UNITS CONVERSION FACTORS

This section includes a particularly useful and comprehensive table to aid students and teachers in converting between systems of units.

The problems and their solutions in this book use SI (International System) as well as English units. Both of these units are in extensive use throughout the world, and therefore students should develop a good facility to work with both sets of units until a single standard of units has been found acceptable internationally.

In working out or solving a problem in one system of units or the other, essentially only the numbers change. Also, the conversion from one unit system to another is easily achieved through the use of conversion factors that are given in the subsequent table. Accordingly, the units are one of the least important aspects of a problem. For these reasons, a student should not be concerned mainly with which units are used in any particular problem. Instead, a student should obtain from that problem and its solution an understanding of the underlying principles and solution techniques that are illustrated there.

To convert	To	Multiply by	For the reverse, multiply by
acres	square feet	4.356×10^4	2.296×10^{-5}
acres	square meters	4047	2.471×10^{-4}
ampere-hours	coulombs	3600	2.778×10^{-4}
ampere-turns	gilberts	1.257	0.7958
ampere-turns per cm.	ampere-turns per inch	2.54	0.3937
angstrom units	inches	3.937×10^{-9}	2.54×10^8
angstrom units	meters	10^{-10}	10^{10}
atmospheres	feet of water	33.90	0.02950
atmospheres	inch of mercury at 0°C	29.92	3.342×10^{-2}
atmospheres	kilogram per square meter	1.033×10^4	9.678×10^{-5}
atmospheres	millimeter of mercury at 0°C	760	1.316×10^{-3}
atmospheres	pascals	1.0133×10^5	0.9869×10^{-5}
atmospheres	pounds per square inch	14.70	0.06804
bars	atmospheres	9.870×10^{-7}	1.0133
bars	dynes per square cm.	10^6	10^{-6}
bars	pascals	10^5	10^{-5}
bars	pounds per square inch	14.504	6.8947×10^{-2}
Btu	ergs	1.0548×10^{10}	9.486×10^{-11}
Btu	foot-pounds	778.3	1.285×10^{-3}
Btu	joules	1054.8	9.480×10^{-4}
Btu	kilogram-calories	0.252	3.969
calories, gram	Btu	3.968×10^{-3}	252
calories, gram	foot-pounds	3.087	0.324
calories, gram	joules	4.185	0.2389
Celsius	Fahrenheit	$(°C \times 9/5) + 32 = °F$	$(°F - 32) \times 5/9 = °C$

To convert	To	Multiply	For the reverse, multiply by
Celsius	kelvin	°C + 273.1 = K	K − 273.1 = °C
centimeters	angstrom units	1×10^{8}	1×10^{-8}
centimeters	feet	0.03281	30.479
centistokes	square meters per second	1×10^{-6}	1×10^{6}
circular mils	square centimeters	5.067×10^{-6}	1.973×10^{5}
circular mils	square mils	0.7854	1.273
cubic feet	gallons (liquid U.S.)	7.481	0.1337
cubic feet	liters	28.32	3.531×10^{-2}
cubic inches	cubic centimeters	16.39	6.102×10^{-2}
cubic inches	cubic feet	5.787×10^{-4}	1728
cubic inches	cubic meters	1.639×10^{-5}	6.102×10^{4}
cubic inches	gallons (liquid U.S.)	4.329×10^{-3}	231
cubic meters	cubic feet	35.31	2.832×10^{-2}
cubic meters	cubic yards	1.308	0.7646
curies	coulombs per minute	1.1×10^{12}	0.91×10^{-12}
cycles per second	hertz	1	1
degrees (angle)	mils	17.45	5.73×10^{-2}
degrees (angle)	radians	1.745×10^{-2}	57.3
dynes	pounds	2.248×10^{-6}	4.448×10^{5}
electron volts	joules	1.602×10^{-19}	0.624×10^{18}
ergs	foot-pounds	7.376×10^{-8}	1.356×10^{7}
ergs	joules	10^{-7}	10^{7}
ergs per second	watts	10^{-7}	10^{7}
ergs per square cm.	watts per square cm.	10^{-3}	10^{3}
Fahrenheit	kelvin	(°F + 459.67)/1.8	1.8K − 459.67
Fahrenheit	Rankine	°F + 459.67 = °R	°R − 459.67 = °F
faradays	ampere-hours	26.8	3.731×10^{-2}
feet	centimeters	30.48	3.281×10^{-2}
feet	meters	0.3048	3.281
feet	mils	1.2×10^{4}	8.333×10^{-5}
fermis	meters	10^{-15}	10^{15}
foot candles	lux	10.764	0.0929
foot lamberts	candelas per square meter	3.4263	0.2918
foot-pounds	gram-centimeters	1.383×10^{4}	1.235×10^{-5}
foot-pounds	horsepower-hours	5.05×10^{-7}	1.98×10^{6}
foot-pounds	kilogram-meters	0.1383	7.233
foot-pounds	kilowatt-hours	3.766×10^{-7}	2.655×10^{6}
foot-pounds	ounce-inches	192	5.208×10^{-3}
gallons (liquid U.S.)	cubic meters	3.785×10^{-3}	264.2
gallons (liquid U.S.)	gallons (liquid British Imperial)	0.8327	1.201
gammas	teslas	10^{-9}	10^{9}
gausses	lines per square cm.	1.0	1.0
gausses	lines per square inch	6.452	0.155
gausses	teslas	10^{-4}	10^{4}
gausses	webers per square inch	6.452×10^{-8}	1.55×10^{7}
gilberts	amperes	0.7958	1.257
grads	radians	1.571×10^{-2}	63.65
grains	grams	0.06480	15.432
grains	pounds	$^{1}/_{7000}$	7000
grams	dynes	980.7	1.02×10^{-3}
grams	grains	15.43	6.481×10^{-2}

