Лајбницова формула: $(u\cdot v)^{(n)}=\sum_{k=0}^{n}\binom{n}{k}\cdot u^{(k)}\cdot v^{(n-k)}$ $\quad u^{(0)}=u$, $v^{(0)}=v$ 10.5.2021.

1. $f(x)=\underbrace{(5x^2-2x+1)}_{u}\cdot \underbrace{e^{-3x}}_{v}$ $\quad f^{(n)}(x)=?\quad f^{(n)}(0)=?$

$\boxed{u}$ $u(x)=5x^2-2x+1$
$u'(x)=10x-2$
$u''(x)=10$
$u^{(k)}(x)=0,\ \forall k\geq 3$

$\boxed{v}$ $v(x)=e^{-3x}$
$v'(x)=(-3)\cdot e^{-3x}$
$v''(x)=(-3)^2\cdot e^{-3x}$
$\dots\ v^{(k)}(x)=(-3)^k\cdot e^{-3x}$

$$f^{(n)}(x)=\sum_{k=0}^{n}\binom{n}{k}\underbrace{u^{(k)}(x)}_{0 \text{ за } k\geq 3}v^{(n-k)}(x)\quad \leftarrow k=0,1,2\ \checkmark \quad \text{за } k\geq 3 \text{ чланови}=0$$

$$=\underbrace{\binom{n}{0}}_{1}\cdot u(x)\cdot v^{(n)}(x)+\underbrace{\binom{n}{1}}_{n}\cdot u'(x)\cdot v^{(n-1)}(x)+\binom{n}{2}u^{(2)}(x)\cdot v^{(n-2)}(x)+0$$

$$=(5x^2-2x+1)\cdot(-3)^n\cdot e^{-3x}+n\cdot(10x-2)\cdot(-3)^{n-1}\cdot e^{-3x}+\frac{n(n-1)}{2}\cdot 10\cdot(-3)^{n-2}\cdot e^{-3x},\ \boxed{\forall x\in\mathbb{R}}$$

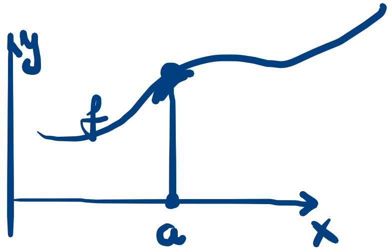
$$f^{(n)}(0)=(-3)^n\cdot 1+n\cdot 2\cdot(-3)^{n-1}\cdot 1+\frac{n(n-1)}{2}\cdot 10\cdot(-3)^{n-2}$$

$$f^{(n)}(0)=(-3)^{n-2}\cdot\left(9+6n+5n(n-1)\right)$$

$$\boxed{f^{(n)}(0)=(-3)^{n-2}\cdot(5n^2+n+9)}$$

Тејлорова формула

I f, $a\in\mathbb{R}$ $\quad f$ непр., $f',\dots,f^{(n)}$ - непр у некој околини тачке a, у тој ок. $\exists\, f^{(n+1)}(x)$



$$f(x)=\underbrace{f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots+\frac{f^{(n)}(a)}{n!}(x-a)^n}_{\text{Тејлоров полином } P_n(x) \text{ у } a}+o\left((x-a)^n\right),\ x\to a$$

$a=0$: Маклоренов пол.: $f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\dots+\frac{f^{(n)}(0)}{n!}x^n+o(x^n),\ X\to 0$

$$\boxed{\sin x}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots+(-1)^n\cdot\frac{x^{2n+1}}{(2n+1)!}+0+o\left(x^{2n+2}\right),\ x\to 0$$

$$\boxed{\cos x}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\dots+(-1)^n\cdot\frac{x^{2n}}{(2n)!}+\underbrace{0+o\left(x^{2n+1}\right)}_{+o(x^{2n})},\ x\to 0$$

$$\boxed{e^x}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots+\frac{x^n}{n!}+o(x^n),\ x\to 0$$

$$\boxed{(1+x)^\alpha}=1+\binom{\alpha}{1}x+\binom{\alpha}{2}x^2+\dots+\binom{\alpha}{n}x^n+o(x^n),\ x\to 0 \quad \leftarrow \quad \alpha\in\mathbb{R}, n\in\mathbb{N}:\ \binom{\alpha}{n}:=\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}$$

$$\boxed{\ln(1+x)}=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots+(-1)^{n-1}\cdot\frac{x^n}{n}+o(x^n),\ x\to 0$$

