17.5.2021.

~ Испитивање функција ~

$f(x) = \dots$

$\boxed{1}$ $D_f = ?$

$\boxed{2}$ парност/непар., периодичност

$\boxed{3}$ нуле, знак

$\boxed{4}$ асимптоте, понашање на крајевима домена

$\boxed{5}$ непр., диф.

$\boxed{6}$ $f'(x)$, интервали монотоније

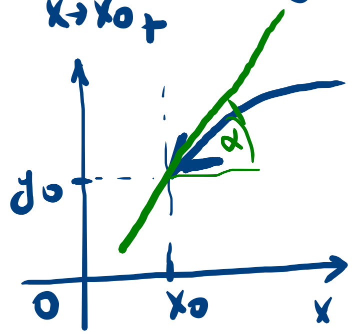
$\boxed{7}$ $f''(x)$, конвексност/конкавност

$\boxed{8}$ Γ_f

Ако

$x_0 \notin D_f$ / f није непр. у x_0

$\lim\limits_{x \to x_0+} f(x) = y_0$



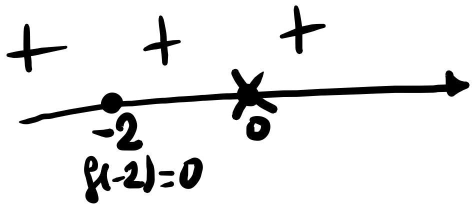
? $\lim\limits_{x \to x_0+} f'(x)$

$\boxed{1.}$ $f(x)=|x+2|\cdot e^{-\frac{1}{x}} = \begin{cases}(x+2)\cdot e^{-1/x}, & x\geq -2,\ x\neq 0\\ -(x+2)\cdot e^{-1/x}, & x<-2\end{cases}$

1° $\boxed{D_f=(-\infty,0)\cup(0,+\infty)}$ $\boxed{f \text{ непр. на } D_f}$

2° П/Н, ПЕР. //

3° НУЛЕ, ЗНАК: $f(x)=0 \Leftrightarrow |x+2|=0 \Leftrightarrow \underline{x=-2}$



$\underline{f(x)\geq 0,\ \forall x\in D_f}$

4° **АСИМПТОТИКА**: $(-\infty,0)\cup(0,+\infty)$

↑ 0_- 0_+ ↑

$x\to+\infty$: $f(x)=(x+2)\cdot e^{\overbrace{-1/x}^{\to 0}} = (x+2)\cdot\left(1-\frac{1}{x}+\frac{1}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right) = \underbrace{x-1}+\frac{1}{2x}+o\left(\frac{1}{x}\right)+\underbrace{2}-\frac{2}{x}+\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)$

$|x+2|=x+2>0$

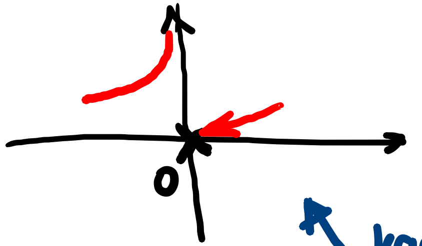
$= x+1\underbrace{-\frac{3}{2}\cdot\frac{1}{x}}_{<0}+o\left(\frac{1}{x}\right)$ $y=x+1$ КА $\underline{x\to+\infty}$ Γ_f испод АС.

$x\to-\infty$ $|x+2|=-(x+2)$ $f(x)=-(x+2)\cdot e^{-1/x}=\dots=-x-1+\overbrace{\frac{3}{2}\cdot\frac{1}{x}}^{<0}+o\left(\frac{1}{x}\right)$ $y=-x-1$ КА $\underline{x\to-\infty}$ Γ_f испод АС.

$\boxed{0}$ $f(x) = |x+2| \cdot e^{-1/x}$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \underbrace{|x+2|}_{\to 2} \, e^{\overbrace{-\frac{1}{x}}^{\to +\infty}} = +\infty$$

$\boxed{\begin{matrix} x=0 \text{ B.A.} \\ x\to 0^- \end{matrix}}$



↖ касније:

под којим углом ?

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \underbrace{|x+2|}_{\to 2} \cdot \underbrace{e^{\overbrace{-\frac{1}{x}}^{\to -\infty}}}_{\to 0} = 0$$

5° $f'(x) = ?$

$x \in (-2,0) \cup (0,+\infty)$: $f'(x) = \left((x+2)\cdot e^{-1/x}\right)' = e^{-1/x} + (x+2)e^{-1/x}\cdot\frac{1}{x^2} = \frac{e^{-1/x}}{x^2}\cdot(x^2+x+2)$

$x \in (-\infty,-2)$: $f'(x) = \left(-(x+2)\cdot e^{-1/x}\right)' = -\frac{e^{-1/x}}{x^2}(x^2+x+2)$

$x=-2$?

$$f'_+(-2) \overset{\text{ако } \exists\,!}{=} \lim_{x\to -2^+} f'(x) = \lim_{x\to -2^+} \frac{e^{-1/x}}{x^2}(x^2+x+2) = \sqrt{e}$$

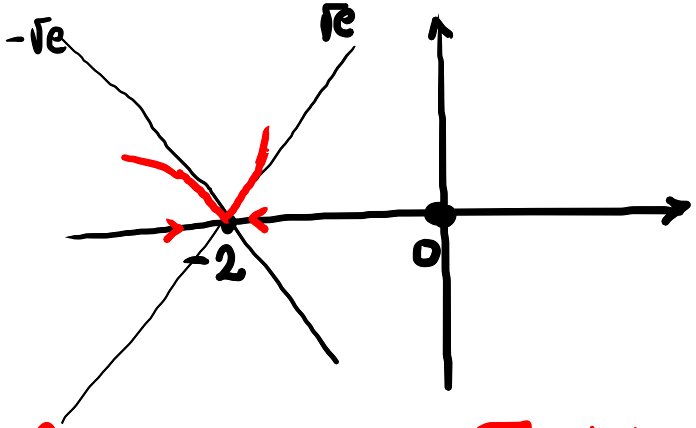
$\neq$ $\boxed{f \text{ није диф. у } -2}$