To convert	To	Multiply	For the reverse, multiply by
grams	ounces (avdp)	3.527×10^{-2}	28.35
grams	poundals	7.093×10^{-2}	14.1
hectares	acres	2.471	0.4047
horsepower	Btu per minute	42.418	2.357×10^{-2}
horsepower	foot-pounds per minute	3.3×10^{4}	3.03×10^{-5}
horsepower	foot-pounds per second	550	1.182×10^{-3}
horsepower	horsepower (metric)	1.014	0.9863
horsepower	kilowatts	0.746	1.341
inches	centimeters	2.54	0.3937
inches	feet	8.333×10^{-2}	12
inches	meters	2.54×10^{-2}	39.37
inches	miles	1.578×10^{-5}	6.336×10^{4}
inches	mils	10^{3}	10^{-3}
inches	yards	2.778×10^{-2}	36
joules	foot-pounds	0.7376	1.356
joules	watt-hours	2.778×10^{-4}	3600
kilograms	tons (long)	9.842×10^{-4}	1016
kilograms	tons (short)	1.102×10^{-3}	907.2
kilograms	pounds (avdp)	2.205	0.4536
kilometers	feet	3281	3.408×10^{-4}
kilometers	inches	3.937×10^{4}	2.54×10^{-5}
kilometers per hour	feet per minute	54.68	1.829×10^{-2}
kilowatt-hours	Btu	3413	2.93×10^{-4}
kilowatt-hours	foot-pounds	2.655×10^{6}	3.766×10^{-7}
kilowatt-hours	horsepower-hours	1.341	0.7457
kilowatt-hours	joules	3.6×10^{6}	2.778×10^{-7}
knots	feet per second	1.688	0.5925
knots	miles per hour	1.1508	0.869
lamberts	candles per square cm.	0.3183	3.142
lamberts	candles per square inch	2.054	0.4869
liters	cubic centimeters	10^{3}	10^{-3}
liters	cubic inches	61.02	1.639×10^{-2}
liters	gallons (liquid U.S.)	0.2642	3.785
liters	pints (liquid U.S.)	2.113	0.4732
lumens per square foot	foot-candles	1	1
lumens per square meter	foot-candles	0.0929	10.764
lux	foot-candles	0.0929	10.764
maxwells	kilolines	10^{-3}	10^{3}
maxwells	webers	10^{-8}	10^{8}
meters	feet	3.28	30.48×10^{-2}
meters	inches	39.37	2.54×10^{-2}
meters	miles	6.214×10^{-4}	1609.35
meters	yards	1.094	0.9144
miles (nautical)	feet	6076.1	1.646×10^{-4}
miles (nautical)	meters	1852	5.4×10^{-4}
miles (statute)	feet	5280	1.894×10^{-4}
miles (statute)	kilometers	1.609	0.6214
miles (statute)	miles (nautical)	0.869	1.1508
miles per hour	feet per second	1.467	0.6818
miles per hour	knots	0.8684	1.152
millimeters	microns	10^{3}	10^{-3}

