10.5.2021.

Лајбницова формула $(u\cdot v)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} u^{(k)} \cdot v^{(n-k)}$

$u^{(0)} = u, \quad v^{(0)} = v$

1. $f(x) = \underbrace{(5x^2-2x+1)}_{u(x)} \cdot \underbrace{e^{-3x}}_{v(x)}$ $\quad n\in\mathbb{N}$ $\quad f^{(n)}(x) = ?$ $\quad f^{(n)}(0) = ?$

$\boxed{u}$

$u(x) = 5x^2-2x+1$

$u'(x) = 10x-2$

$u''(x) = 10$

$u^{(k)}(x) = 0, \ \forall k\geq 3$

$\boxed{v}$

$v(x) = e^{-3x}$

$v'(x) = (-3)\cdot e^{-3x}$

$v''(x) = (-3)^2\cdot e^{-3x}$

$\vdots$

$v^{(k)}(x) = (-3)^k\cdot e^{-3x}, \ \forall k\geq 1$

$\leftarrow (a^x)^{(n)}$

$f^{(n)}(x) = (u\cdot v)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} u^{(k)}\cdot v^{(n-k)}$ $\leftarrow$ $\underline{k=0,1,2}$ $\quad \underline{k\geq 3 \text{ чланови } = 0}$

$= \underbrace{\binom{n}{0}}_{=1} \cdot u(x)\cdot v^{(n)}(x) + \underbrace{\binom{n}{1}}_{=n} u'(x)\cdot v^{(n-1)}(x) + \underbrace{\binom{n}{2}}_{\frac{n(n-1)}{2}} u''(x)\cdot v^{(n-2)}(x) + 0$

$= (5x^2-2x+1)\cdot(-3)^n e^{-3x} + n\cdot(10x-2)\cdot(-3)^{n-1} e^{-3x} + \frac{n(n-1)}{2}\cdot 10\cdot(-3)^{n-2}\cdot e^{-3x}$

$$\boxed{f^{(n)}(x) = e^{-3x}\cdot(-3)^{n-2}\left(9(5x^2-2x+1) + n(10x-2)\cdot(-3) + 5n(n-1)\right)}$$

$f^{(n)}(0) = (-3)^{n-2}(9+6n+5n^2-5n)$

$$\boxed{f^{(n)}(0) = (-3)^{n-2}\cdot(5n^2+n+9)}$$

Тејлоров развој

f - непр., $f', \dots, f^{(n)}$ - непр. у околини a, $\exists f^{(n+1)}(x)$ у тој околини

$$f(x) = \underbrace{f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n}_{\text{Тејлоров полином } P_n(x)} + o((x-a)^n), \quad x \to a$$

$a=0$: Маклоренова формула: $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + o(x^n), \quad x \to 0$

$$\boxed{\sin x} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!} + \underbrace{0 + o(x^{2n+2})}_{o(x^{2n+1})}, \quad x \to 0$$

$$o(x^{2n+2}) = o(x^{2n+1})$$

$$\boxed{\cos x} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \underbrace{0 + o(x^{2n+1})}_{o(x^{2n})}, \quad x \to 0$$

$$\boxed{e^x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + o(x^n), \quad x \to 0$$

$$\boxed{(1+x)^\alpha} = 1 + \binom{\alpha}{1}x + \binom{\alpha}{2}x^2 + \dots + \binom{\alpha}{n}x^n + o(x^n), \quad x \to 0$$

$\alpha \in \mathbb{R}, n \in \mathbb{N}$ $\quad \binom{\alpha}{n} = \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!}$

$$\boxed{\ln(1+x)} = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1} \cdot \frac{x^n}{n} + o(x^n), \quad x \to 0$$

1

① (а) $\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + o(x^n), \; x \to 0$

вежба (б) $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + o(x^n), \; x \to 0$

(а) $\dfrac{1}{1+x} = (1+x)^{-1} \overset{\alpha=-1}{=\!=} 1 + \binom{-1}{1} x + \binom{-1}{2} x^2 + \cdots + \binom{-1}{n} x^n + o(x^n), \; x \to 0$

$$\underbrace{\binom{-1}{k}} = \frac{(-1)(-2)(-3)\cdots(-k)}{k!} = \frac{(-1)^k \cdot k!}{k!} = \underbrace{(-1)^k}$$

$$= 1 + (-1)x + (-1)^2 x^2 + \cdots + (-1)^n x^n + o(x^n), \; x \to 0$$

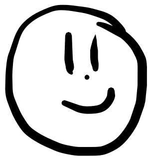
$$= 1 - x + x^2 + \cdots + (-1)^n x^n + o(x^n), \; x \to 0$$

