad \hat{z}' = \hat{z}.$$

Where $\hat{x}$ appeared we now have $-\hat{y}'$; where $\hat{y}$ appeared, we now have $\hat{x}'$, so that

$$A = \hat{x}' - 3\hat{y}' + 2\hat{z}'; \qquad C = -2\hat{y}'.$$

(g) Using the results of part (f), we convert the vectors $\vec{A}$ and $\vec{C}$ into coordinate form in the primed coordinate system, giving us the following dot product:

$$\vec{A} \cdot \vec{C} = (\hat{x}' - 3\hat{y}' + 2\hat{z}') \cdot (0\hat{x}' - 2\hat{y}' + 0\hat{z}') =$$

$$0 + 6 + 0 = 6$$

This is exactly the result obtained in the unprimed system.

(h) Find the vector product $\vec{A} \times \vec{C}$. In the unprimed system $\vec{A} \times \vec{C}$ is defined as

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & 1 & 2 \\ 2 & 0 & 0 \end{vmatrix} = 4\hat{y} - 2\hat{z}.$$

(i) Form the vector $\vec{A} - \vec{C}$. We have

$$\vec{A} - \vec{C} = (3 - 2)\hat{x} + \hat{y} + 2\hat{z} = \hat{x} + \hat{y} + 2\hat{z}.$$

DISPLACEMENT VECTORS

• PROBLEM 1-4

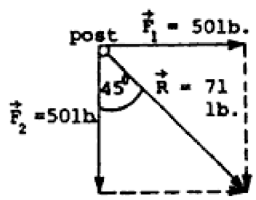
Two hikers set off in an eastward direction. Hiker 1 travels 3 km while hiker 2 travels 6 times the distance covered by hiker 1. What is the displacement of hiker 2?

Solution: From the information given the displacement vector is directed east. The magnitude of the displacement vector for hiker 2 is 6 times the magnitude of the displacement vector for hiker 1. Therefore, its magnitude is

$$6 \times (3 \text{ km}) = 18 \text{ km}$$

• PROBLEM 1-5

Two wires are attached to a corner fence post with the wires making an angle of 90° with each other. If each wire pulls on the post with a force of 50 pounds, what is the resultant force acting on the post? See Figure.



Solution: As shown in the figure, we complete the parallelogram. If we measure R and scale it, we find it is equal to about 71 pounds. The angle of the resultant is 45° from either of the component vectors.

If we use the fact that the component vectors are at right angles to each other, we can write

$$R^2 = 50^2 + 50^2$$

whence

R = 71 pounds approximately at 45° to each wire.

• PROBLEM 1-6

If a person walks 1 km north, 5 km west, 3 km south, and 7 km east, find the resultant displacement vector.

Fig. A

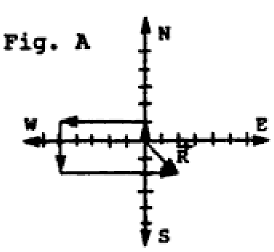
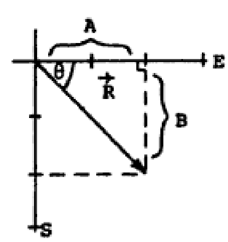


Fig. B



Solution: The vector diagram is shown in figure (a). The resultant displacement vector is labelled $\vec{R}$. The magnitude of this vector is 2.8 km. The direction, as measured with a protractor, is 45° south of east, or the tangent may be used to find the direction, since a right triangle is formed.

We shall also compute the solution analytically.

In figure (b) a closeup of the resultant vector $\vec{R}$ is shown. We can see from the graph that side A and side B each equal 2 km. Thus, by the Pythagorean theorem:

$$R^2 = A^2 + B^2 = (2\ \text{km})^2 + (2\ \text{km})^2 = 8\ \text{km}^2$$

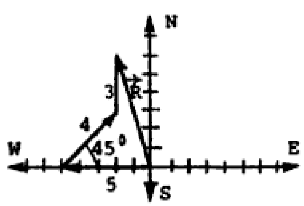
$$R = 2\sqrt{2}\ \text{km} = 2(1.4)\text{km} = 2.8\ \text{km}$$

$$\tan\theta = \frac{2\ \text{km}}{2\ \text{km}} = 1, \qquad \theta = 45°$$

$$\vec{R} = 2.8\ \text{km},\ 45°\ \text{south of east.}$$

• PROBLEM 1-7

An army recruit on a training exercise is instructed to walk due west for 5 mi, then in a northeasterly direction for 4 mi, and finally due north for 3 mi. When he completes his exercise, what is his resultant displacement $\vec{R}$? How far will he be from where he started?