To convert	To	Multiply	For the reverse, multiply by
mils	meters	2.54×10^{-5}	3.94×10^{4}
mils	minutes	3.438	0.2909
minutes (angle)	degrees	1.666×10^{-2}	60
minutes (angle)	radians	2.909×10^{-4}	3484
newtons	dynes	10^{5}	10^{-5}
newtons	kilograms	0.1020	9.807
newtons per sq. meter	pascals	1	1
newtons	pounds (avdp)	0.2248	4.448
oersteds	amperes per meter	7.9577×10	1.257×10^{-2}
ounces (fluid)	quarts	3.125×10^{-2}	32
ounces (avdp)	pounds	6.25×10^{-2}	16
pints	quarts (liquid U.S.)	0.50	2
poundals	dynes	1.383×10^{4}	7.233×10^{-5}
poundals	pounds (avdp)	3.108×10^{-2}	32.17
pounds	grams	453.6	2.205×10^{-3}
pounds (force)	newtons	4.4482	0.2288
pounds per square inch	dynes per square cm.	6.8946×10^{4}	1.450×10^{-5}
pounds per square inch	pascals	6.895×10^{3}	1.45×10^{-4}
quarts (U.S. liquid)	cubic centimeters	946.4	1.057×10^{-3}
radians	mils	10^{3}	10^{-3}
radians	minutes of arc	3.438×10^{3}	2.909×10^{-4}
radians	seconds of arc	2.06265×10^{5}	4.848×10^{-6}
revolutions per minute	radians per second	0.1047	9.549
roentgens	coulombs per kilogram	2.58×10^{-4}	3.876×10^{3}
slugs	kilograms	1.459	0.6854
slugs	pounds (avdp)	32.174	3.108×10^{-2}
square feet	square centimeters	929.034	1.076×10^{-3}
square feet	square inches	144	6.944×10^{-3}
square feet	square miles	3.587×10^{-8}	27.88×10^{6}
square inches	square centimeters	6.452	0.155
square kilometers	square miles	0.3861	2.59
stokes	square meter per second	10^{-4}	10^{4}
tons (metric)	kilograms	10^{3}	10^{-3}
tons (short)	pounds	2000	5×10^{-4}
torrs	newtons per square meter	133.32	7.5×10^{-3}
watts	Btu per hour	3.413	0.293
watts	foot-pounds per minute	44.26	2.26×10^{-2}
watts	horsepower	1.341×10^{-3}	746
watt-seconds	joules	1	1
webers	maxwells	10^{8}	10^{-8}
webers per square meter	gausses	10^{4}	10^{-4}

CHAPTER 1

VECTORS

Basic Attacks and Strategies for Solving Problems in this Chapter

Physics deals with many geometric objects such as scalars and vectors. A scalar *is a quantity which has only a magnitude such as length, temperature, and speed. A* vector *(see Figure 1) is a quantity which has both magnitude and direction such as displacement, velocity, and force. The magnitude is given by the length of the vector and a suitable scale, and the direction by the arrow in the figure. Except in relativity, in physics vectors have two (in two dimensions) or three (in three dimensions) components.*

Consider two displacement vectors represented in Cartesian coordinates as

$$\vec{A} = A_x\hat{x} + A_y\hat{y} = (A_x, A_y), \textit{ and } \vec{B} = B_x\hat{x} + B_y\hat{y} = (B_x, B_y).$$

The first notation is called unit vector notation. *The second is called coordinate, or* point notation. *The magnitude of either vector (see Figure 2) may be found by the theorem of Pythagoras*

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

and the direction by the tangent rule $\tan\theta = A_y/A_x$. *Equivalently, one may use* $\sin\theta = A_y/A$ *or* $\cos\theta = A_x/A$.

