I ГРУПА

1.

$$\begin{array}{l}
\boxed{x} \quad + z - t - 2w = 0 \quad /\cdot(-1) \; /\cdot(-2) \\
x + y - z + t - w = 2 \quad + \\
2x + y \quad + 2t + 3w = 4 \quad + \\
x - 2y + 5z - t + 9w = 1 \quad +
\end{array}$$

$$\begin{array}{l}
\boxed{x} \quad + z - t - 2w = 0 \\
\boxed{y} - 2z + 2t + w = 2 \quad /\cdot(-1) \; /\cdot 2 \\
y - 2z + 4t + 7w = 4 \quad + \\
-2y + 4z \quad + 11w = 1 \quad +
\end{array}$$

$$\begin{array}{l}
\boxed{x} \quad + z - t - 2w = 0 \\
\boxed{y} - 2z + 2t + w = 2 \\
2\boxed{t} + 6w = 2 \quad /\cdot(-2) \\
4t + 13w = 5 \quad +
\end{array}$$

$$\begin{array}{l}
\boxed{x} \quad + z - t - 2w = 0 \\
\boxed{y} - 2z + 2t + w = 2 \\
2\boxed{t} + 6w = 2 \\
\boxed{w} = 1
\end{array}$$

$z = a, \ a \in \mathbb{R}$

$w = 1$

$t = \frac{1}{2}(2 - 6w) = \frac{1}{2}(2-6) = -\frac{4}{2} = -2$

$y = 2 + 2z - 2t - w = 2 + 2a - 2\cdot(-2) - 1 = 2a + 5$

$x = 2w + t - z = 2\cdot 1 - 2 - a = 2 - 2 - a = -a$

$(x, y, z, t, w) = (-a, 2a+5, a, -2, 1), \ a \in \mathbb{R} =$

$= \{(0, 5, 0, -2, 1) + a(-1, 2, 1, 0, 0)\}, \ a \in \mathbb{R}$

2.

$$\left[\begin{array}{cccc|cccc} \boxed{1} & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ -1 & -1 & -5 & -7 & 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & -1 & 0 & 0 & 1 & 0 \\ 2 & 5 & 6 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} /\cdot 1 \; /\cdot(-1) \; /\cdot(-2) \\ + \\ + \\ + \end{array}$$

$$\left[\begin{array}{cccc|cccc} \boxed{1} & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & \boxed{1} & -2 & -3 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -5 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -6 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ /\cdot(-1) \\ + \\ + \end{array}$$

$$\left[\begin{array}{cccc|cccc} \boxed{1} & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & \boxed{1} & -2 & -3 & 1 & 1 & 0 & 0 \\ 0 & 0 & \boxed{1} & -2 & -2 & -1 & 1 & 0 \\ 0 & 0 & 2 & -3 & -3 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ /\cdot(-2) \\ + \end{array}$$

$$\left[\begin{array}{cccc|cccc} \boxed{1} & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & \boxed{1} & -2 & -3 & 1 & 1 & 0 & 0 \\ 0 & 0 & \boxed{1} & -2 & -2 & -1 & 1 & 0 \\ 0 & 0 & 0 & \boxed{1} & 1 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} + \\ /\cdot(-2) \\ + \\ /\cdot 2 \end{array}$$

$$\left[\begin{array}{cccc|cccc} \boxed{1} & 0 & 7 & 10 & -1 & -2 & 0 & 0 \\ 0 & \boxed{1} & -2 & -3 & 1 & 1 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & \boxed{1} & 1 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} + \\ + \\ \\ /\cdot 3 \; /\cdot(-10) \end{array}$$

$$\left[\begin{array}{cccc|cccc} \boxed{1} & 0 & 7 & 0 & -11 & -12 & 20 & -10 \\ 0 & \boxed{1} & -2 & 0 & 4 & 4 & -6 & 3 \\ 0 & 0 & \boxed{1} & 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & \boxed{1} & 1 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} + \\ + \\ /\cdot 2 \; /\cdot(-7) \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -11 & -19 & 41 & -24 \\ 0 & 1 & 0 & 0 & 4 & 6 & -12 & 7 \\ 0 & 0 & 1 & 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -11 & -19 & 41 & -24 \\ 4 & 6 & -12 & 7 \\ 0 & 1 & -3 & 2 \\ 1 & 1 & -2 & 1 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} \boxed{1} & -1 & 1 & 2 \\ 2 & -1 & 3 & 5 \\ 3 & -5 & 2 & 6 \\ 4 & -7 & -1 & 2 \end{bmatrix} \quad /\cdot(-2) \; /\cdot(-3) \; /\cdot(-4)$$

$$\begin{bmatrix} \boxed{1} & -1 & 1 & 2 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & -2 & -1 & 0 \\ 0 & -3 & -5 & -6 \end{bmatrix} \quad /\cdot(2) \; /\cdot 3$$

$$\begin{bmatrix} \boxed{1} & -1 & 1 & 2 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & 0 & \boxed{1} & 2 \\ 0 & 0 & -2 & -3 \end{bmatrix} \quad /\cdot 2$$

$$\begin{bmatrix} \boxed{1} & -1 & 1 & 2 \\ 0 & \boxed{1} & 1 & 9 \\ 0 & 0 & \boxed{1} & 2 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix} \Rightarrow \text{rang } A^{T} = 4$$