Solution: The recruit's path is shown in the figure, where each division on the graph represents one mile.

We find $\vec{R}$ by first adding the components of his individual displacements which we regard as vectors. We will let $\vec{E}$ and $\vec{N}$, representing east and north, be our unit vectors, regarding western and southern displacements as being negative eastern and negative northern displacements, respectively. Assume north and east are given equal weights. Then $\overrightarrow{NE}$ is as shown in the diagram. Thus, the sum of the components is:

$\vec{E}$	$\vec{N}$
-5 mi	0 mi
4 cos 45° mi	4 sin 45° mi
0 mi	3 mi
(4 cos 45° − 5)mi	(4 sin 45° + 3)mi

$$\vec{R} = \left[4\left(\frac{1}{\sqrt{2}}\right) - 5\right] \text{mi } \vec{E} + \left[4\left(\frac{1}{\sqrt{2}}\right) + 3\right] \vec{N}$$

$$= (2.8 - 5)\text{mi } \vec{E} + (2.8 + 3)\text{mi } \vec{N}$$

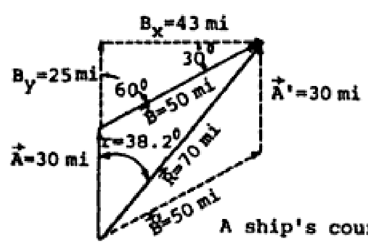
$$= -\,2.2 \text{ mi } \vec{E} + 5.8 \text{ mi } \vec{N}$$

The recruit's final distance from the starting point will be the magnitude of $\vec{R}$:

$$R = \sqrt{(-\,2.2 \text{ mi})^2 + (5.8 \text{ mi})^2} = 6.20 \text{ mi}$$

• PROBLEM 1-8

A ship leaving its port sails due north for 30 miles and then 50 miles in a direction 60° east of north. See the Figure. At the end of this time where is the ship relative to its port?



A ship's course

<u>Solution by Parallelogram Method:</u>

The figure shows the parallelogram completed by the dashed vectors $\vec{A}'$ and $\vec{B}'$. Also shown is the resultant $\vec{R}$ which is found to represent about 70 miles. Angle r is found to be about 38.2° east of north.

<u>Solution by Component Method:</u>

The figure also shows the vector $\vec{B}$ resolved into the components $\vec{B}_x$ and $\vec{B}_y$, which are found to be 43 miles and 25 miles, respectively. (By trigonometry

$\vec{B}_x$ = 50 miles × cos 30° = 43 miles, and

$\vec{B}_y$ = 50 miles × sin 30° = 25 miles).Since $\vec{A}$ and $\vec{B}$ lie along the same direction in this problem, we add them directly to get 30 miles + 25 miles, or 55 miles. We then have a right triangle with one side equal to 55 miles and the other side equal to 43 miles. From these data we find the resultant R according to the equation:

$$R^2 = 55^2 + 43^2$$

whence R = about 70 miles

Solution by the Cosine Law:

In solving this problem by means of the cosine law, we write

$$R^2 = A^2 + B^2 + 2\,AB\cos\theta$$

$$R^2 = 30^2 + 50^2 + 2 \times 30 \times 50 \times 0.5000$$

$$= 4900$$

whence the magnitude of R is

$$R = 70 \text{ miles}$$

$$\tan r = \frac{B\sin\theta}{A + B\cos\theta} = \frac{50 \times 0.866}{30 + 50 \times 0.500}$$

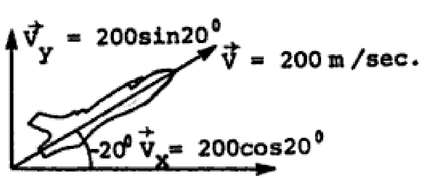
$$= 0.788$$

whence $r = 38.2°$ approximately.

VELOCITY VECTORS

• PROBLEM 1-9

An aircraft is climbing with a steady speed of 200 m/sec at an angle of 20° to the horizontal (see figure). What are the horizontal and vertical components of its velocity?