1: (а) $\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + o(x^n), \ x \to 0$

(б) $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \dots + x^n + o(x^n), \ x \to 0$

веиба

(а) $\dfrac{1}{1+x} = (1+x)^{\overset{\text{"}\alpha\text{"}}{-1}} = 1 + \binom{-1}{1}x + \binom{-1}{2}x^2 + \dots + \binom{-1}{n}x^n + o(x^n), \ x \to 0$

$\binom{-1}{k} = \dfrac{(-1)\cdot(-2)\cdot(-3)\cdot\dots\cdot(-k)}{k!} = \dfrac{(-1)^k \cdot k!}{k!} = (-1)^k$

$= 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + o(x^n), \ x \to 0$ ✓

2. $f(x)=\operatorname{arctg} x$ Тејлоров развој $a=3$, $n=2$

$f(3)=\operatorname{arctg} 3$

$f'(x)=\frac{1}{1+x^2}$ $f'(3)=\frac{1}{10}$

$f''(x)=-\frac{1}{(1+x^2)^2}\cdot 2x$ $f''(3)=\frac{-6}{100}=-\frac{3}{50}$

$$f(x)=\operatorname{arctg}3+\frac{1/10}{1!}\cdot(x-3)+\frac{-\frac{3}{50}}{2!}(x-3)^2+o((x-3)^2),\ x\to 3$$

$$\boxed{\operatorname{arctg} x}=\operatorname{arctg}3+\frac{1}{10}(x-3)-\frac{3}{100}(x-3)^2+o((x-3)^2),\ x\to 3$$

Тејлор: $\underline{\underline{a}}$ $(x-a)^k$, $o((x-a)^k)$, $x\to a$

$\boxed{t=x-a}$

$\downarrow -a$

Маклорен: $\underline{\underline{0}}$ $\underline{\underline{t\to 0}}$ $\underline{\underline{t^k}}$ $o(t^k)$, $t\to 0$

$\smile$ $x \to 0$: $x^n = o(x^m)$, $n > m$

$x^7 = o(x^3)$, $x \to 0$ $\qquad$ $x^7 = \underbrace{x^4}_{\to 0,\, x\to 0} \cdot x^3$

$$\underbrace{x^7}_{=o(x^3)} + \underbrace{x^{15}}_{=o(x^3)} + o(x^3) = o(x^3)$$

$(1+t)^{-1} = 1 - t + t^2 - t^3 + o(t^3)$
$t \to 0$

3. Разложить $f(x) = \operatorname{tg} x$ до ряда 5, $x \to 0$

I начин: $f(x), f', f'', f''', f^{IV}, f^{V} \ldots$

II начин:

$$\operatorname{tg} x = \frac{\sin x}{\cos x} = \sin x \cdot (\cos x)^{-1} = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5)\right) \cdot \Big(1 \overbrace{- \frac{x^2}{2} + \frac{x^4}{4!} + o(x^5)}^{t \to 0}\Big)^{-1}$$

$$= \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)\right) \cdot \left[1 - \overbrace{\left(-\frac{x^2}{2} + \frac{x^4}{4!} + o(x^5)\right)}^{t} + \overbrace{\left(-\frac{x^2}{2} + \frac{x^4}{4!} + o(x^5)\right)^2}^{t^2} - \overbrace{\left(-\frac{x^2}{2} + \frac{x^4}{4!} + o(x^5)\right)^3}^{t^3} + o\Big(\overbrace{\left(-\frac{x^2}{2} + \frac{x^4}{4!} + o(x^5)\right)^3}^{t^3}\Big)\right]$$

$$= \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)\right) \cdot \left[1 + \frac{x^2}{2} - \frac{x^4}{24} + o(x^5) + \frac{x^4}{4} + o(x^5) + o(x^5) + o(x^6)\right]$$

$$= \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)\right) \cdot \left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + o(x^5)\right)$$

$$\operatorname{tg} x = \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)\right) \cdot \left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + o(x^5)\right)$$

$$= x + \frac{x^3}{2} + \frac{5}{24}x^5 + x \cdot o(x^5)$$

$$- \frac{x^3}{6} - \frac{x^5}{12} - \frac{5}{6 \cdot 24}x^7 - \frac{x^3}{6} \cdot o(x^5)$$

$$+ \frac{x^5}{120} + o(x^5)$$

$$+ o(x^5)$$

$$\to o(x^5)$$

$$\boxed{\operatorname{tg} x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + o(x^5), \quad x \to 0}$$