(3.) $U = \{(a,b,c) \in \mathbb{R}^3 \mid 2a - b + 3c = 0\}$

a) $U \overset{?}{\leq} \mathbb{R}^3$

(1) $(0,0,0) \overset{?}{\in} U$

$a=0,\ b=0,\ c=0$

$\Rightarrow 2a - b + 3c = 2\cdot 0 - 0 + 3\cdot 0 = 0$ ✓

$\Rightarrow (0,0,0) \in U$

(2) $u_1, u_2 \in U \overset{?}{\Rightarrow} u_1 + u_2 \in U$

$u_1 = (a_1, b_1, c_1) \in U,\ 2a_1 - b_1 + 3c_1 = 0$

$u_2 = (a_2, b_2, c_2) \in U,\ 2a_2 - b_2 + 3c_2 = 0$

$u_1 + u_2 = (a_1, b_1, c_1) + (a_2, b_2, c_2) =$

$= (a_1 + a_2, b_1 + b_2, c_1 + c_2) \in U$

ако $2(a_1+a_2) - (b_1+b_2) + 3(c_1+c_2) = 0$

$2(a_1+a_2) - (b_1+b_2) + 3(c_1+c_2) =$

$= 2a_1 + 2a_2 - b_1 - b_2 + 3c_1 + 3c_2 =$

$= \underbrace{2a_1 - b_1 + 3c_1}_{=0} + \underbrace{2a_2 - b_2 + 3c_2}_{=0} = 0$ ✓

$\Rightarrow u_1 + u_2 \in U$

(3). $\alpha \in \mathbb{R},\ u \in U \overset{?}{\Rightarrow} \alpha u \in U$

$u = (a, b, c) \in U, \quad 2a - b + 3c = 0$

$\alpha u = \alpha(a, b, c) = (\alpha a, \alpha b, \alpha c) \in U$

ако $2\alpha a - \alpha b + 3\alpha c = 0$

$2\alpha a - \alpha b + 3\alpha c = \alpha\underbrace{(2a - b + 3c)}_{=0} = \alpha \cdot 0 = 0$

$\Rightarrow \alpha u \in U$

$\overset{(1),(2),(3)}{\Rightarrow} U \leq \mathbb{R}^3$

$U = \{(a, b, c) \in \mathbb{R}^3 \mid 2a - b + 3c = 0\}$

$U = \{(a, b, c) \in \mathbb{R}^3 \mid 2a + 3c = b\}$

$U = \{(a, 2a + 3c, c) \in \mathbb{R}^3 \mid a, c \in \mathbb{R}\}$

$U = \{a\underbrace{(1, 2, 0)}_{u_1} + c\underbrace{(0, 3, 1)}_{u_2} \in \mathbb{R}^3 \mid a, c \in \mathbb{R}\}$

$u_1 = (1, 2, 0)$

$u_2 = (0, 3, 1)$

1° $U = \mathcal{L}(u_1, u_2)$ - генератриса за U

2° $\begin{array}{c} u_1 \\ u_2 \end{array}\begin{bmatrix} \boxed{1} & 2 & 0 \\ 0 & \boxed{3} & 1 \end{bmatrix} \Rightarrow$ u_1 и u_2 су линеарно независни вектори

$\overset{1^\circ, 2^\circ}{\Rightarrow}$ $\{u_1, u_2\}$ је база за векторски потпростор U

$\dim U = 2$

б) $W = \{(0, 3c, c) \mid c \in \mathbb{R}\}$

$\mathbb{R}^3 = U \oplus W$?

1° $U + W \subseteq \mathbb{R}^3$ ✓ јер збир 2 вектора из $\mathbb{R}^3$ је вектор који се налази у $\mathbb{R}^3$

2° $\mathbb{R}^3 \subseteq U + W$

Тражимо $v \in \mathbb{R}^3$ тако да је $v = u + w$, $u \in U$, $w \in W$.

$$(\forall v \in \mathbb{R}^3)(\exists u \in U)(\exists w \in W)\ v = u + w$$

$v = (a, b, c) \in \mathbb{R}^3$

$u = (u_1, u_2, u_3) \in U$, $2u_1 - u_2 + 3u_3 = 0$

$w = (w_1, w_2, w_3) \in W$, $w_1 = 0$, $w_2 = 3w_3 \Leftrightarrow w_2 - 3w_3 = 0$

$$(a, b, c) = (u_1, u_2, u_3) + (0, 3w_3, w_3)$$

$$(a, b, c) = (u_1, u_2 + 3w_3, u_3 + w_3)$$

$$\begin{array}{l}
u_1 + w_1 = a \\
u_2 + w_2 = b \\
u_3 + w_3 = c \\
2u_1 - u_2 + 3u_3 = 0 \\
w_1 = 0 \\
w_2 - 3w_3 = 0
\end{array}$$

