1.

$$x+y+2z-t-w=1 \quad /\cdot(-1) \quad /\cdot(-2) \quad /\cdot(-3)$$
$$x+2z+t=7$$
$$2x+y+4z-t+w=4$$
$$3x+3y+6z-2t-4w=6$$

$$-y+2t+w=6 \quad /\cdot(-1)$$
$$-y+t+3w=2$$

$$t-w=3$$

$$\left.\begin{aligned}-t+2w&=-4\\ t-w&=3\end{aligned}\right\}+$$

$$\boxed{w=-1}$$
$$\boxed{t=2}$$

$$x+y+2z-2+1=1$$
$$x+2z+2=7$$
$$2x+y+4z-2-1=4$$
$$3x+3y+6z-4+4=6$$

$$x+y+2z=2$$
$$x+2z=5$$
$$2x+y+4z=7$$
$$\cancel{3x+3y+6z=6} \quad /:3$$

$$x=5-2z \quad , z\in\mathbb{R}$$

$$5-\cancel{2z}+y+\cancel{2z}=2$$
$$2(5-2z)+y+4z=7$$

$$\boxed{y=-3}$$
$$10-4z+y+4z=7$$
$$y=-3$$

$$x=5-2z$$
$$x=-1-2z$$

$$5-2z=-1-2z$$

$(x,y,z,t,w)=$

$$\{\underbrace{(5-2z,-3,z,2,-1)}\mid z\in\mathbb{R}\}=$$

$$\{z(-2,0,1,0,0)+(5,-3,0,2,-1)\mid z\in\mathbb{R}\}$$

2.

$u_1 = (1, -1, 1, 2)$
$u_2 = (1, 0, 2, 6)$
$u_3 = (2, -2, -1, 19)$
$u_4 = (4, -3, 2, 27)$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 1 & 0 & 2 & 6 \\ 2 & -2 & -1 & 19 \\ 4 & -3 & 2 & 27 \end{bmatrix} \begin{matrix} /\cdot(-1) \ /\cdot(-2) \ /\cdot(-4) \end{matrix} =$$

$$\begin{bmatrix} \boxed{1} & -1 & 1 & 2 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & -3 & 15 \\ 0 & 1 & -2 & 19 \end{bmatrix} /\cdot(-1) = \begin{bmatrix} \boxed{1} & -1 & 1 & 2 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & \boxed{-3} & 15 \\ 0 & 0 & -3 & 15 \end{bmatrix} /\cdot(-1) =$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & -3 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$U = \mathcal{L}(u_1, u_2, u_3, u_4)$
$= \mathcal{L}(u_1, u_2, u_3)$

u_4 – lin. zavisan $\quad \dim U = 3$

$\{u_1, u_2, u_3\}$ baza za U

$w_1 = (1, -1, 0, 4)$
$w_2 = (1, -1, 1, 2)$
$w_3 = (-3, 3, -5, 4)$
$w_4 = (-1, 1, -4, 10)$

$$\begin{bmatrix} 1 & -1 & 0 & 4 \\ 1 & -1 & 1 & 2 \\ -3 & 3 & -5 & 4 \\ -1 & 1 & -4 & 10 \end{bmatrix} \begin{matrix} /\cdot(-1) \ /\cdot 3 \ /+ \end{matrix} =$$

$$\begin{bmatrix} \boxed{1} & -1 & 0 & 4 \\ 0 & 0 & \boxed{1} & -2 \\ 0 & 0 & -5 & 16 \\ 0 & 0 & -4 & 14 \end{bmatrix} \begin{matrix} /\cdot 5 \ /\cdot 4 \end{matrix} = \begin{bmatrix} 1 & -1 & 0 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 6 \end{bmatrix} /\cdot(-1)$$

$$\begin{bmatrix} 1 & -1 & 0 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\dim W = 3$