2. $f(x) = \operatorname{arctg} x$ Тејлоров развој $a = 3$, $n = 2$

$$f'(x) = \frac{1}{1+x^2} \qquad f'(3) = \frac{1}{10}$$

$$f''(x) = \left(\frac{1}{1+x^2}\right)' = -\frac{1}{(1+x^2)^2} \cdot 2x \qquad f''(3) = -\frac{1}{100} \cdot 2 \cdot 3 = -\frac{3}{50}$$

$$\boxed{\operatorname{arctg} x} = \overbrace{\operatorname{arctg} 3}^{f(3)} + \frac{1}{\underbrace{10}_{1!}}(x-3) + \frac{-3/50}{2!}(x-3)^2 + o\left((x-3)^2\right), \; x \to 3$$



Тејлор. $\underline{\underline{a}}$ $\quad (x-a)^k, \; x \to a$

смена $t = x - a \quad t \to 0$

Маклорен $\quad t^k, \; t \to 0$

$x\to 0$ $\quad x^n = o(x^m),\ x\to 0 \quad n>m$

$x^7 = o(x^3),\ x\to 0 \qquad x^7 = x^4 x^3,\ x^4 \to 0$

$$x^7 + x^5 + o(x^3) = o(x^3),\ x\to 0 \quad !$$

$x^7 = o(x^3)$, $x^5 = o(x^3)$

③ $f(x) = \operatorname{tg} x \quad x\to 0 \quad$ Маклорен, $n=5$

I начин: $f', f'', f''', f^{IV}, f^{V}$ – за вежбу

$$f(x) = \frac{\sin x}{\cos x} = \sin x (\cos x)^{-1} = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)\right)\cdot\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + o(x^5)\right)^{-1}$$

① (a) $(1+t)^{-1} = 1 - t + t^2 \cdots + (-1)^n t^n + o(t^n)$, $t\to 0$

$t\to 0$

$$= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)\right)\cdot\Bigg(1 - \underbrace{\left(-\frac{x^2}{2!} + \frac{x^4}{4!} + o(x^5)\right)}_{t} + \underbrace{\left(-\frac{x^2}{2!} + \frac{x^4}{4!} + o(x^5)\right)^2}_{t^2} - \underbrace{\left(-\frac{x^2}{2} + \frac{x^4}{4!} + o(x^5)\right)^3}_{t^3} + o\left(\frac{t^3}{x^6}\right)\Bigg),\ x\to 0$$

$$= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)\right)\cdot\left(1 + \frac{x^2}{2} - \frac{x^4}{4!} + o(x^5) + \frac{x^4}{4} + o(x^5) + o(x^5) + o(x^6)\right)$$

$$= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)\right)\cdot\left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + o(x^5)\right),\ x\to 0$$

$$\operatorname{tg} x = \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^6)\right) \cdot \left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + o(x^5)\right), \quad x \to 0$$

$$= x + \frac{x^3}{2} + \frac{5}{24}x^5 + \underbrace{x \cdot o(x^5)}_{= o(x^6) = o(x^5)}$$

$$- \frac{x^3}{6} - \frac{x^5}{12} \underbrace{- \frac{5}{6 \cdot 24}x^7}_{= o(x^5)} + \underbrace{\frac{-x^3}{6} \cdot o(x^5)}_{= o(x^5)}$$

$$+ \frac{x^5}{120} + o(x^5)$$

$$+ o(x^6)$$

$$= o(x^5)$$

$$\boxed{\operatorname{tg} x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^5), \quad x \to 0}$$

4. $f(x)=(1+x)^{\frac{1}{x}}$ развити до: $f(x)=a+bx+cx^2+o(x^2),\ x\to 0$

$$f(x)=(1+x)^{\frac{1}{x}}=e^{\frac{1}{x}\ln(1+x)}$$

← први покушај до x^2 :)

мора више

$$=e^{\frac{1}{x}\cdot\left(x-\frac{x^2}{2}+o(x^2)\right)}$$

$$=e^{1-\frac{x}{2}+\frac{1}{x}\cdot o(x^2)}\quad \frac{1}{x}\cdot o(x^2)=o(x)\qquad =e^{1-\frac{x}{2}+o(x)}$$

није довољно!