$$f'_-(-2) \overset{\text{ако } \exists}{=} \lim_{x\to -2^-} f'(x) = \lim_{x\to -2^-} -\frac{e^{-1/x}}{x^2}(x^2+x+2) = -\sqrt{e}$$

f диф. на $(-\infty,-2)\cup(-2,0)\cup(0,+\infty)$

$y\ -2$: $f'_-(-2)=-\sqrt{e}$ $f'_+(-2)=\sqrt{e}$

$$f'(x)=\begin{cases} e^{-1/x}\cdot\dfrac{x^2+x+2}{x^2}, & x\in(-2,0)\cup(0,+\infty)\\ -e^{-\frac{1}{x}}\cdot\dfrac{x^2+x+2}{x^2}, & x\in(-\infty,-2)\end{cases}$$



Знак $f'(x)$: $x^2+x+2 \overset{?}{\gtreqless} 0$ $D=1-4\cdot 2<0$

$\forall x\in\mathbb{R}$ $x^2+x+2>0$

$\Rightarrow$ $\boxed{\begin{array}{l} f'(x)>0 \text{ за } x\in(-2,0)\cup(0,+\infty)\\ f'(x)<0 \text{ за } x\in(-\infty,-2)\end{array}}$

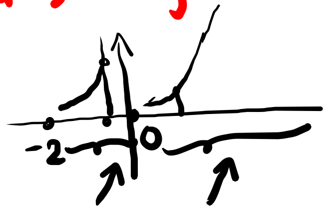
не!

~~$\Rightarrow f\uparrow$ на $(-2,0)\cup(0,+\infty)$.~~

$\Rightarrow f\uparrow$ на $(-2,0)$ и $f\uparrow$ на $(0,+\infty)$

$\boxed{\text{Т}}$ $f'(x)>0$, $\forall x\in(a,b)$

$\Rightarrow f\uparrow$ на (a,b)



$\Rightarrow$ $\boxed{\begin{array}{l} f \text{ расте на } (-2,0) \text{ и } f \text{ расте на } (0,+\infty)\\ f \text{ опада на } (-\infty,-2)\end{array}}$

Како се приближава нули (угао)?

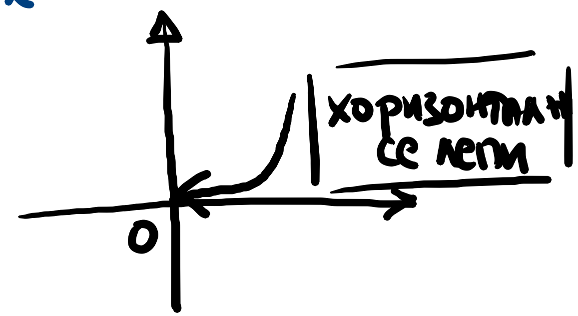
$$f'(x)=\begin{cases} e^{-\frac{1}{x}}\cdot \dfrac{x^2+x+2}{x^2}, & x\in(-2,0)\cup(0,+\infty)\\ -e^{-1/x}\,\dfrac{x^2+x+2}{x^2}, & x\in(-\infty,0)\end{cases}$$

$$\lim_{x\to 0+} f'(x) = \lim_{x\to 0+} e^{-\frac{1}{x}}\cdot\frac{x^2+x+2}{x^2} = 2\cdot\lim_{x\to 0+}\frac{e^{-1/x}}{x^2} \quad \text{„}\frac{0}{0}\text{"}$$

$$\boxed{t=\tfrac{1}{x}\to+\infty}$$

$$= 2\lim_{t\to+\infty}\frac{e^{-t}}{\left(\frac{1}{t}\right)^2} = 2\lim_{t\to+\infty}\frac{t^2}{e^t} \overset{t^2\ll e^t}{=} \boxed{0}$$

„tgα"



$\boxed{16°}$ $f''(x)=?$ ...

$$f''(x)=\begin{cases} e^{-\frac{1}{x}}\cdot\dfrac{(2-3x)}{x^4}, & x\in(-2,0)\cup(0,+\infty)\\ e^{-\frac{1}{x}}\cdot\dfrac{3x-2}{x^4}, & x\in(-\infty,-2)\end{cases}$$