④ Развити: $f(x)=(1+x)^{1/x}$ у облику $f(x)=a+bx+cx^2+o(x^2),\ x\to 0$

$$f(x)=(1+x)^{1/x}=e^{\frac{1}{x}\cdot\ln(1+x)}$$

први покушај реда 2 развој

треба више

$$=e^{\frac{1}{x}\cdot\left(x-\frac{x^2}{2}+o(x^2)\right)}=e^{1-\frac{x}{2}+\frac{1}{x}\cdot o(x^2)}=e^{1-\frac{x}{2}+o(x)}$$

није довољно!

⇒ $\ln(1+x)$ мора до реда 3 јер га множи $\frac{1}{x}$

! $e^t=\dots$ $t\to 0$

$$f(x)=e^{\frac{1}{x}\cdot\ln(1+x)}=e^{\frac{1}{x}\cdot\left(x-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)\right)}=e^{\overbrace{1-\frac{x}{2}+\frac{x^2}{3}+o(x^2)}^{\to 1\ \neq 0}}$$

$$=e\cdot e^{\overbrace{-\frac{x}{2}+\frac{x^2}{3}+o(x^2)}^{t\to 0}}$$

$$=e\cdot\left(1+\left(-\frac{x}{2}+\frac{x^2}{3}+o(x^2)\right)+\underbrace{\frac{1}{2!}\left(-\frac{x}{2}+\frac{x^2}{3}+o(x^2)\right)^2}_{\frac{1}{8}x^2+o(x^2)}+\underbrace{o\left(\left(-\frac{x}{2}+\frac{x^2}{3}+o(x^2)\right)^2\right)}_{=o(x^2)}\right)$$

$$=e\cdot\left(1-\frac{x}{2}+\frac{x^2}{3}+\frac{1}{8}x^2+o(x^2)\right)=\underbrace{e-\frac{e}{2}x+e\cdot\frac{11}{24}x^2+o(x^2)}\ ,\ x\to 0$$

5. $L = \lim\limits_{x\to 0} \dfrac{e^{\sin x}\cdot\cos(\sin x) - 1 - x}{x^3}$ "$\dfrac{0}{0}$" $\boxed{\sin x} = x - \dfrac{x^3}{6} + o(x^3),\ x\to 0 \quad (o(x^4))$

$$L = \lim_{x\to 0} \frac{e^{\overbrace{x-\frac{x^3}{6}+o(x^3)}^{t\to 0}}\cdot\cos\left(x-\frac{x^3}{6}+o(x^3)\right) - 1 - x}{x^3}$$

$$\overset{e^t,\, t\to 0}{=} \lim_{x\to 0}\frac{1}{x^3}\left[\left(1+\overbrace{\left(x-\frac{x^3}{6}+o(x^3)\right)}^{t}+\underbrace{\frac{1}{2}\overbrace{\left(x-\frac{x^3}{6}+o(x^3)\right)^2}^{t^2}}_{\frac{1}{2}x^2+o(x^3)}+\underbrace{\frac{1}{3!}\cdot\overbrace{\left(x-\frac{x^3}{6}+o(x^3)\right)^3}^{t^3}}_{\frac{1}{6}x^3+o(x^3)}+o(\overset{t^3}{x^3})\right)\cdot\cos\left(x-\frac{x^3}{6}+o(x^3)\right)-1-x\right]$$

$$= \lim_{x\to 0}\frac{1}{x^3}\left[\left(1+x-\cancel{\frac{x^3}{6}}+\frac{1}{2}x^2+\cancel{\frac{1}{6}x^3}+o(x^3)\right)\cdot\cos\underbrace{\left(x-\frac{x^3}{6}+o(x^3)\right)}_{t\to 0}-1-x\right]$$