$\ln(1+x)$ мора ред 3 :

$$f(x)=(1+x)^{\frac{1}{x}}=e^{\frac{1}{x}\ln(1+x)}=e^{\frac{1}{x}\cdot\left(x-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)\right)}=e^{\overset{\to 1}{1}-\frac{x}{2}+\frac{x^2}{3}+o(x^2)}=e\cdot e^{-\frac{x}{2}+\frac{x^2}{3}+o(x^2)}\qquad t\to 0$$

e^{t}, $t\to 0$ за развој

$$=e\cdot\left(1+\overbrace{\left(-\frac{x}{2}+\frac{x^2}{3}+o(x^2)\right)}^{t}+\frac{1}{2!}\left(-\frac{x}{2}+\frac{x^2}{3}+o(x^2)\right)^2+\underbrace{o\left(\overbrace{\left(-\frac{x}{2}+\frac{x^2}{3}+o(x^2)\right)^2}^{t^2}\right)}_{o(x^2)}\right)$$

$\frac{x^2}{4},\ x\to 0$

$$=e\cdot\left(1-\frac{x}{2}+\frac{x^2}{3}+o(x^2)+\frac{1}{8}x^2+o(x^2)\right)$$

$$=e\cdot\left(1-\frac{x}{2}+\frac{11}{24}x^2+o(x^2)\right)=e-\frac{e}{2}\cdot x+\frac{11e}{24}x^2+o(x^2),\quad x\to 0\qquad\blacksquare$$

5. $$L=\lim_{x\to 0}\frac{e^{\sin x}\cdot\cos(\sin x)-1-x}{x^3}=?$$ "$\frac{0}{0}$"

$\sin x = x-\frac{x^3}{6}+o(x^3),\ x\to 0$ $(o(x^4))$

$$L=\lim_{x\to 0}\frac{e^{x-\frac{x^3}{6}+o(x^3)}\cdot\cos\left(x-\frac{x^3}{6}+o(x^3)\right)-1-x}{x^3}$$

$$\overset{e^t,\ t\to 0}{=}\lim_{x\to 0}\frac{1}{x^3}\left[\left(1+\left(x-\cancel{\frac{x^3}{6}}+o(x^3)\right)+\frac{1}{2!}\cdot\underbrace{\left(x-\frac{x^3}{6}+o(x^3)\right)^2}_{\frac{1}{2}x^2+o(x^3)}+\frac{1}{3!}\cdot\underbrace{\left(x-\frac{x^3}{6}+o(x^3)\right)^3}_{\cancel{\frac{1}{6}x^3}+o(x^3)}+o(\overset{t^3}{x^3})\right)\cdot\cos\left(x-\frac{x^3}{6}+o(x^3)\right)-1-x\right]$$

$$=\lim_{x\to 0}\frac{1}{x^3}\cdot\left[\left(1+x+\frac{1}{2}x^2+o(x^3)\right)\cdot\cos\left(x-\frac{x^3}{6}+o(x^3)\right)-1-x\right]$$

$$\overset{\cos t,\ t\to 0}{=}\lim_{x\to 0}\frac{1}{x^3}\left[\left(1+x+\frac{1}{2}x^2+o(x^3)\right)\cdot\left(1\underbrace{-\frac{1}{2}\left(x-\frac{x^3}{6}+o(x^3)\right)^2}_{-\frac{1}{2}x^2+o(x^3)}+\underbrace{\frac{1}{4}\left(x-\frac{x^3}{6}+o(x^3)\right)^4+o(\overset{t^4}{x^4})}_{o(x^3)}\right)-1-x\right]$$

$$=\lim_{x\to 0}\frac{1}{x^3}\left[\left(1+x+\frac{1}{2}x^2+o(x^3)\right)\cdot\left(1-\frac{1}{2}x^2+o(x^3)\right)-1-x\right]$$

$$=\lim_{x\to 0}\frac{1}{x^3}\left[\cancel{1}-\frac{1}{2}x^2+o(x^3)+\cancel{x}-\frac{1}{2}x^3+x\cdot o(x^3)+\frac{1}{2}x^2-\frac{1}{4}x^4+\frac{1}{2}x^2\cdot o(x^3)+o(x^3)-\cancel{1}-\cancel{x}\right]$$