		-2		0		$\frac{2}{3}$	
f''	$-$	•	$+$	×	$+$	•	$-$
f	∩		∪		∪		∩

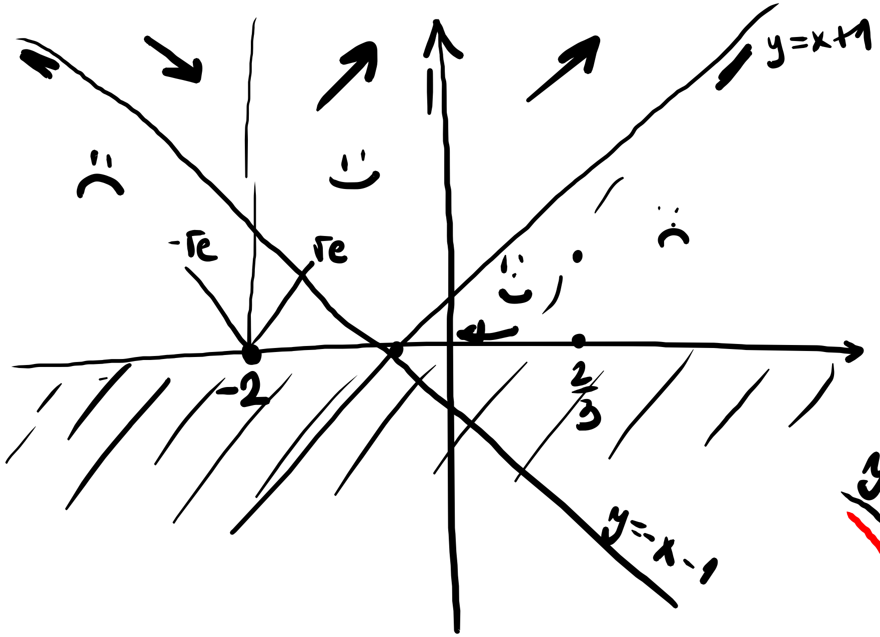
$x=-2$ превојна тачка

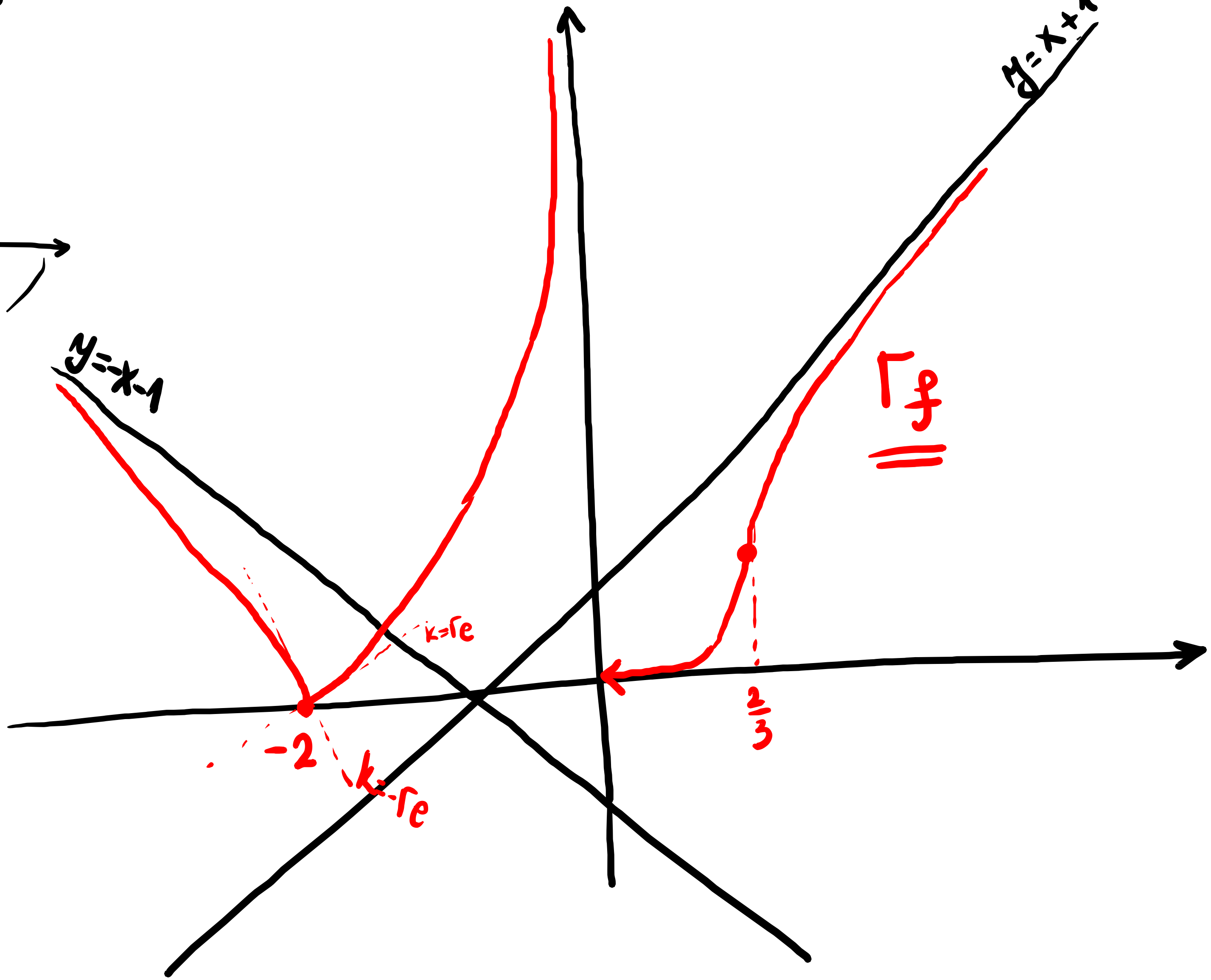
$x=\frac{2}{3}$ превојна тачка

$x<-2$: $f''(x)<0 \Rightarrow$ $\boxed{f \text{ конкавна на } (-\infty,-2)}$

$x\in(-2,0)\cup\left(0,\frac{2}{3}\right)$: $2-3x>0$ $f''(x)>0$ | f конв. на $(-2,0)$ и f конв. на $(0,2/3)$

$x\in\left(\frac{2}{3},+\infty\right)$: $f''(x)<0$ $\boxed{f \text{ конкавна на } \left(\frac{2}{3},+\infty\right)}$


y=x+1
-Γe
Γe
-2
2/3
y=-x-1


y=x+1
y=-x-1
Γf
k=Γe
-2
k=-Γe
2/3

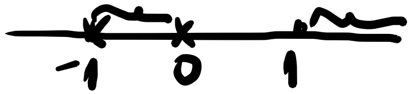
[2.] $f(x) = -x + \ln\left(\frac{|x|-1}{x}\right)$

1° D_f: $x \neq 0$ $\quad \frac{|x|-1}{x} > 0$

$\rightarrow$ $|x|-1>0$ и $x>0$ $\iff$ $x\in(1,+\infty)$

$\rightarrow$ $|x|-1<0$ и $x<0$ $\iff$ $x\in(-1,0)$

$\boxed{D_f = (-1,0)\cup(1,+\infty)}$



$\boxed{f \text{ непр на } D_f}$

2° П/Н/пер // (домен није симетричан $\Rightarrow$ ~~П/Н~~)

3° Нуле и знак: $-x+\ln\left(\frac{|x|-1}{x}\right)=0$ $\quad x=?$ не знамо да решимо