$$\overset{\cos t,\, t\to 0}{=} \lim_{x\to 0}\frac{1}{x^3}\left[\left(1+x+\frac{1}{2}x^2+o(x^3)\right)\cdot\left(1\underbrace{-\frac{1}{2}\left(x-\frac{x^3}{6}+o(x^3)\right)^2}_{-\frac{1}{2}x^2+o(x^3)}+\underbrace{\frac{1}{4!}\cdot\left(x-\frac{x^3}{6}+o(x^3)\right)^4+o(\overset{t^4}{x^4})}_{o(x^3)}\right)-1-x\right]$$

$$= \lim_{x\to 0}\frac{1}{x^3}\left[\left(1+x+\frac{1}{2}x^2+o(x^3)\right)\cdot\left(1-\frac{1}{2}x^2+o(x^3)\right)-1-x\right]$$

$$= \lim_{x\to 0}\frac{1}{x^3}\left[\cancel{1}-\frac{1}{2}x^2+o(x^3)+\cancel{x}-\frac{1}{2}x^3+\underbrace{x\cdot o(x^3)}_{o(x^4)}+\frac{1}{2}x^2-\underbrace{\frac{1}{4}x^4}_{o(x^3)}+\frac{1}{2}x^2 o(x^3)+o(x^3)-\cancel{1}-\cancel{x}\right]$$

скрате се

$$L = \lim_{x\to 0}\frac{-\frac{1}{2}x^3+o(x^3)}{x^3}$$

$$= \lim_{x\to 0}\left(-\frac{1}{2}+o(1)\right)$$

$$\boxed{L = -1/2}$$

⑥ $\lim\limits_{x\to+\infty}\left(\sqrt[5]{x^5+5x^4}-\sqrt{x^2-2x}\right)$ „$\infty-\infty$"

☺ $x\to+\infty\,/\,-\infty$

$\frac{1}{x}\to 0$

$\left(1+\frac{c}{x}\right)^{\alpha}$ ← биномни развој

$$= \lim_{x\to+\infty}\left(x\cdot\sqrt[5]{1+\frac{5}{x}} - \underbrace{|x|}_{x>0}\cdot\sqrt{1-\frac{2}{x}}\right)$$

$$= \lim_{x\to+\infty}\left(x\cdot\Big(1+\underbrace{\frac{5}{x}}_{t\to 0}\Big)^{1/5} - x\cdot\Big(1\underbrace{-\frac{2}{x}}_{\to 0}\Big)^{1/2}\right)$$

$$= \lim_{x\to+\infty}\left(x\cdot\left(1+\underbrace{\underbrace{\binom{1/5}{1}}_{1/5}\cdot\frac{5}{x}}_{1/x}+o\left(\frac{5}{x}\right)\right) - x\cdot\left(1+\underbrace{\underbrace{\binom{1/2}{1}}_{1/2}\cdot\frac{-2}{x}}_{-\frac{1}{x}}+o\left(\frac{-2}{x}\right)\right)\right)$$

$$= \lim_{x\to+\infty}\left(\cancel{x}+1+\underbrace{x\cdot o\left(\frac{5}{x}\right)}_{=o(5)=o(1)\to 0}-\cancel{x}+1+\underbrace{x\cdot o\left(\frac{-2}{x}\right)}_{o(1)}\right)$$

$$= \lim_{x\to+\infty}\left(2+o(1)\right) = \boxed{2}$$

■

7. $f(x)=|3x+1|\cdot e^{\frac{1}{x-2}}$ за $x\to+\infty$, $x\to-\infty$ наћи асимптоте и са које стране ф-ја f прилази асимптотама

Ако: $f(x)=\underbrace{ax+b}_{y=ax+b \text{ асимптота}}+\overset{\to 0}{\frac{c}{x}}+o\left(\frac{1}{x}\right)$, $x\to+\infty$ / $x\to-\infty$