$$=\lim_{x\to 0}\frac{-\frac{1}{2}x^3+o(x^3)}{x^3}$$

$$=\lim_{x\to 0}\left(-\frac{1}{2}+o(1)\right)$$

$$=\boxed{-\frac{1}{2}}$$ □

⑥ $\lim\limits_{x\to+\infty}\left(\sqrt[5]{x^5+5x^4}-\sqrt{x^2-2x}\right)=$ $(\infty-\infty)$

$$=\lim_{x\to+\infty}\left(x\cdot\sqrt[5]{1+\frac{5}{x}}-\underbrace{|x|}_{\substack{=x\\ x>0}}\cdot\sqrt{1-\frac{2}{x}}\right)$$

$x\to+\infty$

!! $x\to+\infty$

$\frac{1}{x}\to 0$

$$=\lim_{x\to+\infty}\left(x\cdot\left(1+\underset{\to 0}{\frac{5}{x}}\right)^{1/5}-x\cdot\left(1-\underset{\to 0}{\frac{2}{x}}\right)^{1/2}\right)$$

$$=\lim_{x\to+\infty}\left(x\cdot\left(1+\underbrace{\binom{1/5}{1}}_{1/5}\cdot\frac{5}{x}+o\left(\frac{5}{x}\right)\right)-x\cdot\left(1+\underbrace{\binom{1/2}{1}}_{1/2}\cdot\frac{-2}{x}+o\left(-\frac{2}{x}\right)\right)\right)$$

$\ulcorner x\cdot o\left(\frac{5}{x}\right)=o(5)\to 0$

$=o(1)$

$$=\lim_{x\to+\infty}\left(\cancel{x}+1+o(1)-\cancel{x}+1+o(1)\right)$$

$$=\lim_{x\to+\infty}\left(2+o(1)\right)=\boxed{2}$$

□

7. $f(x) = |3x+1| \cdot e^{\frac{1}{x-2}}$ асимптоте ? за $x \to +\infty$, $x \to -\infty$

са које стране ф-ја прилази асимптоти ?

$$f(x) = \underbrace{ax+b}_{y=ax+b \text{ АС.}} + \overset{o(1)}{\frac{c}{x}} + o\left(\frac{1}{x}\right), \quad x \to +\infty \,/\, x \to -\infty$$

$\frac{c}{x} > 0$ Γ_f изнад асимптоте

$\frac{c}{x} < 0$ Γ_f испод асимптоте

$$e^{\frac{1}{x-2}} = e^{\frac{1}{x(1-2/x)}} = e^{\frac{1}{x}\cdot\left(1-\frac{2}{x}\right)^{-1}} = e^{\frac{1}{x}\left(1+\frac{2}{x}+\frac{4}{x^2}+o\left(\frac{1}{x^2}\right)\right)} = e^{\frac{1}{x}+\frac{2}{x^2}+\underbrace{\frac{4}{x^3}+o\left(\frac{1}{x^3}\right)}_{o\left(\frac{1}{x^2}\right)}}$$

$x \to +\infty / x \to -\infty$

$\frac{1}{x} \to 0$

$$= e^{\overset{t \to 0}{\frac{1}{x}+\frac{2}{x^2}+o\left(\frac{1}{x^2}\right)}}$$

$$= 1 + \left(\frac{1}{x}+\frac{2}{x^2}+o\left(\frac{1}{x^2}\right)\right) + \underbrace{\frac{1}{2}\left(\frac{1}{x}+\frac{2}{x^2}+o\left(\frac{1}{x^2}\right)\right)^2}_{\frac{1}{2}\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)} + o\left(\frac{1}{x^2}\right)$$

$$\boxed{e^{\frac{1}{x-2}} = 1+\frac{1}{x}+\frac{5}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)} \quad \begin{matrix} x \to +\infty \\ x \to -\infty \end{matrix}$$

