(1)

$$\begin{aligned} \boxed{x} + y + z &= 1 \quad /(-1) \\ x + ay + z &= b \\ x + a^2y + z &= b \end{aligned}$$

$a, b \in \mathbb{R}$

$$\begin{aligned} \boxed{x} + y + z &= 1 \\ (a-1)y &= b-1 \\ (a^2-1)y &= b-1 \end{aligned}$$

1° $a=1$

$$\begin{aligned} \boxed{x} + y + z &= 1 \\ 0 &= b-1 \\ 0 &= b-1 \end{aligned}$$

1.1° $a=1, b=1$

$\boxed{x} + y + z = 1$

$\boxed{y=\alpha, \alpha \in \mathbb{R} \\ z=\beta, \beta\in\mathbb{R} \\ x = 1-\alpha-\beta}$

∞ РЕШ.

1.2° $a=1, b\neq 1$

$0 = b-1 \neq 0$ ↯

НЕМА РЕШ.

2° $a \neq 1$

$$\begin{aligned} \boxed{x} + y + z &= 1 \\ (a-1)\boxed{y} &= b-1 \quad /\cdot(-a-1) \\ (a^2-1)y &= b-1 \end{aligned}$$

$$\begin{aligned} \boxed{x} + y + z &= 1 \\ (a-1)y &= b-1 \\ 0 &= -(b-1)a \end{aligned}$$

2.1° $a=0$

$$\begin{aligned} \boxed{x} + y + z &= 1 \\ -\boxed{y} &= b-1 \\ 0 &= 0 \end{aligned}$$

$z=\beta, \beta\in\mathbb{R}$
$y = 1-b$
$x = 1-(1-b)-\beta = b-\beta$

∞ РЕШ.

2.2° $a\neq 1, a\neq 0, b=1$

$$\begin{aligned} x + y + z &= 1 \\ (a-1)y &= 0 \\ 0 &= 0 \end{aligned}$$

$z = \beta, \beta\in\mathbb{R}$
$y = 0$
$x = 1-\beta$

∞ РЕШ

2.3° $a\neq 1, a\neq 0, b\neq 1$

$0 = (b-1)a \neq 0$ ↯

НЕМА РЕШ.

$$(x,y,z) \in \begin{cases} (1-\alpha-\beta, \alpha, \beta), \alpha,\beta\in\mathbb{R} & \text{ако } a=1 \wedge b=1 \\ \emptyset & \text{ако } a=1 \wedge b\neq 1 \\ (b-\beta, 1-b, \beta), \beta\in\mathbb{R} & \text{ако } a=0 \\ (1-\beta, 0, \beta), \beta\in\mathbb{R} & \text{ако } (a\neq 0 \wedge a\neq 1)\wedge b=1 \\ \emptyset & \text{ако } (a\neq 0\wedge a\neq 1)\wedge b\neq 1 \end{cases}$$

(5) 1)

$$\det A = \begin{vmatrix} a & b & b & \cdots & b \\ b & a & b & \cdots & b \\ b & b & a & \cdots & b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \cdots & a \end{vmatrix}_{n\times n} = \begin{vmatrix} a & b & b & \cdots & b \\ b-a & a-b & 0 & \cdots & 0 \\ b-a & 0 & a-b & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b-a & 0 & 0 & \cdots & a-b \end{vmatrix} = \begin{vmatrix} a+(n-1)b & b & b & \cdots & b \\ 0 & a-b & 0 & \cdots & 0 \\ 0 & 0 & a-b & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a-b \end{vmatrix}$$

горње ▽

$$= \boxed{(a+(n-1)b)(a-b)^{n-1}}$$

2) $a=1, b=0 \rightarrow \det A \neq 0 \Rightarrow$ врсте лин. нез. $\Rightarrow \rho(A) = n$

(6) $\boxed{\alpha_1(u+v) + \alpha_2(u-v) + \alpha_3(u-2v+w) = \vec{0}} \Rightarrow (\alpha_1+\alpha_2-\alpha_3)u + (\alpha_1-\alpha_2-2\alpha_3)v + \alpha_3 w = \vec{0}$

u, v, w лин. нез.

$$\overset{\text{def}}{\Rightarrow} \begin{aligned} \boxed{\alpha_1} + \alpha_2 - \alpha_3 &= 0 \quad /(-1) \\ \alpha_1 - \alpha_2 - 2\alpha_3 &= 0 \\ \alpha_3 &= 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} \boxed{\alpha_1} + \alpha_2 - \alpha_3 &= 0 \\ -2\boxed{\alpha_2} - 3\alpha_3 &= 0 \\ \boxed{\alpha_3} &= 0 \end{aligned} \Rightarrow \boxed{\alpha_1 = \alpha_2 = \alpha_3 = 0}$$

Дакле,
$u+v$, $u-v$
$u-2v+w$
су лин. нез.