$w_1 = 0$

$u_1 = a - w_1 = a - 0 = a$

$u_2 - 3u_3 = 2a$

$$
\begin{aligned}
\boxed{u_1} & & &= a \quad / \cdot (-2)\\
u_2 & & w_2 &= b\\
u_3 & & +w_3 &= c\\
2u_1 - u_2 + 3u_3 & & &= 0\\
& & w_2 - 3w_3 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\boxed{u_1} & & &= a\\
\boxed{u_2} & +w_2 & &= b \quad /\\
u_3 & & +w_3 &= c\\
-u_2 + 3u_3 & & &= -2a\\
& & w_2 - 3w_3 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\boxed{u_1} & & &= a\\
\boxed{u_2} & +w_2 & &= b\\
\boxed{u_3} & & +w_3 &= c \quad / \cdot (-3)\\
3u_3 & +w_2 & &= -2a+b\\
& & w_2 - 3w_3 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\boxed{u_1} & & &= a\\
\boxed{u_2} & +w_2 & &= b\\
\boxed{u_3} & & +w_3 &= c\\
& \boxed{w_2} - 3w_3 & &= -2a+b-3c \quad / \cdot (-1)\\
& w_2 - 3w_3 & &= 0
\end{aligned}
$$

$$
S: \left\{
\begin{aligned}
\boxed{u_1} & & &= a\\
\boxed{u_2} & +w_2 & &= b\\
\boxed{u_3} & & +w_3 &= c\\
& \boxed{w_2} - 3w_3 & &= -2a+b-3c\\
& & 0 &= 2a-b+3c
\end{aligned}
\right.
$$

Систем S није увек задовољен. Пример:

$v = (a,b,c) = (1,1,1) \Rightarrow 2a - b + 3c = 2 - 1 + 3 = 4 \neq 0$

$\Rightarrow \mathbb{R}^3 \neq U + W \Rightarrow \mathbb{R}^3 \neq U \oplus W$

4. $u_1 = (1, 2, 3, 1)$ $\quad w_1 = (1, 1, 1, -2)$

$u_2 = (3, 5, 7, 0)$ $\quad w_2 = (1, 2, 2, -1)$

$u_3 = (1, 1, 1, -2)$ $\quad w_3 = (2, 5, 5, -1)$

$u_4 = (1, 0, -2, -8)$

$U = \mathcal{L}(u_1, u_2, u_3, u_4$

$$\begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{bmatrix} \boxed{1} & 2 & 3 & 1 \\ 3 & 5 & 7 & 0 \\ 1 & 1 & 1 & -2 \\ 1 & 0 & -2 & -8 \end{bmatrix} \begin{array}{l} /\cdot(-3) \; /\cdot(-1) \\ + \\ + \\ + \end{array} \sim \begin{bmatrix} \boxed{1} & 2 & 3 & 1 \\ 0 & \boxed{-1} & -2 & -3 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & -5 & -9 \end{bmatrix} \begin{array}{l} \\ /\cdot(-1) \; /\cdot(-2) \\ + \\ + \end{array}$$

$$\sim \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{bmatrix} \boxed{1} & 2 & 3 & 1 \\ 0 & \boxed{-1} & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{-1} & -3 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 2 & 3 & 1 \\ 0 & \boxed{-1} & -2 & -3 \\ 0 & 0 & \boxed{-1} & -3 \end{bmatrix}$$

$$U = \mathcal{L}(u_1, u_2, u_3, u_4) = \mathcal{L}(u_1, u_2, u_4)$$

генератриса за U, $\{u_1, u_2, u_4\}$ су међусобно линеарно независни па је $\{u_1, u_2, u_4\}$ база за U

$\Rightarrow \dim U = 3$

$W = \mathcal{L}(w_1, w_2, w_3)$

$$\begin{array}{c} w_1 \\ w_2 \\ w_3 \end{array} \begin{bmatrix} \boxed{1} & 1 & 1 & -2 \\ 1 & 2 & 2 & -1 \\ 2 & 5 & 5 & -1 \end{bmatrix} \begin{array}{l} /\cdot(-1) \; /\cdot(-2) \\ + \\ + \end{array} \sim \begin{bmatrix} \boxed{1} & 1 & 1 & -2 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & 3 & 3 & 3 \end{bmatrix} \begin{array}{l} \\ /\cdot(-3) \\ \end{array} \sim$$

$$\sim \begin{bmatrix} \boxed{1} & 1 & 1 & -2 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 1 & 1 & -2 \\ 0 & \boxed{1} & 1 & 1 \end{bmatrix} \begin{array}{c} w_1 \\ w_2 \end{array}$$