$\underline{x\to+\infty}$ $\quad f(x)=\underbrace{|3x+1|}_{3x+1>0}\cdot e^{\frac{1}{x-2}}=(3x+1)\cdot\left(1+\frac{1}{x}+\frac{5}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)$

$$=3x+3+\frac{15}{2}\cdot\frac{1}{x}+3x\cdot o\left(\frac{1}{x^2}\right)+1+\frac{1}{x}+\frac{5}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)$$

$$=\underbrace{3x+4}_{}+\underbrace{\frac{17}{2}\frac{1}{x}}_{>0}+o\left(\frac{1}{x}\right)$$

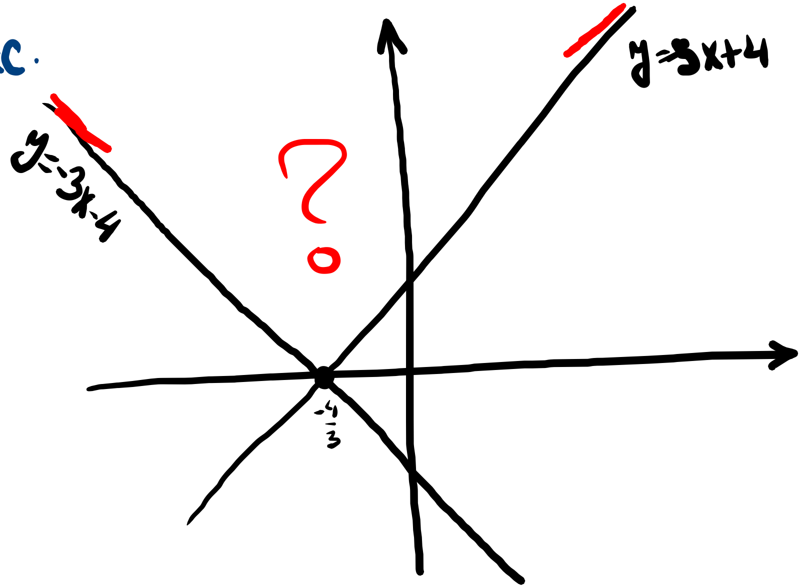
$y=3x+4$ К.А. $x\to+\infty$ $\qquad \Gamma_f$ изнад ас.

$\underline{x\to-\infty}$ $\quad f(x)=\underbrace{|\overbrace{3x+1}^{<0}|}_{-(3x+1)}\cdot e^{\frac{1}{x-2}}=-(3x+1)\cdot e^{\frac{1}{x-2}}$

$$=\dots$$

$$=\underbrace{-3x-4}_{}\underbrace{-\frac{17}{2}\underbrace{\frac{1}{x}}_{<0}}_{>0,\ x\to-\infty}+o\left(\frac{1}{x}\right)$$

$y=-3x-4$ $x\to-\infty$ К.А. $\qquad \Gamma_f$ изнад ас.



~ Испитивање функција ~

$f(x) = \dots$

$\boxed{1}$ $D_f = ?$

$\boxed{2}$ парност/непар., периодичност

$\boxed{3}$ нуле, знак

$\boxed{4}$ асимптоте, понашање на крајевима домена $(\downarrow \; \downarrow) \cup (\downarrow \; \downarrow)$

$\boxed{5}$ непр., диф.

$\boxed{6}$ $f'(x)$, интервали монотоније

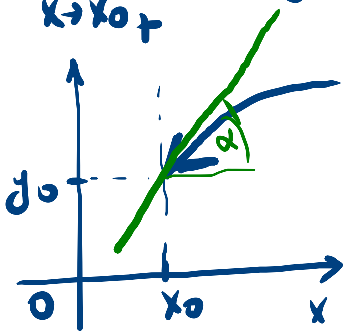
$\boxed{7}$ $f''(x)$, конвексност/конкавност

$\boxed{8}$ Γ_f

Ако

$x_0 \notin D_f$ / f није непр. у x_0

$\lim_{x \to x_0^+} f(x) = y_0$



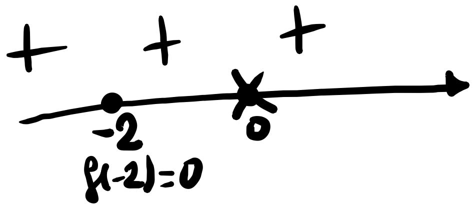
? $\lim_{x \to x_0^+} f'(x)$

8. $f(x)=|x+2|\cdot e^{-\frac{1}{x}} = \begin{cases} (x+2)\cdot e^{-1/x}, & x\geq -2,\ x\neq 0 \\ -(x+2)\cdot e^{-1/x}, & x<-2 \end{cases}$

