①

$$\begin{aligned} x+(2\beta-2)y+\boxed{z}&=4 \quad /\cdot(-1) \\ (\alpha+1)x+\quad y+z&=4 \\ x+(\beta-1)y+z&=3 \end{aligned}$$

$$\begin{aligned} x+(2\beta-2)y+\boxed{z}&=4 \\ \alpha x+(3-2\beta)y\quad&=0 \\ (1-\beta)y\quad&=-1 \end{aligned}$$

$\underline{1^\circ\ \alpha=0}$

$$\begin{aligned} x+(2\beta-2)y+\boxed{z}&=4 \\ (3-2\beta)y\quad&=0 \\ (1-\beta)y\quad&=-1 \end{aligned}$$

$\underline{1.1^\circ\ \beta=1\ (\alpha=0)}$

$$\begin{aligned} x+\quad\boxed{z}&=4 \\ y\quad&=0 \\ \boxed{0\quad=-1} \end{aligned}$$

$\Rightarrow$ **НЕМА РЕШЕЊА**

$\underline{1.2^\circ\ \beta\neq1\ (\alpha=0)}$

$$\begin{aligned} x+(2\beta-2)y+\boxed{z}&=4 \\ (1-\beta)\boxed{y}\quad&=-1 \quad /\cdot\left(-\frac{3-2\beta}{1-\beta}\right) \\ (3-2\beta)y\quad&=0 \end{aligned}$$

$$\begin{aligned} x+(2\beta-2)y+\boxed{z}&=4 \\ (1-\beta)\boxed{y}\quad&=-1 \\ 0\quad&=\frac{3-2\beta}{1-\beta} \end{aligned}$$

$\underline{1.2.1^\circ\ \beta=\frac{3}{2}\ (\beta\neq1,\alpha=0)}$

$$\begin{aligned} x+y+\boxed{z}&=4 \\ -\tfrac{1}{2}\boxed{y}\quad&=-1 \\ 0&=0 \end{aligned}$$

$x=a,\ a\in\mathbb{R}$

$y=2$

$z=4-a-2=2-a$

$$\boxed{(x,y,z)\in\{(a,2,2-a)\mid a\in\mathbb{R}\}}$$

$\underline{1.2.2^\circ\ \beta\neq\frac{3}{2}\ (\beta\neq1,\alpha=0)}$

$0=\frac{3-2\beta}{1-\beta}\neq0$

НЕМА РЕШЕЊА

$\underline{2^\circ\ \alpha\neq0}$

$$\begin{aligned} x+(2\beta-2)y+\boxed{z}&=4 \\ \alpha\boxed{x}+(3-2\beta)y\quad&=0 \\ (1-\beta)y\quad&=-1 \end{aligned}$$

$\underline{2.1^\circ\ \beta=1\ (\alpha\neq0)}$

$$\begin{aligned} x+\quad\boxed{z}&=4 \\ \alpha\boxed{x}+y\quad&=0 \\ \boxed{0\quad=-1} \end{aligned}$$

НЕМА РЕШЕЊА

$\underline{2.2^\circ\ \beta\neq1\ (\alpha\neq0)}$

$$\begin{aligned} x+(2\beta-2)y+\boxed{z}&=4 \\ \alpha\boxed{x}+(3-2\beta)y\quad&=0 \\ (1-\beta)\boxed{y}\quad&=-1 \end{aligned}$$

ЈЕДИНСТВЕНО РЕШЕЊЕ

$y=\frac{1}{\beta-1}$

$x=-\frac{3-2\beta}{\alpha}y=\frac{2\beta-3}{\alpha(\beta-1)}$

$z=4-\frac{2\beta-3}{\alpha(\beta-1)}-\underbrace{(2\beta-2)\frac{1}{\beta-1}}_{}=2-\frac{2\beta-3}{\alpha(\beta-1)}=\frac{2\alpha\beta-2\alpha-2\beta+3}{\alpha(\beta-1)}$

$$\boxed{(x,y,z)\in\left\{\left(\frac{2\beta-3}{\alpha(\beta-1)},\frac{1}{\beta-1},\overset{=2}{2}-\frac{2\beta-3}{\alpha(\beta-1)}\right)\right\}}$$