The two vectors $\vec{A}$ *and* $\vec{B}$ *may be added (See Figure 3) to get the resultant, or sum*

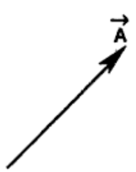


Figure 1

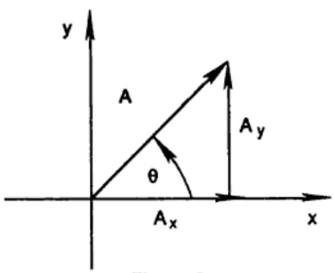


Figure 2

$$\vec{R} = (R_x, R_y) = (A_x + B_x, A_y + B_y).$$

The direction is again given by finding $\theta = Arctan\ (R_y / R_x)$ *such that* $\tan \theta = R_y / R_x$. *A similar rule applies for finding the difference. Note that this is equivalent to walking first along vector* $\vec{A}$ *and then along vector* $\vec{B}$ *to get to the resultant location* $\vec{R}$. *This method of adding the components and using the Pythagorean theorem and trigonometry is called the* component method. *It can easily be generalized to adding more than two vectors or dealing with higher dimensions. For example, in three dimensions, one would get*

$$\vec{R} = (A_x + B_x + C_x)\hat{x} + (A_y + B_y + C_y)\hat{y} + (A_z + B_z + C_z)\hat{z}$$

in unit vector notation.

The other equivalent way to add vectors is by Newton's parallelogram rule. *One connects the vectors head to tail, just as in the component method. One could then find the magnitude of* $\vec{R}$ *and the direction angle* θ *graphically using a ruler and protractor as in a force table laboratory exercise. Analytically, one can use geometry and the laws of cosines and sines. From geometry and Figure 3,* $\angle R = \theta_A + 180° - \theta_B$. *The law of cosines then gives*

$$R = \sqrt{A^2 + B^2 + -2AB \cos \angle R}\,.$$

The law of sines states that $\sin \angle R / R = \sin \angle B / B$. *One may thus find* $\angle B$ *and from it get the direction of the resultant* $\theta = \theta_A + \angle B$.

Vectors cannot be multiplied or divided as scalars can. However, there are two special products: the dot product and the cross product. The dot product, *or two vectors* $\vec{A}$ *and* $\vec{B}$, *is a scalar* $C = \vec{A} \cdot \vec{B} = AB \cos \theta = A_xB_x + A_yB_y + A_zB_z$. *The dot product may be used to find the work done by a force exerted over a certain distance, for example. The cross product is more complicated and is given by* $\vec{C} = \vec{A} \times \vec{B}$ *where*

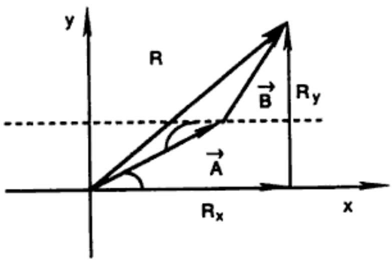


Figure 3

$$\vec{C} = (A_yB_z - A_zB_y)\hat{x} + (A_zB_x - A_xB_z)\hat{y} + (A_xB_y - A_yB_x)\hat{z}$$

For example, if A *points in the* x*–direction and* B *in the* y*–direction, then* $\vec{C} = A_xB_y\hat{z} = AB\,\hat{z}$*. More conveniently, the cross product has a magnitude given by* $C = AB \sin\theta$, θ *being the angle from* A *to* B*, and a direction given by the* right hand rule. *Coil the fingers of your right hand from* $\vec{A}$ *to* $\vec{B}$ *and stick out the thumb; your thumb then points in the direction of* C. *For example, if* $\vec{A}$ *and* $\vec{B}$ *are in the xy plane (Figure 4), then the cross product points in the z–direction. The cross product is used in physics to find the torque exerted by a force acting at a certain position.*

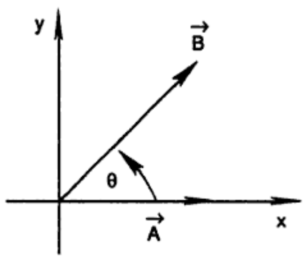


Figure 4

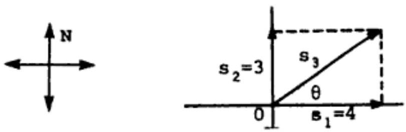
CHAPTER 1

VECTORS

VECTOR FUNDAMENTALS

• **PROBLEM 1-1**

Find the resultant of the vectors $\vec{S}_1$ and $\vec{S}_2$ specified in the figure.