1° $\boxed{D_f=(-\infty,0)\cup(0,+\infty)}$ $\boxed{f \text{ непр. на } D_f}$

2° П/Н, ПЕР. //

3° НУЛЕ, ЗНАК: $f(x)=0 \iff |x+2|=0 \iff \underline{x=-2}$



$\underline{f(x)\geq 0,\ \forall x\in D_f}$

4° **АСИМПТОТИКА**: $(-\infty,0)\cup(0,+\infty)$

$\uparrow \quad 0_- \quad 0_+ \quad \uparrow$

$x\to+\infty$ $\quad f(x)=(x+2)\cdot e^{-1/x} = (x+2)\cdot\left(1+\frac{-1}{x}+\frac{1}{2}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right) = x-1+\frac{1}{2x}+o\left(\frac{1}{x}\right)+2-\frac{2}{x}+\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)$

$|x+2|=x+2>0$

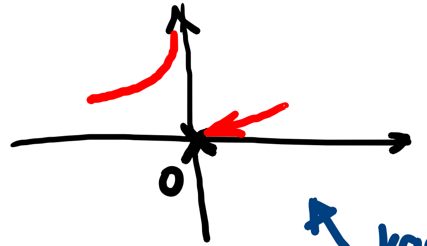
$= x+1\underbrace{-\frac{3}{2}\cdot\frac{1}{x}}_{<0}+o\left(\frac{1}{x}\right)$ $\quad y=x+1$ К.А. $x\to+\infty$ Γ_f испод АС.

$x\to-\infty$ $\quad |x+2|=-(x+2)$ $\quad f(x)=-(x+2)\cdot e^{-1/x}=\dots=-x-1+\overbrace{\frac{3}{2}\cdot\frac{1}{x}}^{<0}+o\left(\frac{1}{x}\right)$ $\quad y=-x-1$ К.А. $x\to-\infty$ Γ_f испод АС.

$\boxed{0}$ $f(x)=|x+2|\cdot e^{-1/x}$

$$\lim_{x\to 0-} f(x)=\lim_{x\to 0-} \underbrace{|x+2|}_{\to 2}\, e^{\overbrace{-\frac{1}{x}}^{\to +\infty}} = +\infty$$

$\boxed{x=0 \text{ В.А. } x\to 0-}$



↑ касније: под којим углом ?

$$\lim_{x\to 0+} f(x)=\lim_{x\to 0+} \underbrace{|x+2|}_{\to 2}\cdot \underbrace{e^{\overbrace{-\frac{1}{x}}^{\to -\infty}}}_{\to 0} = 0$$

5° $f'(x)=?$

$x\in(-2,0)\cup(0,+\infty)$: $f'(x)=\left((x+2)\cdot e^{-1/x}\right)' = e^{-1/x}+(x+2)e^{-1/x}\cdot\frac{1}{x^2} = \frac{e^{-1/x}}{x^2}\cdot(x^2+x+2)$

$x\in(-\infty,-2)$: $f'(x)=\left(-(x+2)\cdot e^{-1/x}\right)' = -\frac{e^{-1/x}}{x^2}(x^2+x+2)$

$x=-2$?

$$f'_+(-2) \overset{\text{ако } \exists}{=} \lim_{x\to -2+} f'(x) = \lim_{x\to -2+} \frac{e^{-1/x}}{x^2}(x^2+x+2) = \sqrt{e}$$

$\neq$ $\boxed{f \text{ није диф. у } -2}$

$$f'_-(-2) \overset{\text{ако } \exists}{=} \lim_{x\to -2-} f'(x) = \lim_{x\to -2-} -\frac{e^{-1/x}}{x^2}(x^2+x+2) = -\sqrt{e}$$

f диф. на $(-\infty,-2)\cup(-2,0)\cup(0,+\infty)$

$$f'(x) = \begin{cases} e^{-1/x}\cdot\dfrac{x^2+x+2}{x^2}, & x\in(-2,0)\cup(0,+\infty) \\ -e^{-\frac{1}{x}}\cdot\dfrac{x^2+x+2}{x^2}, & x\in(-\infty,-2) \end{cases}$$

у -2: $f'_-(-2) = -\sqrt{e}$ $\quad f'_+(-2) = \sqrt{e}$