ако $\frac{c}{x}>0 \to \Gamma_f$ изнад асимптоте

ако $\frac{c}{x}<0 \to \Gamma_f$ испод −||−

$$\boxed{e^{\frac{1}{x-2}}}=e^{\frac{1}{x(1-2/x)}}=e^{\frac{1}{x}\cdot\left(1-\frac{2}{x}\right)^{-1}}=e^{\frac{1}{x}\cdot\left(1+\frac{2}{x}+\frac{4}{x^2}+o\left(\frac{1}{x^2}\right)\right)}=e^{\frac{1}{x}+\frac{2}{x^2}+\overbrace{\frac{4}{x^3}+o\left(\frac{1}{x^3}\right)}^{o\left(\frac{1}{x^2}\right)}}$$

$$=e^{\underbrace{\frac{1}{x}+\frac{2}{x^2}+o\left(\frac{1}{x^2}\right)}_{\to 0}}=1+\left(\frac{1}{x}+\frac{2}{x^2}+o\left(\frac{1}{x^2}\right)\right)+\underbrace{\frac{1}{2!}\left(\frac{1}{x}+\frac{2}{x^2}+o\left(\frac{1}{x^2}\right)\right)^2}_{\frac{1}{2x^2}+o\left(\frac{1}{x^2}\right)}+\underbrace{o\left(\left(\frac{1}{x}+\frac{2}{x^2}+o\left(\frac{1}{x^2}\right)\right)^2\right)}_{o\left(\frac{1}{x^2}\right)}$$

$$=1+\frac{1}{x}+\frac{2}{x^2}+\frac{1}{2x^2}+o\left(\frac{1}{x^2}\right)=\boxed{1+\frac{1}{x}+\frac{5}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)}$$

$x\to+\infty$

$x\to-\infty$

$f(x)=|3x+1|\,e^{\frac{1}{x-2}}$

$x\to+\infty$: $|3x+1|=3x+1>0$

$f(x)=(3x+1)\cdot\left(1+\frac{1}{x}+\frac{5}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)$

$=3x+3+\frac{15}{2}\cdot\frac{1}{x}+\underbrace{3x\cdot o\left(\frac{1}{x^2}\right)}_{o\left(\frac{1}{x}\right)}+1+\frac{1}{x}+\frac{5}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)$