$W = \mathcal{L}(w_1, w_2, w_3, w_4) = \mathcal{L}(w_1, w_2, w_3)$

$\{w_1, w_2, w_3\}$ baza za W

$$\begin{array}{c} u_1 \\ u_2 \\ u_3 \\ w_1 \\ w_2 \\ w_3 \end{array}\begin{bmatrix} 1 & -1 & 1 & 2 \\ 1 & 0 & 2 & 6 \\ 2 & -2 & -1 & 19 \\ 1 & -1 & 0 & 4 \\ 1 & -1 & 1 & 2 \\ -3 & 3 & -5 & 4 \end{bmatrix} \begin{array}{l} /\cdot(-1)\ /\cdot(-2)\ /\cdot 3 \\ \\ \\ \\ \\ \\ \end{array} =$$

$$\begin{array}{c} u_1 \\ u_2-u_1 \\ u_3-2u_1 \\ w_1-u_1 \\ w_2-u_1 \\ w_3+3u_1 \end{array}\begin{bmatrix} \boxed{1} & -1 & 1 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & -3 & 15 \\ 0 & 0 & \boxed{-1} & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 10 \end{bmatrix} /:3 = \begin{array}{c} u_1 \\ u_2-u_1 \\ \frac{u_3-2u_1}{3} \\ w_1-u_1 \\ w_2-u_1 \\ w_3+3u_1 \end{array}\begin{bmatrix} \boxed{1} & -1 & 1 & 2 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & \boxed{-1} & 5 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 10 \end{bmatrix} /\cdot(-1)\ /\cdot(-2)$$

$$\begin{array}{c} u_1 \\ u_2-u_1 \\ \frac{u_3-2u_1}{3} \\ w_1-u_1-\frac{u_3-2u_1}{3} \\ w_3+3u_1-2\cdot\frac{u_3-2u_1}{3} \end{array}\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\dim(U+W)=4$ ✓

$(U+W) = \mathcal{L}(u_1,u_2,u_3,w_1,w_2,w_3) = \mathcal{L}(u_1,u_2,u_3,w_1)$

$\{u_1,u_2,u_3,w_1\}$ baza za $(U+W)$

$\dim(U\cap W) = \dim U + \dim W - \dim(U+W) = 3+3-4 = 2$

$w_2 - u_1 = 0$

$w_2 = u_1$

$(1,-1,1,2) = \underbrace{(1,-1,1,2)}_{e_1}$ ✓

$w_3 + 3u_1 - 2\cdot\frac{u_3-2u_1}{3} = 0 \ /\cdot 3$

$3w_3 + 9u_1 - 2u_3 + 4u_1 = 0$

$3w_3 = 2u_3 - 13u_1$

$3(-3,3,-5,4) = 2(2,-2,-1,19) - 13(1,-1,1,2)$

$(-9,9,-15,12) = (4,-4,-2,38) + (-13,13,-13,-26)$

$(-9,9,-15,12) = \underbrace{(-9,9,-15,12)}_{e_2}$

$\{e_1,e_2\}$ baza za $U\cap W$ ✓

9) 3. $L: \mathbb{R}^3 \to \mathbb{R}^3$

$$L(x,y,z) = (x-2z, -y, x-y-z)$$

1) aditivnost $(\forall u_1, u_2 \in \mathbb{R}^3)$

$$L(u_1+u_2) \stackrel{?}{=} L(u_1) + L(u_2)$$

L: $u_1 = (x_1, y_1, z_1)$ $\quad x_1, y_1, z_1 \in \mathbb{R}$

$u_2 = (x_2, y_2, z_2)$ $\quad x_2, y_2, z_2 \in \mathbb{R}$

$$L(u_1+u_2) = L(x_1+x_2, y_1+y_2, z_1+z_2)$$

$$\Rightarrow (x_1+x_2 - 2(z_1+z_2), -(y_1+y_2), x_1+x_2-(y_1+y_2)-(z_1+z_2))$$

$$= (x_1+x_2-2z_1-2z_2, -y_1-y_2, x_1+x_2-y_1-y_2-z_1-z_2)$$

D: $L(u_1)+L(u_2) \Rightarrow (x_1-2z_1, -y_1, x_1-y_1-z_1) +$

$$(x_2-2z_2, -y_2, x_2-y_2-z_2)$$

$$= (x_1-2z_1+x_2-2z_2, -y_1-y_2, x_1-y_1-z_1+x_2-y_2-z_2)$$

$$L = D \quad \checkmark$$

2) homogenost $(\forall \alpha \in \mathbb{R})(\forall u \in \mathbb{R})$

$$L(\alpha u) \stackrel{?}{=} \alpha \cdot L(u)$$

L: $u = (x_1, y_1, z_1)$ $\quad x_1, y_1, z_1 \in \mathbb{R}$

$$L(\alpha(x_1, y_1, z_1)) = L(\alpha x_1, \alpha y_1, \alpha z_1)$$

$$\Rightarrow (\alpha x_1 - 2\alpha z_1, -\alpha y_1, \alpha x_1 - \alpha y_1 - \alpha z_1)$$

