Асимптоте - мало 😊

$f: \mathbb{R} \to \mathbb{R}$

$x \to +\infty$ $\quad y = ax + b$ асимптота $\Leftrightarrow$ $f(x) = ax + b + o(1),\ x \to +\infty$

$$a = \lim_{x \to +\infty} \frac{f(x)}{x}$$

$$b = \lim_{x \to +\infty} (f(x) - a \cdot x)$$

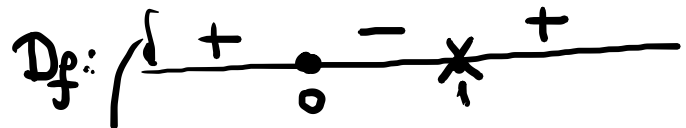
а, вето

$a \neq 0$ → коса асимптота (К.А.) $\quad a = 0$ $\quad y = b$ — хоризонтална ас. (Х.А.)

$x \to -\infty$ - аналогично

① $f(x) = \sqrt{\frac{x^3}{x-1}}$ асимптота? $\quad x \to +\infty,\ x \to -\infty$

D_f: $\frac{x^3}{x-1}$



$D_f = (-\infty, 0] \cup (1, +\infty)$

$f(x)=\sqrt{\frac{x^3}{x-1}}$ $\sim \sqrt{\frac{x^3}{x}} \sim \sqrt{x^2}=|x|$

$f(x)=ax+b+o(1)$

$f(x)=\sqrt{x^2\cdot\frac{x}{x-1}}=|x|\cdot\sqrt{\frac{x}{x-1}}=|x|\cdot\sqrt{\frac{1}{1-1/x}}$

$=|x|\cdot\left(1-\frac{1}{x}\right)^{-1/2}$ $t=-\frac{1}{x}\to 0$ $x\to+\infty$ $x\to-\infty$

$=|x|\cdot\left(1+\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{x}\right)+o\left(-\frac{1}{x}\right)\right)$ $x\to+\infty$ $x\to-\infty$

$=|x|\cdot\left(1+\frac{1}{2x}+o\left(\frac{1}{x}\right)\right)$

$(1+t)^\alpha \underset{t\to 0}{=} 1+\alpha t+o(t)$

$x\to+\infty/-\infty$, $\frac{1}{x}\to 0$, t

$x\to+\infty$: $|x|=x$

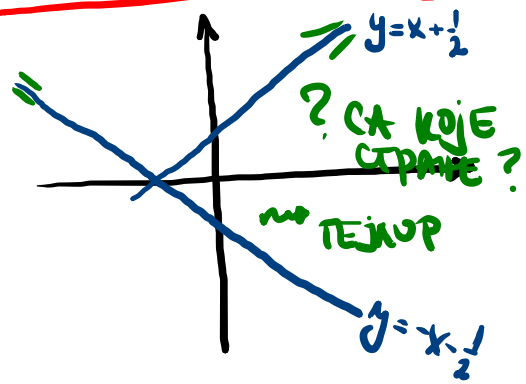
$f(x)\overset{\downarrow}{=} x\cdot\left(1+\frac{1}{2x}+o\left(\frac{1}{x}\right)\right)=x+\frac{1}{2}+\underbrace{x\cdot o\left(\frac{1}{x}\right)}_{=o(1)}=\boxed{x+\frac{1}{2}}+o(1)$

$\boxed{y=x+\frac{1}{2}\ \text{K.A.}\ x\to+\infty}$

$x\to-\infty$: $|x|=-x$

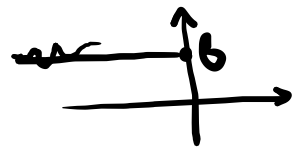
$f(x)\overset{\downarrow}{=}-x\cdot\left(1+\frac{1}{2x}+o\left(\frac{1}{x}\right)\right)=\boxed{-x-\frac{1}{2}}+o(1)$

$\boxed{y=-x-\frac{1}{2}\ \text{K.A.}\ x\to-\infty}$



② Асимптоти, ако ∃, $x\to+\infty$ и $x\to-\infty$: $f(x)=2\ln|e^x-1|-\ln|e^x-2|$

Домен ? - за вежбу



① $x\to-\infty$: $y=b$ хор. А., $x\to-\infty \iff \lim\limits_{x\to-\infty} f(x)=b$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty} 2\cdot\ln|e^x-1|-\ln|e^x-2| = 2\cdot\frac{\ln 1}{0}-\ln 2 = -\ln 2 \Rightarrow \boxed{y=-\ln 2 \text{ хор. А. } x\to-\infty}$$

($e^x\to 0$: $e^x-1\to -1$, $e^x-2\to -2$)

② $x\to+\infty$: $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\ln\frac{(e^x-1)^2}{e^x-2}\overset{\cdots}{=}+\infty \Rightarrow$ нема хор. А. кад $x\to+\infty$

($\frac{(e^x-1)^2}{e^x-2}\to+\infty$, $x\to+\infty$)

$$f(x)=2\ln(e^x-1)-\ln(e^x-2)=2\cdot\ln\left(e^x\left(1-\frac{1}{e^x}\right)\right)-\ln\left(e^x\left(1-\frac{2}{e^x}\right)\right)$$