$=\underline{3x+4}+\frac{17}{2}\cdot\frac{1}{x}+o\left(\frac{1}{x}\right)$

$\frac{17}{2}\cdot\frac{1}{x}>0,\ x>0$

$\boxed{y=3x+4 \text{ к.а. } x\to+\infty}$

$\boxed{\Gamma_f \text{ изнад АС.}}$

$x\to-\infty$: $|3x+1|=-(3x+1)$

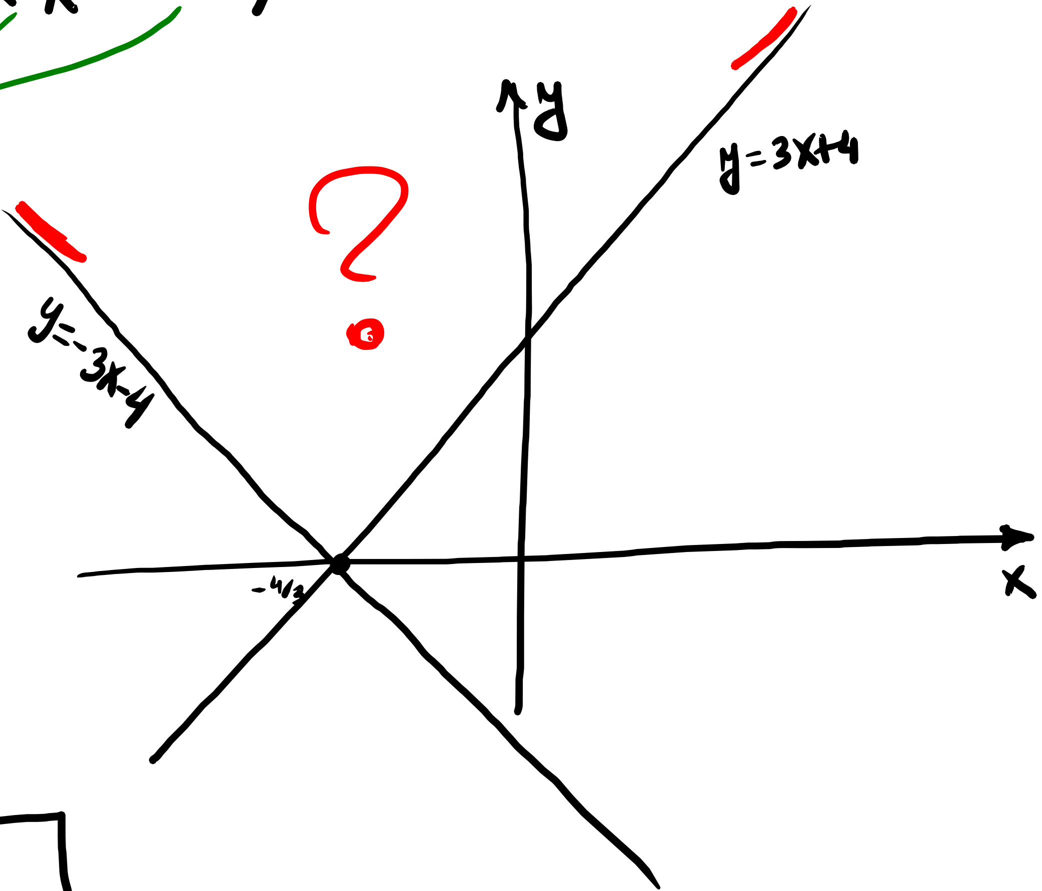
$f(x)=-(3x+1)\cdot(\sim\sim)$

$=\underline{-3x-4}-\frac{17}{2}\cdot\frac{1}{x}+o\left(\frac{1}{x}\right)$

$-\frac{17}{2}\cdot\frac{1}{x}>0,\ x<0$

$\boxed{y=-3x-4 \text{ к.а. } x\to-\infty}$

$\boxed{\Gamma_f \text{ изнад АС.}}$



~ Испитивање функција ~

$f(x) = \dots$

1° D_f

2° парност / неп. ; периодичност

3° нуле & знак

4° асимптоте (Х.А., К.А., В.А.) и понашање у крајевима $D_f = (\_,\_) \cup (\_,\_) \cup (\_,\_)$

5° непр. & диф.

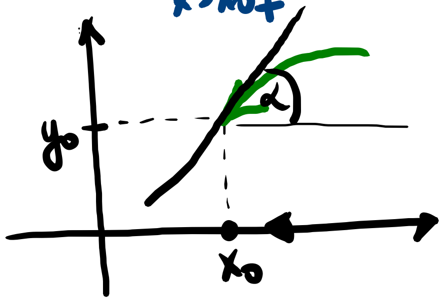
6° $f'(x) = \dots$, монотонија?

7° $f''(x) = \dots$, конвексност / конкавност

8° Γ_f

3°

(!) $x_0 \in \mathbb{R}$ на рубу домена / f није непр. у x_0 али $\exists \lim\limits_{x \to x_0+} f(x) = y_0$



(?) $\lim\limits_{x \to x_0+} f'(x)$

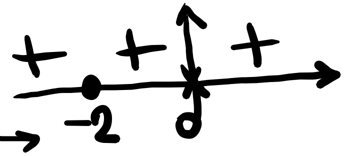
8 $f(x)=|x+2|\cdot e^{-\frac{1}{x}} = \begin{cases} (x+2)\cdot e^{-\frac{1}{x}}, & x\geq -2,\ x\neq 0 \\ -(x+2)\cdot e^{-\frac{1}{x}}, & x<-2 \end{cases}$

1° $D_f:\ x\neq 0$ $\boxed{D_f=(-\infty,0)\cup(0,+\infty)}$ $\boxed{f \text{ непр. на } D_f}$

2° п/н, пер. //

3° Нуле & знак: $f(x)=0 \Leftrightarrow |x+2|=0 \Leftrightarrow x=-2$

$f(x)\geq 0,\ \forall x\in D_f$



4° Асимптотика: $(-\infty,0)\cup(0,+\infty)$

$\uparrow \quad 0- \quad 0+ \quad \uparrow$

$x\to+\infty$

$|x+2|=x+2$

$f(x)=(x+2)\cdot e^{-\frac{1}{x}} = (x+2)\cdot\left(1-\frac{1}{x}+\frac{1}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)$

$= x-1+\frac{1}{2x}+o\left(\frac{1}{x}\right)+2-\frac{2}{x}+\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)$

$= \boxed{x+1}-\underbrace{\frac{3}{2}\cdot\frac{1}{x}}_{<0}+o\left(\frac{1}{x}\right),\ x\to+\infty$

$x\to+\infty$: К.А. $y=x+1$

Гф испод АС.