$W = \mathcal{L}(w_1, w_2) \to$ генератриса за W,

$\{w_1, w_2\}$ су линеарно независни вектори

$\Rightarrow$ па је тај скуп $\{w_1, w_2\}$ база за W

$\dim W = 2$

$$U + W = \mathcal{L}(u_1, u_2, u_4, w_1, w_2)$$

$$\begin{array}{l} \begin{bmatrix} 1 & 2 & 3 & 1 \\ 3 & 5 & 7 & 0 \\ 1 & 0 & -2 & -8 \\ 1 & 1 & 1 & -2 \\ 1 & 2 & 2 & -1 \end{bmatrix} \end{array} \begin{array}{l} u_1 \ /\cdot(-3) \ /\cdot(-1) \\ u_2 \\ u_4 \\ w_1 \\ w_2 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & -5 & -9 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & -1 & -2 \end{bmatrix} \begin{array}{l} u_1 \\ u_2 - 3u_1 \ /\cdot(-2) \ /\cdot(-1) \\ u_4 - u_1 \\ w_1 - u_1 \\ w_2 - u_1 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 \end{bmatrix} \begin{array}{l} u_1 \\ u_2 - 3u_1 \\ u_4 - u_1 - 2(u_2 - 3u_1) \ /\cdot(-1) \\ w_1 - u_1 - u_2 + 3u_1 = \vec{0} \\ w_2 - u_1 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} u_1 \\ u_2 - 3u_1 \\ u_4 - u_1 - 2u_2 + 6u_1 \\ w_2 - u_1 - u_4 - 5u_1 + 2u_2 \end{array}$$

$\omega_1 = u_1 + u_2 - 3u_1 = u_2 - 2u_1$

$U + W = \mathcal{L}(u_1, u_2, u_4, w_2) \to$ генератриса за $U+W$, $\{u_1, u_2, u_4, w_2\}$ су линеарно независни $\Rightarrow \{u_1, u_2, u_4, w_2\}$ једна база за $U+W$

$\dim U+W = 4$

$\dim U \cap W = \dim U + \dim W - \dim U+W =$
$= 3 + 2 - 4 = 1$

$w_1 - u_2 + 2u_1 = \vec{0}$

$(*)\ \underbrace{w_1}_{\in W} = \underbrace{u_2 - 2u_1}_{\in U}$

$v \in U \cap W \Leftrightarrow v \in U \wedge v \in W$

из $(*)$ $v = \underbrace{w_1}_{\in W} = \underbrace{u_2 - 2u_1}_{\in U}$

$v = w_1 \wedge v = u_2 - 2u_1$

$\dim U \cap W = \underline{1} \Rightarrow U \cap W = \mathcal{L}(v) = \mathcal{L}(w_1)$

$\Downarrow$

$\{w_1\}$ je база за $U \cap W \Leftarrow$ генератриса за $U \cap W$

$\{(1, 1, 1, -2)\}$

5. $f_1 = (1, 1, 2, 4)$

$f_2 = (1, -1, 7, 13)$

$f_3 = (9, 7, 1, 1)$

$V = \mathcal{L}(f_1, f_2, f_3)$

$$\begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & -1 & 7 & 13 \\ 9 & 7 & 1 & 1 \end{bmatrix} \begin{matrix} \cdot(-1) \quad \cdot(-9) \\ + \\ + \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & \boxed{-2} & 5 & 9 \\ 0 & -2 & -17 & -35 \end{bmatrix} \begin{matrix} \\ \cdot(-1) \\ + \end{matrix}$$

$$\begin{bmatrix} \boxed{1} & 1 & 2 & 4 \\ 0 & \boxed{-2} & 5 & 9 \\ 0 & 0 & \boxed{-22} & -44 \end{bmatrix} \Rightarrow$$ $\{f_1, f_2, f_3\}$ су линеарно незави- сни, dim $V = 3 \Rightarrow \{f_1, f_2, f_3\}$ je база за V

$\hat{e}_1 = f_1 = (1, 1, 2, 4)$

$$\hat{e}_2 = f_2 - \frac{\langle f_2, \hat{e}_1 \rangle}{\langle \hat{e}_1, \hat{e}_1 \rangle} \hat{e}_1 =$$

$$= (1, -1, 7, 13) - \frac{(1, -1, 7, 13) \circ (1, 1, 2, 4)}{(1, 1, 2, 4) \circ (1, 1, 2, 4)} (1, 1, 2, 4) =$$

$$= (1, -1, 7, 13) - \frac{\cancel{1} - \cancel{1} + 14 + 52}{1^2 + 1^2 + 2^2 + 4^2} (1, 1, 2, 4)$$

$$= (1, -1, 7, 13) - \frac{66}{22} (1, 1, 2, 4)$$

$$= (1, -1, 7, 13) - 3(1, 1, 2, 4)$$

$$= (1, -1, 7, 13) + (-3, -3, -6, -12) = (1-3, -1-3, 7-6, 13-12) =$$

$$= (-2, -4, 1, 1)$$

$$\hat{e}_3 = f_3 - \frac{\langle f_3, \hat{e}_1 \rangle}{\langle \hat{e}_1, \hat{e}_1 \rangle}\hat{e}_1 - \frac{\langle f_3, \hat{e}_2 \rangle}{\langle \hat{e}_2, \hat{e}_2 \rangle}\hat{e}_2 =$$