$e^x-1>0$, $e^x>2$, $x>\ln 2$

$$=\underbrace{2\ln e^x}_{=2x}+2\cdot\ln\underbrace{\left(1-\frac{1}{e^x}\right)}_{\to 1,\,x\to+\infty}-\underbrace{\ln e^x}_{=x}-\ln\underbrace{\left(1-\frac{2}{e^x}\right)}_{\to 1} = 2x+o(1)-x-o(1),\ x\to+\infty$$

($\ln\left(1-\frac{1}{e^x}\right)\to 0$, $\ln\left(1-\frac{2}{e^x}\right)\to 0$, $x\to+\infty$)

$$=x+o(1),\ x\to+\infty$$

$$\boxed{y=x \text{ хор. А. } x\to+\infty}$$

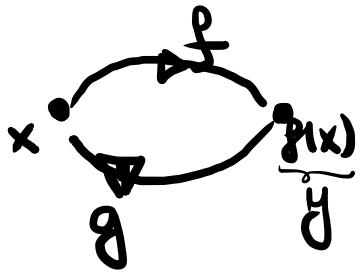
* Како налазимо инверзну фју ?

$$f(x)=\begin{cases} x+1, & x\le -1 \\ \sqrt{1-x^2}, & -1<x<0 \\ 2^x, & x\ge 0 \end{cases}$$

$f:\mathbb{R}\to\mathbb{R}$

1-1, НА ✓

Бијекција

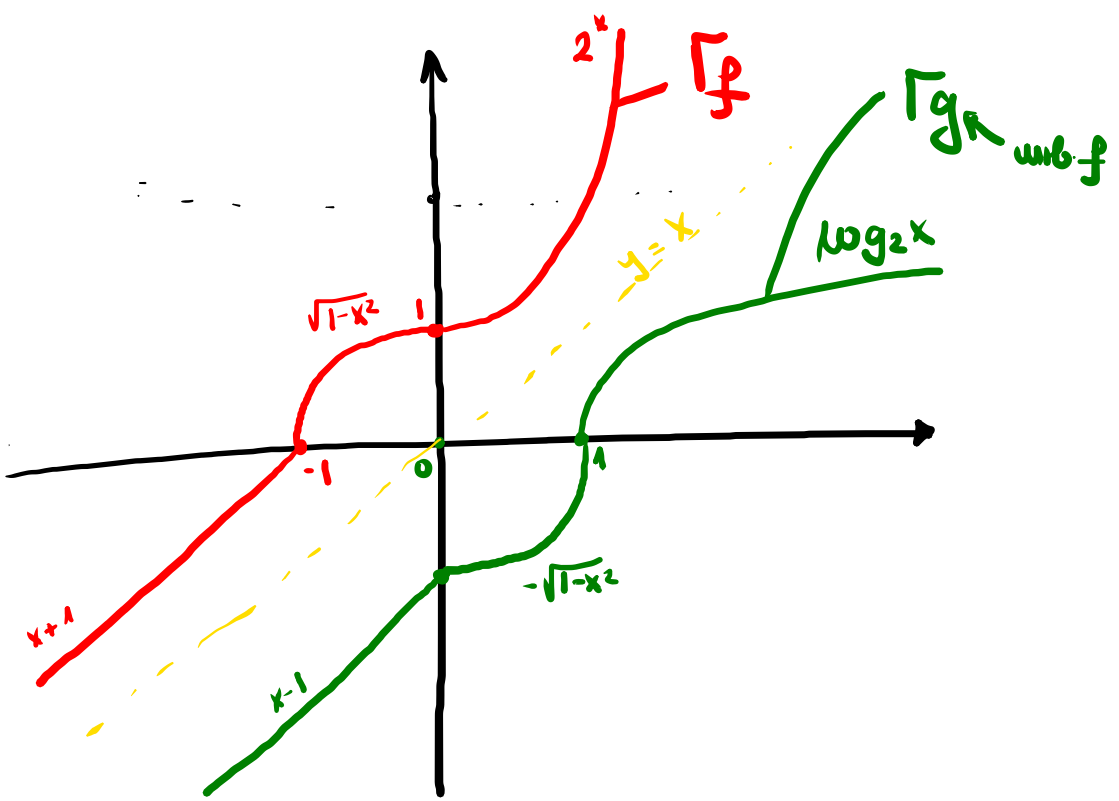


$g:\mathbb{R}\to\mathbb{R}$ инверзна фја

$g(f(x))=x$

$g(y)=x$

! изражавамо x преко y



① $x\le -1$ $f(x)=x+1=y$

$x\in(-\infty,-1] \to y\in(-\infty,0]$

$x+1=y$

$x=y-1$

$g(y)=x \Rightarrow$ $g(y)=y-1$, $y\in(-\infty,0]$

② $x \in (-1,0)$

$f(x) = \sqrt{1-x^2} = y$ $\quad g(y) = x$

$1-x^2 = y^2$

$x^2 = 1-y^2$

$x < 0 \quad x = -\sqrt{1-y^2}$

čemu je y? $\quad y = \sqrt{1-x^2}, \ x \in (-1,0)$

$y \in (0,1)$

$$g(y) = -\sqrt{1-y^2}, \ y \in (0,1)$$

③ $x \in [0,+\infty)$

$f(x) = 2^x \rightarrow y \in [1,+\infty)$

y

$x = \log_2 y$

$$g(y) = \log_2 y$$

fja inverzna f:

$$g(y) = \begin{cases} y-1, & y \in (-\infty, 0] \\ -\sqrt{1-y^2}, & y \in (0,1) \\ \log_2 y, & y \in [1,+\infty) \end{cases}$$