$\boxed{\text{На крају}}$

4° рубови домена

$$\boxed{\lim_{x\to -1_+} f(x)} = \lim_{x\to -1_+} \underbrace{-x}_{\to 1} + \underbrace{\ln \frac{\overbrace{|x|-1}^{\to 0_-}}{\underbrace{x}_{\to -1}}}_{\to -\infty} = \boxed{-\infty}$$

$\overset{\to 0_-}{} \to 0_+$

$\boxed{x=-1 \text{ вертикална асимптота кад } x\to -1_+ \quad f(x)\to -\infty}$

$$\boxed{\lim_{x\to 0_-} f(x)} = \lim_{x\to 0_-} -x + \underbrace{\ln \frac{|x|-1}{\underbrace{x}_{\to 0_-}}}_{\to +\infty} = \boxed{+\infty}$$

$\to +\infty$

$\boxed{x=0 \text{ В.А. кад } x\to 0_- \quad f(x)\to +\infty}$

$$\boxed{\lim_{x\to 1^+} f(x)} = \lim_{x\to 1^+}\Big(-x+\underbrace{\ln\Big(\underbrace{\frac{|x|-1}{x}}_{\to 0^+}\Big)}_{\to -\infty}\Big) = \boxed{-\infty}$$

$\boxed{x=1 \text{ В.А. кад } x\to 1^+,\ f(x)\to -\infty}$

$$f(x) = -x + \ln\frac{|x|-1}{x}$$

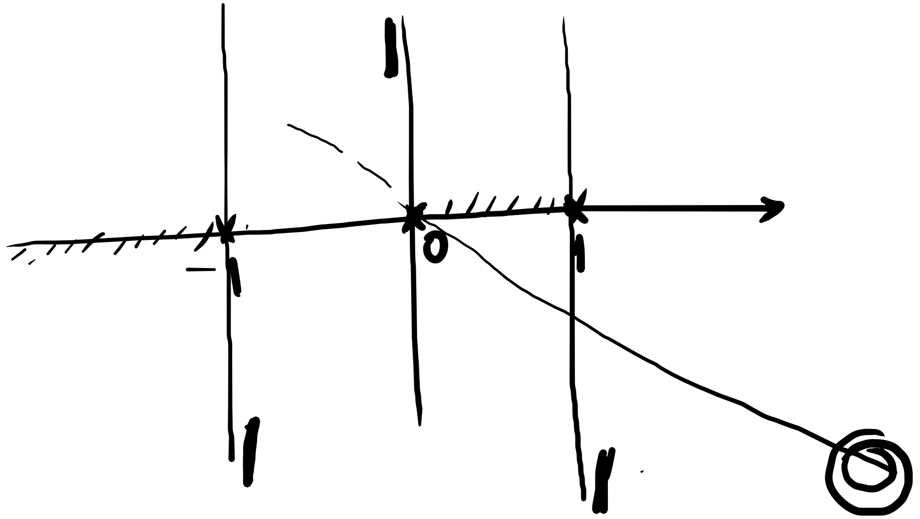
$$\boxed{\lim_{x\to +\infty} f(x)} = \lim_{x\to +\infty}\Big(-x + \underbrace{\ln\Big(\frac{|x|-1}{x}\Big)}_{\to 0}\Big) = -\infty \Rightarrow f \text{ нема Х.А. кад } x\to+\infty$$

К.А. ?

$$a = \lim_{x\to+\infty}\frac{f(x)}{x} = \lim_{x\to+\infty}\left(-1+\frac{\ln\left(\frac{|x|-1}{x}\right)}{x}\right) = -1$$

$$b = \lim_{x\to+\infty}(f(x)-ax) = \lim_{x\to+\infty}\ln\frac{|x|-1}{x} = 0$$

$a=-1,\ b=0 \Rightarrow \boxed{y=-x \text{ К.А. } x\to+\infty}$



5° $f'(x)=?$

$$f(x) = -x + \ln\left(\frac{|x|-1}{x}\right) = \begin{cases} -x+\ln\left(\frac{-x-1}{x}\right), & x\in(-1,0) \\ -x+\ln\left(\frac{x-1}{x}\right), & x\in(1,+\infty) \end{cases}$$

$x\in(-1,0)$ $\quad f'(x) = -1 + \dfrac{1}{\frac{-x-1}{x}}\cdot\dfrac{1}{x^2} = -1 - \dfrac{1}{(x+1)\cdot x} = -\dfrac{x^2+x+1}{x(x+1)}$

$$\boxed{f'(x) = -\frac{\overbrace{x^2+x+1}^{D<0,\ >0}}{\underbrace{x}_{<0}\underbrace{(x+1)}_{>0}}}$$

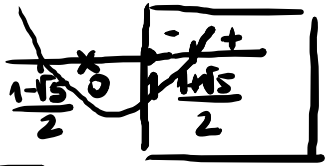
$f'(x)>0,\ \forall x\in(-1,0)$

$\boxed{f\uparrow \text{ на } (-1,0)}$

$x\in(1,+\infty)$ $\quad f'(x) = -1 + \dfrac{1}{\frac{x-1}{x}}\cdot\dfrac{1}{x^2} = \dfrac{-x^2+x+1}{x(x-1)}$

$$\boxed{f'(x) = -\frac{x^2-x-1}{x(x-1)}}$$

$x^2-x-1=0 \quad x_{1,2} = \dfrac{1\pm\sqrt5}{2}$



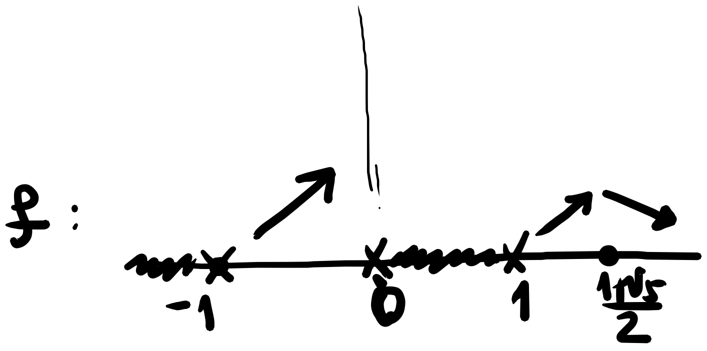
$x\in\left(1,\frac{1+\sqrt5}{2}\right)$: $f'(x)>0$