$x\to-\infty$

$|x+2|=-(x+2)$

$f(x)=-(x+2)\cdot e^{-1/x}$

$= \dots$

$= \boxed{-x-1}+\underbrace{\frac{3}{2}\cdot\frac{1}{x}}_{<0}+o\left(\frac{1}{x}\right)$

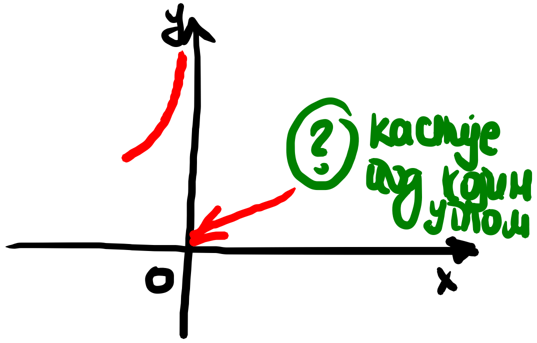
$x\to-\infty$ К.А. $y=-x-1$

Гф испод АС.

4° $f(x)=|x+2|\cdot e^{-\frac{1}{x}}$

$x\to 0+$: $\lim\limits_{x\to 0+} f(x)=\lim\limits_{x\to 0+}\underbrace{|x+2|}_{\to 2}\cdot\underbrace{e^{\overbrace{-\frac{1}{x}}^{\to-\infty}}}_{\to 0}=2\cdot 0=\boxed{0}$

$x\to 0-$: $\lim\limits_{x\to 0-} f(x)=\lim\limits_{x\to 0-}\underbrace{|x+2|}_{\to 2}\cdot\underbrace{e^{\overbrace{-\frac{1}{x}}^{\to+\infty}}}_{\to+\infty}=\boxed{+\infty}$ ⟵ Верт ас. $x=0$ код $x\to 0-$



5° $f'(x)=?$

$x\in(-2,0)\cup(0,+\infty)$: $f'(x)=\left((x+2)\cdot e^{-\frac{1}{x}}\right)'=e^{-\frac{1}{x}}+(x+2)\cdot e^{-\frac{1}{x}}\cdot\frac{1}{x^2}=\frac{e^{-\frac{1}{x}}}{x^2}\cdot(x^2+x+2)$

$x\in(-\infty,-2)$: $f'(x)=\left(-(x+2)\cdot e^{-\frac{1}{x}}\right)'=-\frac{e^{-\frac{1}{x}}}{x^2}\cdot(x^2+x+2)$

$x=-2$?

$f'_+(-2)\overset{\text{ако }\exists}{=}\lim\limits_{x\to-2+} f'(x)=\lim\limits_{x\to-2+}\frac{e^{-\frac{1}{x}}}{x^2}(x^2+x+2)=\underline{\sqrt{e}}$ $\neq$ $\boxed{f \text{ није диф. у } -2}$

$f'_-(-2)\overset{\text{ако }\exists}{=}\lim\limits_{x\to-2-} f'(x)=\lim\limits_{x\to-2-}\frac{-e^{-\frac{1}{x}}}{x^2}(x^2+x+2)=\underline{-\sqrt{e}}$

$\Rightarrow$ $\boxed{f \text{ диф. на } (-\infty,-2)\cup(-2,0)\cup(0,+\infty)}$

Заключак за f':

$$f'(x) = \begin{cases} e^{-\frac{1}{x}} \cdot \dfrac{x^2+x+2}{x^2}, & x \in (-2,0) \cup (0,+\infty) \\ -e^{-\frac{1}{x}} \cdot \dfrac{x^2+x+2}{x^2}, & x \in (-\infty,-2) \end{cases}$$

$f'_-(-2) = -\sqrt{e}$, $f'_+(-2) = \sqrt{e}$