$$= (9,7,1,1) - \frac{(9,7,1,1)\circ(1,1,2,4)}{(1,1,2,4)\circ(1,1,2,4)}(1,1,2,4) -$$

$$- \frac{(9,7,1,1)\circ(-2,-4,1,1)}{(-2,-4,1,1)\circ(-2,-4,1,1)}(-2,-4,1,1) =$$

$$= (9,7,1,1) - \frac{9+7+2+4}{1+1+4+16}(1,1,2,4) - \frac{-18-28+1+1}{4+16+1+1}(-2,-4,1,1) =$$

$$= (9,7,1,1) - \overset{=1}{\frac{22}{22}}(1,1,2,4) - \frac{-44}{22}(-2,-4,1,1) =$$

$$= (9,7,1,1) + (-1,-1,-2,-4) + 2(-2,-4,1,1) =$$

$$= (8,6,-1,-3) + (-4,-8,2,2) = (4,-2,1,-1)$$

$S = \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ - ортогоналан скуп, $\dim S = 3$, $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ - линеарно независни вектори $\Rightarrow$ S је ортогонална база за V

$$e_1 = \frac{1}{\|\hat{e}_1\|}\hat{e}_1 = \frac{1}{\sqrt{\langle \hat{e}_1, \hat{e}_1 \rangle}}\hat{e}_1 = \frac{1}{\sqrt{1^2+1^2+2^2+4^2}}(1,1,2,4) =$$

$$= \frac{1}{\sqrt{22}}(1,1,2,4) = \left(\frac{1}{\sqrt{22}}, \frac{1}{\sqrt{22}}, \frac{2}{\sqrt{22}}, \frac{4}{\sqrt{22}}\right)$$

$$e_2 = \frac{1}{\|\hat{e}_2\|}\hat{e}_2 = \frac{1}{\sqrt{\langle \hat{e}_2, \hat{e}_2 \rangle}}\hat{e}_2 = \frac{1}{\sqrt{(-2,-4,1,1)\circ(-2,-4,1,1)}}(-2,-4,1,1)$$

$$= \frac{1}{\sqrt{4+16+1+1}}(-2,-4,1,1) = \frac{1}{\sqrt{22}}(-2,-4,1,1) =$$

$$= \left(-\frac{2}{\sqrt{22}}, -\frac{4}{\sqrt{22}}, \frac{1}{\sqrt{22}}, \frac{1}{\sqrt{22}}\right)$$

$$e_3 = \frac{1}{\|\hat{e}_3\|}\hat{e}_3 = \frac{1}{\sqrt{\langle \hat{e}_3, \hat{e}_3 \rangle}}\hat{e}_3 = \frac{1}{\sqrt{4^2+(-2)^2+1^2+(-1)^2}}(4,-2,1,-1) =$$

$$= \frac{1}{\sqrt{22}}(4,-2,+1,-1) = \left(\frac{4}{\sqrt{22}}, -\frac{2}{\sqrt{22}}, \frac{1}{\sqrt{22}}, -\frac{1}{\sqrt{22}}\right)$$

$\{e_1, e_2, e_3\}$ – ортонормиран скуп вектора димензије 3, вектори су линеарно независни $\Rightarrow$

$$\left\{\left(\frac{1}{\sqrt{22}}, \frac{1}{\sqrt{22}}, \frac{2}{\sqrt{22}}, \frac{4}{\sqrt{22}}\right), \left(-\frac{2}{\sqrt{22}}, -\frac{4}{\sqrt{22}}, \frac{1}{\sqrt{22}}, \frac{1}{\sqrt{22}}\right),\right.$$

$$\left.\left(\frac{4}{\sqrt{22}}, -\frac{2}{\sqrt{22}}, \frac{1}{\sqrt{22}}, -\frac{1}{\sqrt{22}}\right)\right\}$$ је ортонормирана

база за V

II група

1.

$$\begin{array}{l} x + y + t + 4w = 1 \quad /\cdot 3 \\ -3x - 2y + z + t - 5w = -2 \\ x + 2y + z + 2t + 3w = 1 \\ -x + y + 2z + 3t - 2w = 1 \end{array}$$

$$\begin{array}{l} \boxed{x} + y + t + 4w = 1 \\ \boxed{y} + z + 4t + 7w = 1 \quad /\cdot(-1) \\ y + z + t - w = 0 \\ 2y + 2z + 4t + 2w = 2 \quad /:2 \end{array}$$

$$\begin{array}{l} \boxed{x} + y + t + 4w = 1 \\ \boxed{y} + z + 4t + 7w = 1 \\ -3t - 8w = -1 \\ -2t - 6w = 0 \quad /:(-2) \end{array}$$

$$\begin{array}{l} \boxed{x} + y + t + 4w = 1 \\ \boxed{y} + z + 4t + 7w = 1 \\ \boxed{t} + 3w = 0 \quad /\cdot 3 \\ -3t - 8w = -1 \end{array}$$

$$\begin{array}{l} \boxed{x} + y + t + 4w = 1 \\ \boxed{y} + z + 4t + 7w = 1 \\ \boxed{t} + 3w = 0 \\ \boxed{w} = -1 \end{array}$$