$\Rightarrow \boxed{f\nearrow \text{ на } \left(1,\frac{1+\sqrt5}{2}\right)}$

$x\in\left(\frac{1+\sqrt5}{2},+\infty\right)$: $f'(x)<0$

$\boxed{f\downarrow \text{ на } \left(\frac{1+\sqrt5}{2},+\infty\right)}$

$\boxed{x=\frac{1+\sqrt5}{2} \text{ лок. max}}$



$\boxed{f \text{ диф. на } D_f}$

6° $f''(x) =$

$x\in(-1,0)$: $f''(x) = \left(-\dfrac{x^2+x+1}{x(x+1)}\right)' \overset{\dots}{=} \dfrac{\overset{-\frac{1}{2}}{2x+1}}{x^2(x+1)^2}$

$x\in(1,+\infty)$: $f''(x) = \left(-\dfrac{x^2-x-1}{x(x-1)}\right)' \overset{\dots}{=} -\dfrac{(2x-1)^{>0}}{x^2(x-1)^2}$

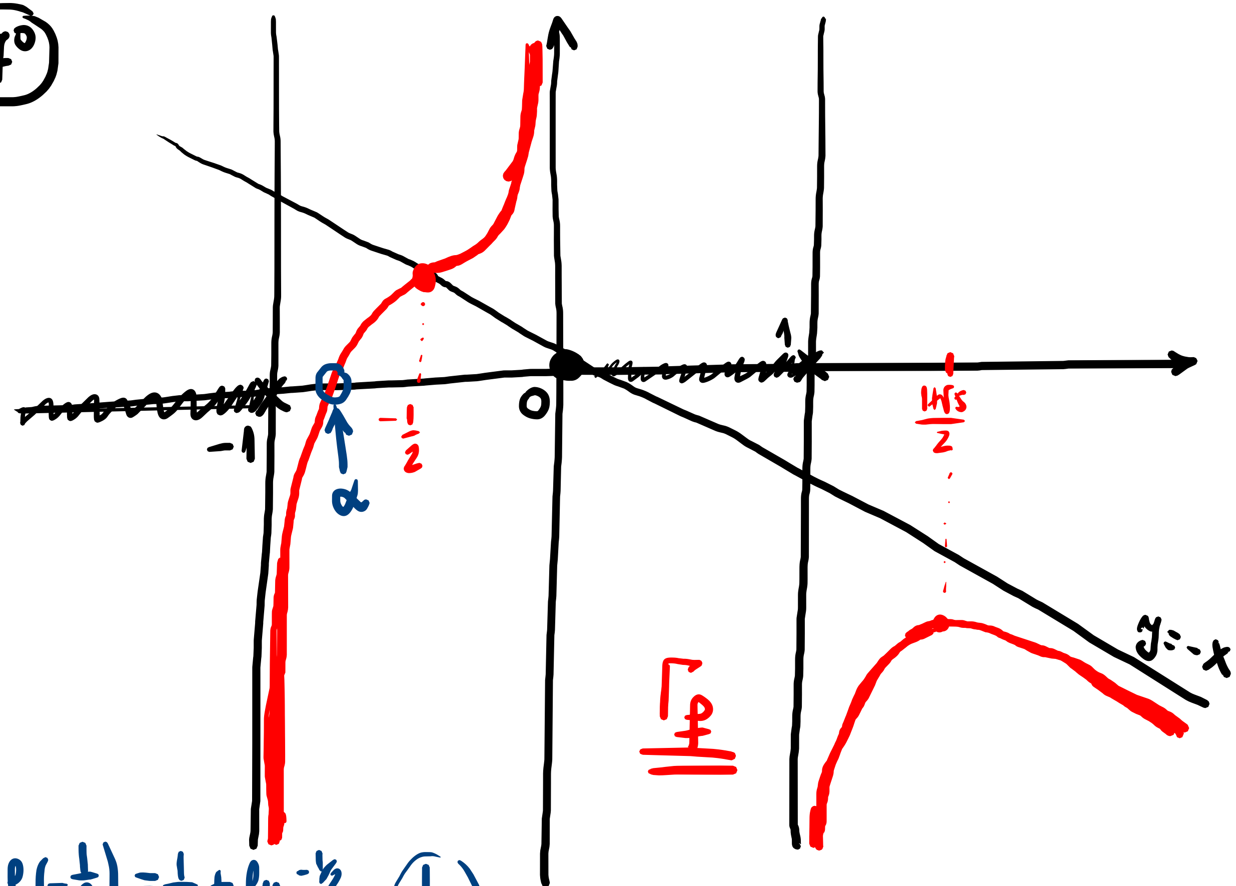
$x\in(-1,-\frac{1}{2})$: $f''(x)<0$ f $\frown$

$x\in(-\frac{1}{2},0)$: $f''(x)>0$ f $\smile$

$x=-\frac{1}{2}$ превојна т.