<u>Solution.</u> From the Pythagorean theorem, $S_1^2 + S_2^2 = S_3^2$, or $4^2 + 3^2 = S_3^2$, and so we get $S_3 = 5$ units. The direction of S_3 may be specified by the angle θ which it makes with S_1.

$$\sin\theta = \frac{S_2}{S_3} = 0.60 \quad \text{gives } \theta = 37°.$$

Resultant $\vec{S}_3$ therefore represents a displacement of 5 units from 0 in the direction 37° north of east.

• **PROBLEM 1-2**

Three forces acting at a point are $\vec{F}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{F}_2 = -\hat{i} + 3\hat{j} + 2\hat{k}$, and $\vec{F}_3 = -\hat{i} + 2\hat{j} - \hat{k}$. Find the directions and magnitudes of $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$, $\vec{F}_1 - \vec{F}_2 + \vec{F}_3$, and $\vec{F}_1 + \vec{F}_2 - \vec{F}_3$.

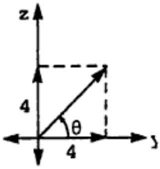


Fig. A

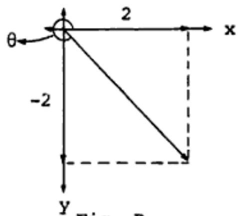


Fig. B

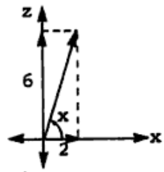


Fig. C

<u>Solution:</u> When vectors are added (or subtracted), their components in the directions of the unit vectors add (or subtract) algebraically. Thus since

$$\vec{F}_1 = 2\hat{i} - \hat{j} + 3\hat{k},\quad \vec{F}_2 = -\hat{i} + 3\hat{j} + 2\hat{k},\quad \vec{F}_3 = -\hat{i} + 2\hat{j} - \hat{k},$$

then it follows that

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (2 - 1 - 1)\hat{i} + (-1 + 3 + 2)\hat{j} + (3 + 2 - 1)\hat{k}$$
$$= 0\hat{i} + 4\hat{j} + 4\hat{k}.$$

Similarly,

$$\vec{F}_1 - \vec{F}_2 + \vec{F}_3 = \left[2 - (-1) - 1\right]\hat{i} + \left[-1 - (3) + 2\right]\hat{j} + \left[3 - (2) - 1\right]\hat{k}$$
$$= 2\hat{i} - 2\hat{j} + 0\hat{k}$$

and

$$\vec{F}_1 + \vec{F}_2 - \vec{F}_3 = \left[2 - 1 - (-1)\right]\hat{i} + \left[-1 + 3 - (2)\right]\hat{j} + \left[3 + 2 - (-1)\right]\hat{k}$$
$$= 2\hat{i} + 0\hat{j} + 6\hat{k}$$

The vector $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ thus has no component in the x-direction, one of 4 units in the y-direction, and one of 4 units in the z-direction. It therefore has a magnitude of $\sqrt{4^2 + 4^2}$ units = $4\sqrt{2}$ units = 5.66 units, and lies in the y- z plane, making an angle θ with the y - axis, as shown in figure (a), where tan θ = 4/4 = 1. Thus θ = 45°.

Similarly, $\vec{F}_1 - \vec{F}_2 + \vec{F}_3$ has a magnitude of $2\sqrt{2}$ units = 2.82 units, and lies in the x - y plane, making an angle ϕ with the x-axis, as shown in figure (b), where tan ϕ = + 2/-2 = - 1. Thus ϕ = 315°.

Also, $\vec{F_1} + \vec{F_2} - \vec{F_3}$ has a magnitude of $\sqrt{2^2 + 6^2}$ units $= 2\sqrt{10}$ units $= 6.32$ units, and lies in the x - z plane at an angle χ to the x-axis, as shown in figure (c), where $\tan \chi = 6/2 = 3$. Thus $\chi = 71°34'$.