$z = a, \quad a \in R$

$w = -1$

$t = -3w = -3(-1) = 3$

$y = 1 - 7w - 4t - z$

$= 1 + 7 - 12 - a$

$y = -4 - a$

$x = 1 - 4w - t - y$

$x = 1 + 4 - 3 + 4 + a$

$x = 6 + a$

систем има бесконачно решења

$(x, y, z, t, w) = \{(6+a, -4-a, a, 3, -1) \mid a \in R\}$

(2) $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 3 & 5 & 7 & 0 \\ 2 & 5 & 7 & 3 \\ 1 & 0 & -2 & -8 \end{bmatrix}$

инверз

$\longrightarrow$ (на крају)

$\rho(A^T) = ?$

$$A^T = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & 5 & 5 & 0 \\ 3 & 7 & 7 & -2 \\ 1 & 0 & 3 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & -1 & 1 & -2 \\ 0 & -2 & 1 & -5 \\ 0 & -3 & 1 & -9 \end{bmatrix} / \cdot(-2) \quad / \cdot(-3)$$

$$= \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -2 & -3 \end{bmatrix} / \cdot(-2) \,]+ \quad = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$\rho(A^T) = 4$ јер има 4 независне врсте!

(3) $U = \{(a,b,c) \in R^3 \mid 3a+2b-c=0\}$

a) $U \overset{?}{\leq} R^3$ $\qquad c = 3a+2b$

база, dim = ?

$U = \{(a, b, 3a+2b) \in R^3 \mid a, b \in R\}$

0) $U \subseteq R^3$ ✓ тривијално

1) $\vec{0} \overset{?}{\in} U$

$\vec{0} = (0,0,0)$ $\qquad 3a+2b-c=0$

$3 \cdot 0 + 2 \cdot 0 - 0 = 0$ ✓ $\qquad \vec{0} \in U$

$0 = 0$

2) $\vec{u} + \vec{v} \overset{?}{\in} U$

$\vec{u} \in U \quad \vec{u} = (a_1, b_1, c_1), \, 3a_1+2b_1-c_1=0 \qquad \vec{u} = (a_1, b_1, 3a_1+2b_1)$

$\vec{v} \in U \quad \vec{v} = (a_2, b_2, c_2), \, 3a_2+2b_2-c_2=0 \qquad \vec{v} = (a_2, b_2, 3a_2+2b_2)$

$\vec{u} + \vec{v} = (a_1, b_1, 3a_1+2b_1) + (a_2, b_2, 3a_2+2b_2)$

$\vec{u} + \vec{v} = (\underbrace{a_1+a_2}_{a}, \underbrace{b_1+b_2}_{b}, \underbrace{3(a_1+a_2)+2(b_1+b_2)}_{c})$

$\vec{u} + \vec{v} = (a, b, 3a+2b) \in U$ ✓ припада простору U

3) $\alpha \cdot \vec{u} \overset{?}{\in} U$

$\alpha, \vec{u} = (a_1, b_1, 3a_1+2b_1)$

$\alpha \in R$

$\alpha \cdot \vec{u} = \alpha (a_1, b_1, 3a_1+2b_1)$

$= (\alpha a_1, \underbrace{\alpha b_1}_{b}, 3\alpha a_1 + 2\underbrace{\alpha b_1}_{b})$

$= (a, b, 3a+2b) \in U$ ✓ припада простору!

$(0,1,2,3) \Rightarrow U \leq R^3$

$U = \{(a, b, 3a+2b) \mid a, b \in R\}$

$U = \{a(1,0,3) + b(0,1,2) \mid a, b \in R\}$

$U = [\{(1,0,3), (0,1,2)\}]$ → база за U

$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ - вектори су лин. независни

$\{(1,0,3), (0,1,2)\}$

$\dim U = 2$

б) $W = \{(0, b, 2b) \mid b \in R\} = \{(a,b,c) \mid a=0, c=2b\}$

$R^3 \overset{?}{=} U \oplus W$

$R^3 = U \oplus W \Leftrightarrow U+W = R^3 \wedge U \cap W = \{\vec{0}\}$

$U \cap W = \left\{(a,b,c) \,\middle|\, \begin{array}{l} 3a+2b-c=0 \\ a=0 \\ c=2b \end{array}\right\}$

$U \cap W = \left\{(a,b,c) \in R^3 \,\middle|\, \begin{array}{l} 3a+2b-c=0 \\ -2b+C=0 \\ a=0 \;/\cdot(-3) \end{array}\right\}$

$\begin{array}{l} \boxed{a} = 0 \\ -2b+c=0 \\ 2b-c=0 \end{array}$

$\begin{array}{l} a = 0 \\ 0=0 \end{array}$

$c = t,\ a = 0,\ b = t/2,\ t \in R$

систем има бесконачно решења.