$x\in(1,+\infty)$: $f''(x)<0$ f $\frown$

7°



$f(-\frac{1}{2}) = \frac{1}{2} + \ln\frac{-1/2}{-1/2} = \frac{1}{2}$

3° $\underline{(-1,0)}$ $-\infty \nearrow +\infty$

Болцано-Коши $\Rightarrow \exists \alpha\in(-1,0)$ $f(\alpha)=0$

$x\in(-1,\alpha)$: $f(x)<0$

$x\in(\alpha,0)$: $f(x)>0$

$\underline{(1,+\infty)}$ $-\infty \nearrow \bullet \searrow -\infty$

$f(\frac{1+\sqrt{5}}{2}) = -\frac{1+\sqrt{5}}{2} + \ln(\dots) \underline{<0}$

$\Rightarrow f(x)<0,\ \forall x\in(1,+\infty)$

3. $f(x) = -\frac{1}{|x|} + \operatorname{arctg}\frac{2x}{x^2-1}$

1° D_f: $x \neq 0$, $x \neq -1, 1$ $\quad D_f = (-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,+\infty)$

2° //

3° нуле & знак $\rightsquigarrow$ на крају

4° $f'(x)$: $f'(x) = \left(-\frac{1}{|x|} + \operatorname{arctg}\frac{2x}{x^2-1}\right)'$

$$= \underbrace{\frac{1}{|x|^2}\cdot \operatorname{sgn} x}_{\frac{\operatorname{sgn} x}{x^2}} + \underbrace{\frac{1}{1+\left(\frac{2x}{x^2-1}\right)^2}\cdot\frac{2(x^2-1)-2x\cdot 2x}{(x^2-1)^2}}_{\dots\; -\frac{2}{x^2+1}}, \quad x \neq 0, 1, -1$$

$|x| = \operatorname{sgn} x \cdot x$

$|x|' = \operatorname{sgn} x$

$\uparrow$

$x \neq 0$

$$\Rightarrow \boxed{f'(x)} = \begin{cases} \frac{1}{x^2} - \frac{2}{x^2+1}, & x \in (0,1)\cup(1,+\infty) \\ -\frac{1}{x^2} - \frac{2}{x^2+1}, & x \in (-\infty,-1)\cup(-1,0) \end{cases}$$

<0

$\boxed{f \text{ диференцијабилна на } D_f}$

$x \in (-\infty,-1)\cup(-1,0)$: $f'(x) < 0$ $\quad \boxed{\begin{matrix} f \downarrow (-\infty,-1) \\ f \downarrow (-1,0) \end{matrix}}$

$x \in (0,1)\cup(1,+\infty)$: $f'(x) = \frac{1-x^2}{x^2(x^2+1)}$

$x \in (0,1)$: $f'(x) > 0$ $\quad \boxed{f \nearrow (0,1)}$

$x \in (1,+\infty)$: $f'(x) < 0$ $\quad \boxed{f \downarrow (1,+\infty)}$

f: $\searrow$ -1 $\searrow$ 0 $\nearrow$ 1 $\searrow$

$$f(x) = -\frac{1}{|x|} + \operatorname{arctg}\frac{2x}{x^2-1}$$

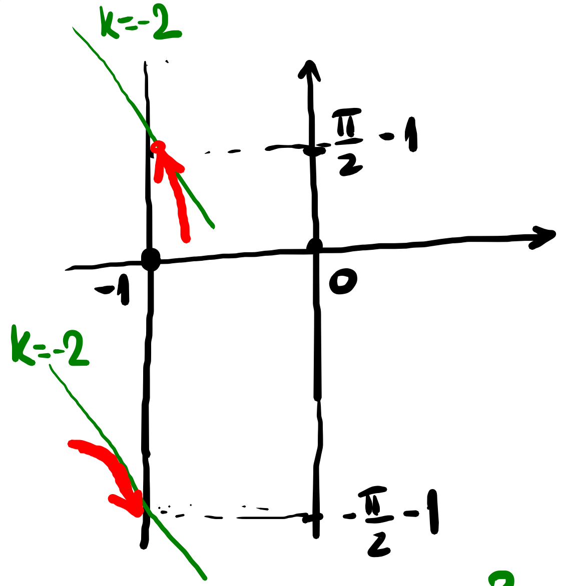
5° асимптоте & рубове домена:

$\boxed{-\infty}$: $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}\Big(\underbrace{-\frac{1}{|x|}}_{\to 0} + \operatorname{arctg}\underbrace{\frac{2x}{x^2-1}}_{\to 0}\Big) = \boxed{0} \Rightarrow \boxed{y=0 \text{ х.А. } x\to-\infty}$

$\boxed{+\infty}$: $\lim\limits_{x\to+\infty} f(x) = \ldots = 0 \Rightarrow \boxed{y=0 \text{ х.А. кад } x\to+\infty}$

$\boxed{-1}$: $\lim\limits_{x\to-1_-} f(x) = \lim\limits_{x\to-1_-}\Big(\underbrace{-\frac{1}{|x|}}_{\to -1} + \underbrace{\operatorname{arctg}\underbrace{\frac{\overbrace{2x}^{\to -2}}{\underbrace{x^2-1}_{\to 0_+}}}_{\to -\infty}}_{\to -\frac{\pi}{2}}\Big) = \boxed{-1-\frac{\pi}{2}}$

$\lim\limits_{x\to-1_+} f(x) = \lim\limits_{x\to-1_+}\Big(\underbrace{-\frac{1}{|x|}}_{\to -1} + \underbrace{\operatorname{arctg}\underbrace{\frac{\overbrace{2x}^{\to -2}}{\underbrace{x^2-1}_{\to 0}}}_{\to +\infty}}_{\to \frac{\pi}{2}}\Big) = \boxed{-1+\frac{\pi}{2}}$



под којим условима?