$$(x,y,z)\in\begin{cases}\left\{\left(\frac{2\beta-3}{\alpha(\beta-1)},\frac{1}{\beta-1},2-\frac{2\beta-3}{\alpha(\beta-1)}\right)\right\}, & \text{за } \alpha\neq0 \text{ и } \beta\neq1 \\ \{(a,2,2-a)\mid a\in\mathbb{R}\}, & \text{за } \alpha=0 \text{ и } \beta=\frac{3}{2} \\ \emptyset, & \text{за } \underline{(\alpha=0 \text{ и } \beta=1) \text{ или } (\alpha=0 \text{ и } \beta\neq1 \text{ и } \beta\neq\frac{3}{2}) \text{ или } (\alpha\neq0 \text{ и } \beta=1)} \end{cases}$$

$\Leftrightarrow \underline{(\beta=1) \text{ или } (\beta\neq\frac{3}{2} \text{ и } \beta\neq1 \text{ и } \alpha=0)}$

5. (Задак из књиге проф. Ракића)

$$\det A = \begin{vmatrix} 0 & 1 & 2 & 3 & \dots & n-2 & n-1 \\ 1 & 0 & 1 & 2 & \dots & n-3 & n-2 \\ 2 & 1 & 0 & 1 & \dots & n-4 & n-3 \\ 3 & 2 & 1 & 0 & \dots & n-5 & n-4 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-2 & n-3 & n-4 & n-5 & \dots & 0 & 1 \\ n-1 & n-2 & n-3 & n-4 & \dots & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 & 3 & \dots & n-2 & n-1 \\ 1 & 1 & 3 & 5 & \dots & 2n-5 & 2n-3 \\ 2 & 2 & 2 & 4 & \dots & 2n-6 & 2n-4 \\ 3 & 3 & 3 & 3 & \dots & 2n-7 & 2n-5 \\ \vdots & \vdots & \vdots & \vdots & & & \\ n-2 & n-2 & n-2 & n-2 & \dots & n-2 & n \\ n-1 & n-1 & n-1 & n-1 & \dots & n-1 & n-1 \end{vmatrix} =$$

$$= (n-1) \begin{vmatrix} 0 & 1 & 2 & 3 & \dots & n-2 & n-1 \\ 1 & 1 & 3 & 5 & \dots & 2n-5 & 2n-3 \\ 2 & 2 & 2 & 4 & \dots & 2n-6 & 2n-4 \\ 3 & 3 & 3 & 3 & \dots & 2n-7 & 2n-5 \\ \vdots & \vdots & \vdots & \vdots & & & \vdots \\ n-2 & n-2 & n-2 & n-2 & \dots & n-2 & n \\ \boxed{1} & 1 & 1 & 1 & \dots & 1 & 1 \end{vmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ /(2-n) \quad /(-3) \quad /(-2) \quad /(-1) \end{matrix} = (n-1) \begin{vmatrix} 0 & 1 & 2 & 3 & \dots & n-2 & n-1 \\ 0 & 0 & 2 & 4 & \dots & 2n-6 & 2n-4 \\ 0 & 0 & 0 & 2 & \dots & 2n-8 & 2n-6 \\ 0 & 0 & 0 & 0 & \dots & 2n-10 & 2n-8 \\ \vdots & & & & & & \\ 0 & 0 & 0 & 0 & \dots & 0 & 2 \\ \boxed{1} & 1 & 1 & 1 & \dots & 1 & 1 \end{vmatrix}_{n\times n} =$$

Лапласов развој

$$= (n-1)(-1)^{n+1} \begin{vmatrix} 1 & 2 & 3 & \dots & n-2 & n-1 \\ 0 & 2 & 4 & \dots & 2n-6 & 2n-4 \\ 0 & 0 & 2 & \dots & 2n-8 & 2n-6 \\ \vdots & \vdots & \vdots & \ddots & & \\ 0 & 0 & 0 & \dots & 2 & 4 \\ 0 & 0 & 0 & \dots & 0 & 2 \end{vmatrix}_{(n-1)\times(n-1)} = (n-1)(-1)^{n+1}\cdot 2^{n-2} = \boxed{-(n-1)(-2)^{n-2}}$$