D: $\alpha \cdot L(u) = \alpha \cdot L(x_1, y_1, z_1) = \alpha \cdot (x_1 - 2z_1, -y_1, x_1 - y_1 - z_1)$

$$= (\alpha x_1 - 2\alpha z_1, -\alpha y_1, \alpha x_1 - \alpha y_1 - \alpha z_1)$$

$$L = D \quad \checkmark$$

b) $\ker L = \{\vec{0}\}$

$$\ker L = \{ u \in \mathbb{R}^3 \mid L(u) = \{\vec{0}\} \}$$
$$= \left\{ (a,b,c) \in \mathbb{R}^3 \;\middle|\; \begin{matrix} a - 2c = 0 \\ -b = 0 \\ a - b - c = 0 \end{matrix} \right\} = \{0,0,0\}$$

$$b = 0$$
$$\begin{aligned} a - 2c &= 0 \quad / \cdot (-1) \\ a - c &= 0 \end{aligned}$$
$$c = 0, \quad a = 0$$

$\Rightarrow$ L ima inverz

$$e = \{ e_1 = (1,0,0),\ e_2 = (0,1,0),\ e_3 = (0,0,1) \}$$
$$L(e_1) = (1,0,1) = 1e_1 + 0e_2 + 1e_3$$
$$L(e_2) = (0,-1,-1) = 0e_1 - 1e_2 - 1e_3$$
$$L(e_3) = (-2,0,-1) = -2e_1 + 0e_2 - 1e_3$$

$$[L]_e = \begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} \boxed{1} & 0 & -2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 & 0 & 1 \end{array}\right] / \cdot(-1) \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & \boxed{-1} & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right] / \cdot(-1) =$$

$[L]_e \qquad E$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & \boxed{1} & -1 & -1 & 1 \end{array}\right] / \cdot 2 = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -2 & +2 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right]$$

$E \qquad [L^{-1}]_e$

$$[L^{-1}]_e = \begin{bmatrix} -1 & -2 & +2 \\ 0 & -1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & +2 \\ 0 & -1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. $A = \begin{bmatrix} -5 & 0 & 6 \\ 0 & -2 & 0 \\ -3 & 0 & 4 \end{bmatrix}$

$\varphi_A(t) = \det(A - tE) = \begin{vmatrix} -5-t & 0 & 6 \\ 0 & -2-t & 0 \\ -3 & 0 & 4-t \end{vmatrix} \begin{matrix} -5-t & 0 \\ 0 & -2-t \\ -3 & 0 \end{matrix} =$

$(-5-t)(-2-t)(4-t) - (-18(-2-t)) =$

$(5+t)(2+t)(4-t) - (18(2+t)) =$

$(2+t)((5+t)(4-t) - 18) =$

$(2+t)(-t^2 - t + 20 - 18) =$

$(2+t)(-t^2 - t + 2) =$

$-(2+t)(t^2 + t - 2) =$

$-(2+t)(2+t)(t-1) =$

$-(2+t)^2(t-1)$

$\lambda_1 = -2 \quad \lambda_2 = 1 \implies$ sopstvene vrednosti

$\lambda_1 = -2$

$v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$Av = v\lambda$

$\begin{bmatrix} -5 & 0 & 6 \\ 0 & -2 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot (-2)$

$-5x + 6z = -2x$

$-2y = -2y$

$-3x + 4z = -2z$

$-3x + 6z = 0$

$-3x + 6z = 0$

$z \in \mathbb{R} \setminus \{0\}$

$6z = 3x \; / :3$

$x = 2z$

$y \in \mathbb{R} \setminus \{0\}$

$v = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2z \\ y \\ z \end{bmatrix} = z \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$y, z \in \mathbb{R} \setminus \{0\}$

A'