Fig. A


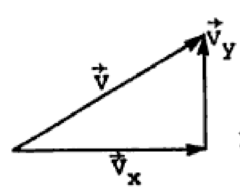


Fig. B

Solution: Using trigonometric relations for right triangles, the velocity can be broken down into two components perpendicular to each other.
Horizontal component = 200 cos 20°

Vertical component = 200 sin 20°.

Trigonometric tables tell us that

cos 20° = 0.9397 and sin 20° = 0.3420

Therefore, horizontal component = 200 × 0.9397
= 187.94 m/sec

Vertical component = 200 × 0.3420
= 68.40 m/sec.

Notice that the sum of 187.94 and 68.40 is not 200, but you can check that $(187.94)^2 + (68.40)^2 = (200)^2$. This occurs because the horizontal and vertical components, $\vec{V}_x$ and $\vec{V}_y$, of the velocity are vectors and must be added accordingly. Since they are pernependicular to each other, forming a right triangle with $\vec{V}$ as the hypotenuse,

$$V_x^2 + V_y^2 = V^2$$

• PROBLEM 1-10

An automobile driver, A, traveling relative to the earth at 65 mi/hr on a straight, level road, is ahead of motorcycle officer B, traveling in the same direction at 80 mi/hr. What is the velocity of B relative to A? Find the same quantity if B is ahead of A.

Solution: The velocity of B relative to A is equal to the velocity of B relative to the earth minus the velocity of A relative to the earth, or

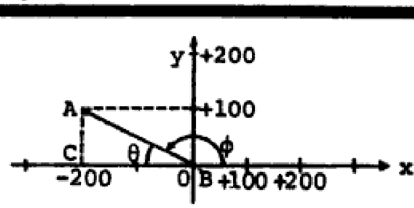
$$V_{BA} = V_{BE} - V_{AE} = 80 \text{ mi/hr} - 65 \text{ mi/hr} = 15 \text{ mi/hr}$$

If B is ahead of A, the velocity of B relative to A is still the velocity of B relative to the earth minus the velocity of A relative to the earth or 15 mi/hr.

In the first case, B is overtaking A, and, in the second, B is pulling ahead of A.

• PROBLEM 1-11

City A is 100 miles north and 200 miles west of city B. An airplane flies in a direct line between the cities in a time of one hour. What are the vectors that describe the distance of A from B, and the velocity of the airplane?



Solution: We will define first a coordinate system with B at the origin (see the figure below). The x-direction is east and the y-direction is north. The vector BA is specified by its coordinates

$$x = -200 \text{ mi}$$
$$y = 100 \text{ mi}$$

or by its magnitude and direction

$$(BA)^2 = x^2 + y^2$$
$$= ((-200)^2 + (100)^2)\text{mi}^2$$

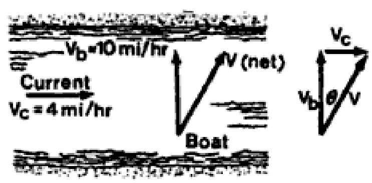
$$BA = 100\sqrt{5} \text{ mi}$$
$$\sin\theta = \frac{CA}{BA} = \frac{100}{100}\sqrt{5} = \frac{1}{\sqrt{5}}$$
$$\theta = 26.5^\circ$$
$$\phi = 180^\circ - \theta = 153.5^\circ$$

The velocities are given in a similar way. Since they are constant

$$v_x = \frac{x}{1 \text{ hr}} = \frac{-200 \text{ mi}}{1 \text{ hr}} = -200 \text{ mi/hr}$$
$$v_y = \frac{y}{1 \text{ hr}} = \frac{100 \text{ mi}}{1 \text{ hr}} = 100 \text{ mi/hr}$$
$$v^2 = v_x^2 + v_y^2 = ((-200)^2 + (100)^2)\text{mi}^2/\text{hr}^2$$
$$v = 100\sqrt{5} \text{ mi/hr}$$
$$\phi = 153.5^\circ$$

• PROBLEM 1-12

A certain boat can move at a speed of 10 mi/hr in still water. The helmsman steers straight across a river in which the current is 4 mi/hr. What is the velocity of the boat?



Solution: The boat has a speed of v_b = 10 mi/hr perpendicular to the river due to the power of the boat. The current gives it a speed of v_c = 4 mi/hr in the direction of flow of the river. The boat's resultant velocity (having both magnitude and direction) can be found through vector addition.