II начин (Никола Станојевић 92/2016)

$$\det A = \begin{vmatrix} 0 & 1 & 2 & 3 & \dots & n-2 & n-1 \\ 1 & 0 & 1 & 2 & \dots & n-3 & n-2 \\ 2 & 1 & 0 & 1 & \dots & n-4 & n-3 \\ 3 & 2 & 1 & 0 & \dots & n-5 & n-4 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-2 & n-3 & n-4 & n-5 & \dots & 0 & 1 \\ n-1 & n-2 & n-3 & n-4 & \dots & 1 & 0 \end{vmatrix} \begin{matrix} /(-1) \\ /(-1) \\ /(-1) \\ /(-1) \\ /(-1) \\ /(-1) \\ \end{matrix} = \begin{vmatrix} 0 & 1 & 2 & 3 & \dots & n-2 & n-1 \\ 1 & -1 & -1 & -1 & \dots & -1 & -1 \\ 1 & 1 & -1 & -1 & \dots & -1 & -1 \\ 1 & 1 & 1 & -1 & \dots & -1 & -1 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & 1 & \dots & -1 & -1 \\ 1 & 1 & 1 & 1 & \dots & 1 & -1 \end{vmatrix} =$$

$$= \begin{vmatrix} n-1 & n & n+1 & n+2 & \dots & n-2 & n-1 \\ 0 & -2 & -2 & -2 & \dots & -2 & -1 \\ 0 & 0 & -2 & -2 & \dots & -2 & -1 \\ 0 & 0 & 0 & -2 & \dots & -2 & -1 \\ \vdots & \vdots & \vdots & \vdots & & & \vdots \\ 0 & 0 & 0 & 0 & \dots & -2 & -1 \\ 0 & 0 & 0 & 0 & \dots & 0 & -1 \end{vmatrix} = \boxed{(n-1)(-2)^{n-2}(-1)}$$

6. $u = (1,2,3,4)$
$v = (1,1,1,1)$
$x = (1,0,-1,-2)$
$y = (-2,-5,-8,-11)$
$z = (0,1,2,3)$

$\mathcal{L}(u,v) \overset{?}{=} \mathcal{L}(x,y,z)$

$\mathcal{L}(u,v) = \{\beta_1 u + \beta_2 v \mid \beta_1, \beta_2 \in \mathbb{R}\}$, $\mathcal{L}(x,y,z) = \{\alpha_1 x + \alpha_2 y + \alpha_3 z \mid \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}\}$

$\mathcal{L}(u,v) = \mathcal{L}(x,y,z) \Leftrightarrow$ сваки вектор првог скупа може да се нађе у другом скупу, и обрнуто

$\Leftrightarrow (\exists \alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2 \in \mathbb{R})\ \alpha_1 x + \alpha_2 y + \alpha_3 z = \beta_1 u + \beta_2 v$ ← тачно је

$\Leftrightarrow \alpha_1(1,0,-1,-2) + \alpha_2(-2,5,-8,-11) + \alpha_3(0,1,2,3) = \beta_1(1,2,3,4) + \beta_2(1,1,1,1)$

$$\Leftrightarrow \begin{aligned} \boxed{\alpha_1} - 2\alpha_2 - \beta_1 - \beta_2 &= 0 \quad /\cdot 2 \\ 5\alpha_2 + \alpha_3 - 2\beta_1 - \beta_2 &= 0 \\ -\alpha_1 - 8\alpha_2 + 2\alpha_3 - 3\beta_1 - \beta_2 &= 0 \quad + \\ -2\alpha_1 - 11\alpha_2 + 3\alpha_3 - 4\beta_1 - \beta_2 &= 0 \end{aligned}$$

$$\begin{aligned} 5\boxed{\alpha_2} + \alpha_3 - 2\beta_1 - \beta_2 &= 0 \\ -10\alpha_2 + 2\alpha_3 - 4\beta_1 - 2\beta_2 &= 0 \\ -15\alpha_2 + 3\alpha_3 - 6\beta_1 - 3\beta_2 &= 0 \end{aligned}$$

исте ј-не