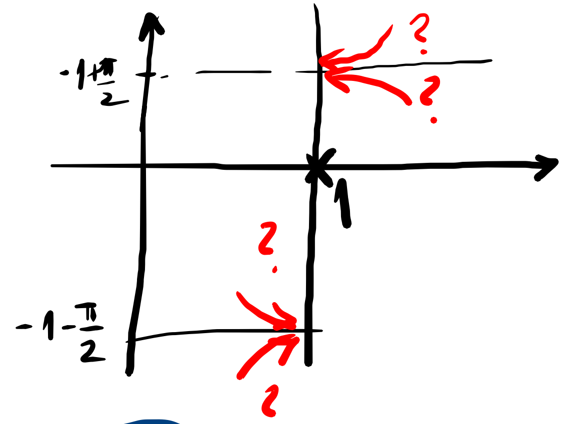
$\lim\limits_{x\to-1_-} f'(x) = \lim\limits_{x\to-1_-}\Big(-\frac{1}{x^2} - \frac{2}{x^2+1}\Big) = -1-1 = -2$

$\lim\limits_{x\to-1_+} f'(x) = -\|- = -2$

1 ~~слично~~ – за вежбу:

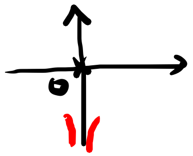
$\lim_{x\to 1+} f(x) = -1+\frac{\pi}{2}$ $\quad \lim_{x\to 1+} f'(x) = 0$

$\lim_{x\to 1-} f(x) = -1-\frac{\pi}{2}$ $\quad \lim_{x\to 1-} f'(x) = 0$



0 $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{-1}{|x|} + \operatorname{arctg}\frac{\overset{\to 0}{2x}}{\underset{\to 0}{x^2-1}} = \boxed{-\infty}$

$\boxed{\begin{array}{l} x=0 \text{ В.А} \\ f\to 0, x\to 0 \end{array}}$



$f(x) = -\frac{1}{|x|} + \operatorname{arctg}\frac{2x}{x^2-1}$

6° $f''(x) = \left(\frac{\operatorname{sgn} x}{x^2} - \frac{2}{x^2+1}\right)' = -2\frac{\operatorname{sgn} x}{x^3} + \frac{2\cdot 2x}{(x^2+1)^2} = \begin{cases} -\frac{2}{x^3} + \frac{4x}{(x^2+1)^2}, & x\in(0,1)\cup(1,+\infty) \\ \frac{2}{x^3} + \frac{4x}{(x^2+1)^2}, & x\in(-\infty,-1)\cup(-1,0) \end{cases}$

$\boxed{x\in(-\infty,-1)\cup(-1,0)}$: $f''(x)<0$

$\boxed{f \text{ конкавна на } (-\infty,-1),\ f \text{ конкавна на } (-1,0)}$

$\boxed{x\in(0,1)\cup(1,+\infty)}$ $\quad f''(x) = \frac{-2(x^2+1)^2+4x^4}{x^3(x^2+1)^2} \overset{?}{>} 0$

$\Leftrightarrow 2x^4-4x^2-2>0 \;/:2$

$\Leftrightarrow x^4-2x^2-1>0 \Leftrightarrow \cancel{\overset{t}{x^2} < \underbrace{1-\sqrt{2}}_{<0}} \lor x^2 > 1+\sqrt{2} \Leftrightarrow \boxed{x>\sqrt{1+\sqrt{2}}}$

$t=x^2$: $\quad t^2-2t-1>0 \quad t_{1,2} = \frac{2\pm\sqrt{8}}{2} = 1\pm\sqrt{2}$

$f'(x)>0$: $x\in(\sqrt{1+\sqrt{2}},+\infty)$ $\quad f$ конвексна!

$f'(x)<0$: $x\in(0,1)$ $\quad f$ конкавна

$x\in(1,\sqrt{1+\sqrt{2}})$ $\quad f$ конкавна

$x=\sqrt{1+\sqrt{2}}$ превојна тачка

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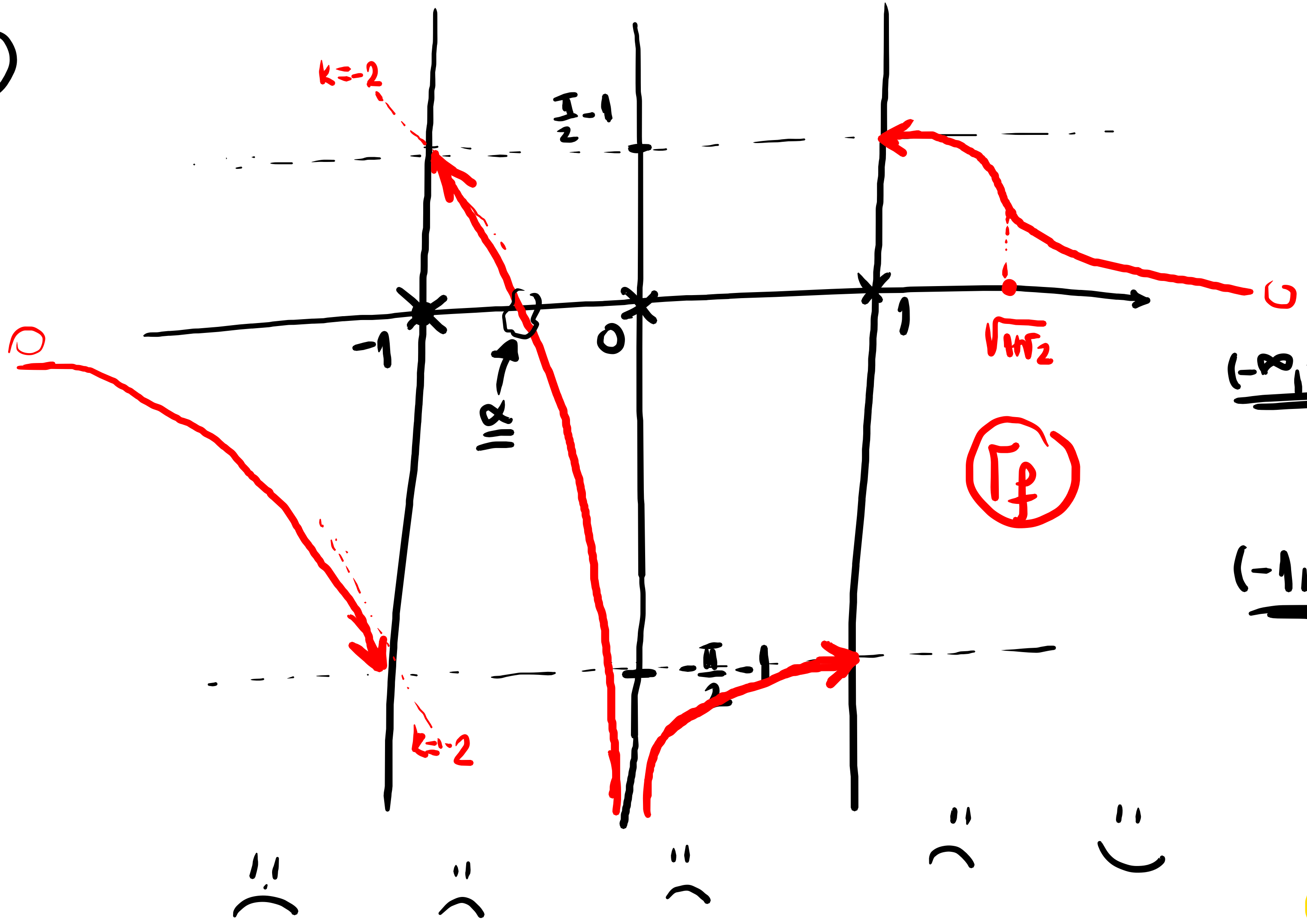