сума није директна.

јер пресек векторских потпростора U и W није тривијалан

④ $U \leq \mathbb{R}^4$

$W \leq \mathbb{R}^4$

$u_1 = (1, 2, 3, 4)$ $\qquad w_1 = (1, 1, 5, 7)$

$u_2 = (-1, -1, -5, -7)$ $\qquad w_2 = (1, 2, 6, 4)$

$u_3 = (2, 5, 6, 2)$ $\qquad w_3 = (2, 3, 11, 11)$

$u_4 = (1, 4, -1, -2)$

база и dim за $U, W, U+W$ и $U \cap W$

$$U: \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & -1 & -5 & -7 \\ 2 & 5 & 6 & 2 \\ 1 & 4 & -1 & -2 \end{bmatrix} \; /\cdot(-2) \; /\cdot(-1)$$

$$\to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & -3 \\ 0 & 1 & 0 & -6 \\ 0 & 2 & -4 & -6 \end{bmatrix} \; /\cdot(-1) \; /\cdot(-2) \longrightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$U = [\{u_1, u_2, u_3, u_4\}] = [\{u_1, u_2, u_3\}]$$

$\{u_1, u_2, u_3\}$ - база за U $\rightarrow$ u_1, u_2, u_3 су лин. независни и генератори па чине базу потпростора U, $\dim U = 3$ (по теореми)

$$W: \begin{array}{c} w_1 \\ w_2 \\ w_3 \end{array} \begin{bmatrix} 1 & 1 & 5 & 7 \\ 1 & 2 & 6 & 4 \\ 2 & 3 & 11 & 11 \end{bmatrix} \; /\cdot(-1) \; /\cdot(-2)$$

$$\to \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 1 & 1 & -3 \\ 0 & 1 & 1 & -3 \end{bmatrix} \; /\cdot(-1) \longrightarrow \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W = [\{w_1, w_2, w_3\}] = [\{w_1, w_2\}]$$

$\{w_1, w_2\}$ - база за W $\rightarrow$ w_1 и w_2 су лин. нез. и чине базу за W, $\dim W = 2$

$U+W$: $\quad U + W = [\{u_1, u_2, u_3\}] + [\{w_1, w_2\}]$

$$= [\{u_1, u_2, u_3, w_1, w_2\}]$$

$$\begin{array}{c} u_1 \\ u_2 \\ u_3 \\ w_1 \\ w_2 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & -1 & -5 & -7 \\ 2 & 5 & 6 & 2 \\ 1 & 1 & 5 & 7 \\ 1 & 2 & 6 & 4 \end{bmatrix} \; /\cdot(-2) \; /\cdot(-1) \; /\cdot(-1) \longrightarrow$$

$$\begin{array}{r} u_1 \\ u_1+u_2 \\ [illegible]-u_1 \\ [illegible]-u_1 \\ w_2-u_1 \end{array}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & -3 \\ 0 & 1 & 0 & -6 \\ 0 & -1 & 2 & 3 \\ 0 & 1 & 3 & 0 \end{bmatrix} /(-1) \longrightarrow \quad \left(-\tfrac{3}{2}\right)$$

$$\begin{array}{r} u_1 \\ u_1+u_2 \\ u_3-2u_1-u_1-u_2 \\ w_1-u_1+u_1+u_2 \\ w_2-u_1 \end{array}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} /(-3/2) \longrightarrow$$

$$\begin{array}{r} u_1 \\ u_1+u_2 \\ u_3-3u_1-u_2 \\ w_1+u_2 \\ w_2-u_1-3/2(u_3-3u_1-u_2) \end{array}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9/2 \end{bmatrix}$$

$U+W = [\{u_1, u_2, u_3, w_1, w_2\}] = [\{u_1, u_2, u_3, w_2\}]$

$\{u_1, u_2, u_3, w_2\}$ база за $U+W$ → вектори u_1, u_2, u_3, w_2 су независни и чине базу за $U+W$, $\dim(U+W) = 4$

$U \cap W$: $\dim U + \dim W = \dim(U+W) + \dim(U\cap W)$

$\dim(U\cap W) = \dim U + \dim W - \dim(U+W)$

$\dim(U \cap W) = 3 + 2 - 4$

$\dim(U\cap W) = 1 \qquad U\cap W = [\{v\}]$

$w_1 + u_2 = \vec{0}$

$w_1 = -u_2$ јер пресек векторских потпростора U и W, што се види из претходног корака (проналажење базе за $U+W$)

$v = \alpha\cdot(-u_2) = \beta \cdot w_1$

$v = \beta\cdot w_1$

$v = \beta\cdot(1,1,5,7)$

$U\cap W = \{\beta(1,1,5,7) \mid \beta \in R\}$

$U \cap W = [\{w_1\}]$

$v = \alpha\cdot(-1)\cdot(-1,-1,-5,-7)$

$v = \alpha\cdot(1,1,5,7)$

$U\cap W = \{\alpha(1,1,5,7) \mid \alpha\in R\}$

$U\cap W = [\{(-u_1)\}]$

w_1 је независан у односу [illegible] па је $\{w_1\}$ база за $U \cap W$

(5) $f_1 = (3,2,1,1)$ $\qquad V = [\{f_1, f_2, f_3\}]$ $\qquad V \leq \mathbb{R}^4$