• PROBLEM 1-3

We consider the vector

$$\vec{A} = 3\hat{x} + \hat{y} + 2\hat{z}$$

(a) Find the length of $\vec{A}$.
(b) What is the length of the projection of $\vec{A}$ on the xy plane?
(c) Construct a vector in the xy plane and perpendicular to $\vec{A}$.
(d) Construct the unit vector $\hat{B}$.
(e) Find the scalar product with $\vec{A}$ of the vector $\vec{C} = 2\hat{x}$.
(f) Find the form of $\vec{A}$ and $\vec{C}$ in a reference frame obtained from the old reference frame by a rotation of $\pi/2$ clockwise looking along the positive z axis.
(g) Find the scalar product $\vec{A} \cdot \vec{C}$ in the primed coordinate system.
(h) Find the vector product $\vec{A} \times \vec{C}$.
(i) Form the vector $\vec{A} - \vec{C}$.

Fig. 1

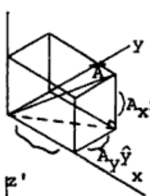


Fig. 2

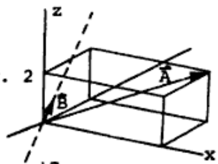


Fig. 3

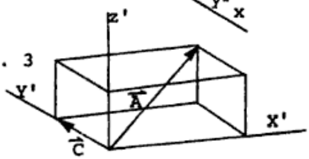
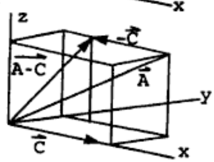


Fig. 4



The primed reference frame x', y', z', is generated from the unprimed system x, y, z, by a rotation of $+\pi/2$ about the z axis.

Solution: (a) When a vector is given in the form $A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$, its length is given by $\sqrt{A_x^2 + A_y^2 + A_z^2}$.

This can be seen from diagram 1. Vector $\vec{A}$ has components in the x, y and z directions. The x and y components form the legs of a right triangle. By the Pythagorean theorem the length of the hypotenuse of this triangle is $\sqrt{A_x^2 + A_y^2}$. But this line segment whose length is $\sqrt{A_x^2 + A_y^2}$ is one leg in a right triangle whose other leg is $A_z\hat{z}$ and whose hypotenuse is vector $\vec{A}$. Applying the Pythagorean theorem again, we find that the length of $\vec{A}$ is $\sqrt{A_x^2 + A_y^2 + A_z^2}$. Substituting our values we have $\sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}$.

(b) We refer again to diagram 1. The projection of $\vec{A}$ on the xy plane is simply the dotted line which is the vector $A_x\hat{x} + A_y\hat{y}$. Its length is $\sqrt{A_x^2 + A_y^2}$ by the Pythagorean theorem. In our problem, the length is $\sqrt{3^2 + 1^2} = \sqrt{10}$.

(c) Construct a vector in the xy plane and perpendicular to A. We want a vector of the form

$$B = B_x\hat{x} + B_y\hat{y}$$

with the property $A \cdot B = 0$ (since $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos \phi$ where ϕ is the angle between $\vec{A}$ and $\vec{B}$). Hence

$$(3\hat{x} + \hat{y} + 2\hat{z}) \cdot (B_x\hat{x} + B_y\hat{y}) = 0.$$

On taking the scalar product we find

$$3B_x + B_y = 0,$$

or
$$\frac{B_y}{B_x} = -3.$$

The length of the vector B is not determined by the specification of the problem. We have therefore determined just the slope of vector B, not its magnitude. See diagram 2.

(d) The unit vector B is the vector in the B direction but with the magnitude 1. It lies in the xy plane, and its slope (B_y/B_x) is equal to -3. Therefore, $\hat{B}$ must satisfy the following two equations:

$$\hat{B}_x^2 + \hat{B}_y^2 = 1$$

$$\frac{\hat{B}_y}{\hat{B}_x} = -3$$

Solving simultaneously we have: $\hat{B}_x^2 + (-3\hat{B}_x)^2 = 1$ or $\hat{B}_x = 1/\sqrt{10}$ and $\hat{B}_y = -3/\sqrt{10}$.

The vector B is then:

$$\hat{B} = (1/\sqrt{10})\hat{x} - (3/\sqrt{10})\hat{y}$$