3° НУЛИ & ЗНАК

$(-\infty,-1)$: $\lim_{x\to-\infty} f(x)=0$, $f\downarrow$ $\Big\}$ $f(x)<0$ за $x\in(-\infty,-1)$

$(-1,0)$: $\lim_{x\to-1+} f(x)>0$, $\lim_{x\to 0-} f(x)=-\infty$, $f\downarrow$ на $(-1,0)$

БК $\Rightarrow$ $\exists!$ $\alpha\in(-1,0)$ $f(\alpha)=0$

$x\in(-1,\alpha)$ $f(x)>0$

$x\in(\alpha,0)$ $f(x)<0$

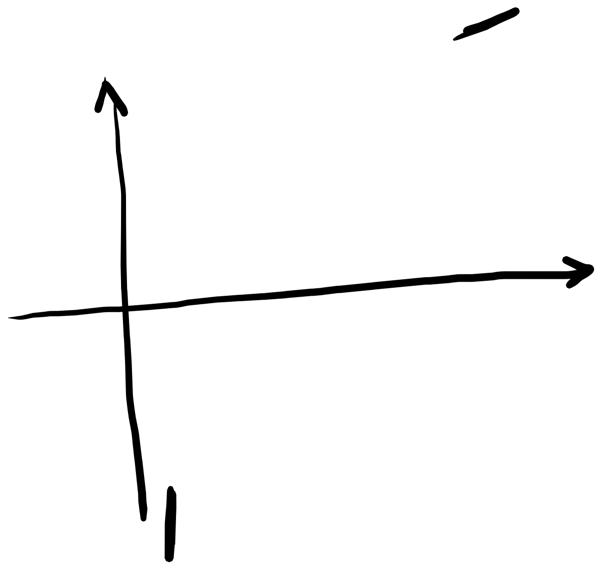
Отговор: $f(x)<0, \forall x\in(0,1)$

$f(x)>0, \forall x\in(1,+\infty)$

$\square$

4. Доказати да једначина $\ln x = \frac{1}{x^2}$ има јединствено решење.

$f(x) = \ln x - \frac{1}{x^2}$ има јединствену нулу



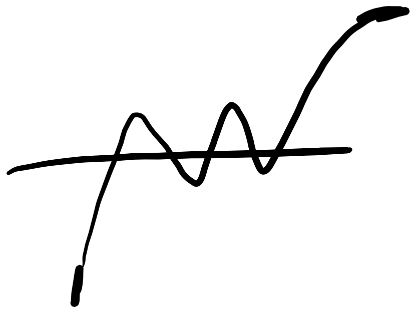
$D_f = (0, +\infty)$

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+}\Big(\underbrace{\ln x}_{\to -\infty} - \underbrace{\frac{1}{x^2}}_{\to +\infty}\Big) = \boxed{-\infty}$$

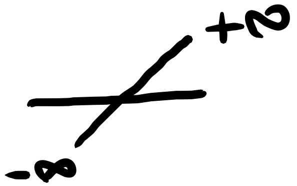
$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty}\Big(\underbrace{\ln x}_{\to +\infty} - \frac{1}{\underbrace{x^2}_{\to +\infty}}\Big) = \boxed{+\infty}$$

f непрекидна

Б.К. $\Rightarrow$ $\boxed{\exists x_0 > 0 \quad f(x_0) = 0}$



$$f'(x) = \frac{1}{x} + 2\cdot\frac{1}{x^3} > 0, \ \forall x > 0 \Rightarrow \boxed{f \nearrow \text{ на } (0, +\infty)}$$



$\Rightarrow$ **нула је јединствена**

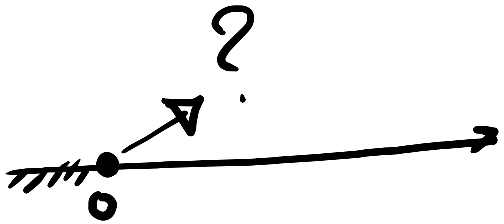
$\square$

* <u>за вежбу</u>: Док. да за $\forall x \geq 0$

$$\operatorname{arctg}\sqrt{x} \geq \frac{\sqrt{x}\cdot(5x+3)}{3(x+1)^2}.$$

$$f(x) = \operatorname{arctg}\sqrt{x} - \frac{\sqrt{x}\cdot(5x+3)}{3(x+1)^2} \overset{?}{\geq} 0 \quad \forall x \geq 0$$

$$f(0) = 0$$



„мало испитамо"

$$f'(x) = \dots = \frac{8x^2}{6\sqrt{x}(1+x)^3} > 0, \quad \forall x > 0$$

$$\left.\begin{array}{l} \Rightarrow \boxed{f \uparrow} \; (0, +\infty) \\ f \text{ непр. на } [0, +\infty) \end{array}\right\} \Rightarrow \boxed{f \uparrow \text{ на } [0, +\infty)}$$

$$f(x) \geq f(0) = 0, \quad \forall x \geq 0 \quad \checkmark$$