$\lambda_2 = 1$

$$v_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$Av = v \cdot \lambda$$

$$\begin{bmatrix} -5 & 0 & 6 \\ 0 & -2 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot 1$$

$$-5x + 6z = x$$
$$-2y = y$$
$$-3x + 4z = z$$

$$-6x + 6z = 0$$
$$3y = 0 \quad , \quad y = 0$$
$$-3x + 3z = 0$$

$$6x = 6z$$
$$x = z \quad , \quad z \in \mathbb{R} \setminus \{0\}$$

$$v_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z \\ 0 \\ z \end{bmatrix} = z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad , \quad z \in \mathbb{R} \setminus \{0\}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right] \qquad P^{-1} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \sim$$

$$A = PDP^{-1} \quad /^{n}$$

$$A^n = (PDP^{-1})(PDP^{-1}) \cdots (PDP^{-1})$$

$$A^n = PD^nP^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1^n & 0 & 0 \\ 0 & -2^n & 0 \\ 0 & 0 & -2^n \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1^n & 0 & -2^n \cdot 2 \\ 0 & -2^n & 0 \\ 1^n & 0 & -2^n \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1^n - 2^n \cdot 2 & 0 & 2 \cdot 1^n + 2^n 2 \\ 0 & -2^n & 0 \\ -1^n - 2^n & 0 & 2 + 2^n \end{bmatrix} = \begin{bmatrix} -1 - 2^{n+1} & 0 & 2 + 2^n \cdot 2 \\ 0 & -2^n & 0 \\ -1 - 2^n & 0 & 2 + 2^n \end{bmatrix}$$

9) 5. $p: \frac{2x}{1} = \frac{3y}{1} = \frac{z+7}{2} = t$

$q: \frac{x-5}{2} = \frac{y-3}{1} = \frac{z-10}{5} = s$

$x = 2s+5$
$y = s+3$
$z = 5s+10$

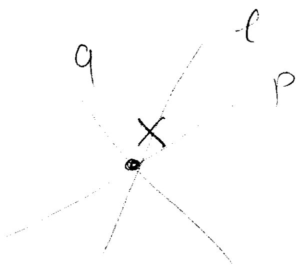
$2x = t$
$3y = t$
$z = 2t-7$

$4s+10 = t \quad / \cdot (-1)$
$3s+9 = t$
$2t-7 = 5s+10$

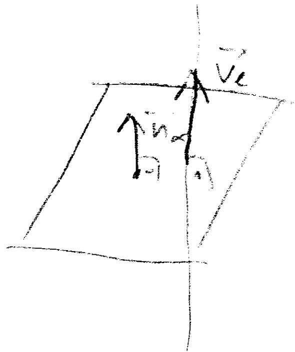
$-s-1=0$
$2t-5s=17$

$s=-1$
$2t+5=17$
$2t=12$
$t=6$

$X(3,2,5)$



$\ell \perp \alpha$
$\Rightarrow \vec{v_\ell} = \vec{n_\alpha}$



$\alpha: 2x-18y+z+46=0$

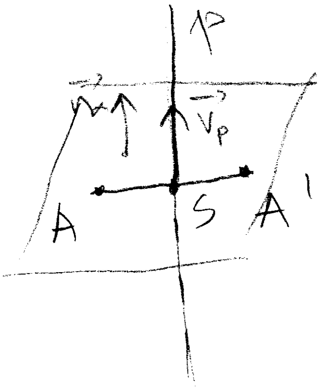
$\vec{n_\alpha} = (2,-18,1)$

$\vec{v_\ell} = (2,-18,1)$

$\ell: \frac{x-3}{2} = \frac{y-2}{-18} = \frac{z-5}{1}$

c) $A(3,-1,0)$

$p: \frac{x-1}{2} = \frac{y-3}{1} = \frac{z}{5}$ $\qquad \vec{v}_p = (2,1,5)$



$\vec{n}_\alpha = (2,1,5)$

$\alpha: 2x + y + 5z + d = 0$

$6 - 1 + 0 + d = 0$

$d = -5$

$\alpha: 2x + y + 5z - 5 = 0$

$p: x = 2t + 1$

$y = t + 3$

$z = 5t$

$p \cap \alpha \Rightarrow S$

$2(2t+1) + t + 3 + 5(5t) - 5 = 0$

$4t + 2 + t + 3 + 25t - \cancel{5} = 0$

$3t = 0 \qquad t = 0$

$S(1,3,0)$

$S - A = A' - S$

$2S = A + A'$

$A' = 2S - A = 2(1,3,0) - (3,-1,0) = (2,6,0) - (3,-1,0)$

$A' = (-1,7,0)$