$f_2 = (5,3,4,5)$

$f_3 = (6,8,4,7)$

$$\begin{bmatrix} 3 & 2 & 1 & \boxed{1} \\ 5 & 3 & 4 & 5 \\ 6 & 8 & 4 & 7 \end{bmatrix} \begin{matrix} /\cdot(-5) \;\; /\cdot(-7) \\ \\ \end{matrix} \longrightarrow$$

$$\begin{bmatrix} 3 & 2 & 1 & \boxed{1} \\ -10 & -7 & -1 & 0 \\ -15 & -6 & -3 & 0 \end{bmatrix} \begin{matrix} \\ /\cdot(-1) \\ /:3 \end{matrix} \longrightarrow$$

$$\begin{bmatrix} 3 & 2 & 1 & \boxed{1} \\ 10 & 7 & \boxed{1} & 0 \\ -5 & -2 & -1 & 0 \end{bmatrix} \begin{matrix} \\ /\cdot 1 \, + \\ \end{matrix} \longrightarrow$$

$$\begin{bmatrix} 3 & 2 & 1 & \boxed{1} \\ 10 & 7 & \boxed{1} & 0 \\ 5 & 5 & 0 & 0 \end{bmatrix} /:5 \longrightarrow \begin{bmatrix} 3 & 2 & 1 & \boxed{1} \\ 10 & 7 & \boxed{1} & 0 \\ 1 & \boxed{1} & 0 & 0 \end{bmatrix}$$

Вектори f_1, f_2, f_3 се линеарно независни, $\dim V = 3$

$\overset{(*)}{\Longrightarrow} \{f_1, f_2, f_3\}$ чини база за V.

Ортогонална база

$\hat{e}_1 = f_1 = (3,2,1,1)$

$$\hat{e}_2 = f_2 - \frac{f_2 \circ \hat{e}_1}{\hat{e}_1 \circ \hat{e}_1} \hat{e}_1$$

$$= (5,3,4,5) - \frac{(5,3,4,5) \circ (3,2,1,1)}{(3,2,1,1) \circ (3,2,1,1)} \cdot (3,2,1,1)$$

$$= (5,3,4,5) - \frac{15+6+4+5}{9+4+1+1} \cdot (3,2,1,1)$$

$$= (5,3,4,5) - \frac{30}{15} (3,2,1,1)$$

$$= (5,3,4,5) - 2 \cdot (3,2,1,1) = (5,3,4,5) - (6,4,2,2)$$

$\hat{e}_2 = (-1,-1,2,3)$

$$\hat{e}_3 = f_3 - \frac{f_3 \circ \hat{e}_2}{\hat{e}_2 \circ \hat{e}_2}\hat{e}_2 - \frac{f_3 \circ \hat{e}_1}{\hat{e}_1 \circ \hat{e}_1}\hat{e}_1$$

$$= (6,8,4,7) - \frac{(6,8,4,7)\circ(-1,-1,2,3)}{(-1,-1,2,3)\circ(-1,-1,2,3)}\cdot(-1,-1,2,3) - \frac{(6,8,4,7)\circ(3,2,1,1)}{(3,2,1,1)\circ(3,2,1,1)}\cdot(3,2,1,1)$$

$$= (6,8,4,7) - \frac{-6-8+8+21}{1+1+4+9}\cdot(-1,-1,2,3) - \frac{18+16+4+7}{9+4+1+1}(3,2,1,1)$$

$$= (6,8,4,7) - \frac{15}{15}(-1,-1,2,3) - \frac{45}{15}(3,2,1,1)$$

$$= (6,8,4,7) - (-1,-1,2,3) - 3\cdot(3,2,1,1)$$

$$= (7,9,2,4) - (9,6,3,3) = (-2,3,-1,1)$$

$\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ - ортогонална база за V

Ортонормирана база

$$e_1 = \frac{1}{\|\hat{e}_1\|}\hat{e}_1 \qquad \|e_1\| = \sqrt{e_1 \circ e_1}$$

$$= \frac{1}{\sqrt{3+4+1+1}}\cdot(3,2,1,1) = \frac{1}{\sqrt{15}}(3,2,1,1) = \frac{\sqrt{15}}{15}(3,2,1,1) = \left(\frac{\sqrt{15}}{5}, \frac{2\sqrt{15}}{15}, \frac{\sqrt{15}}{15}, \frac{\sqrt{15}}{15}\right)$$

$$e_2 = \frac{1}{\|\hat{e}_2\|}\hat{e}_2$$

$$= \frac{1}{\sqrt{1+1+4+9}}(-1,-1,2,3) = \frac{1}{\sqrt{15}}(-1,-1,2,3) = \left(-\frac{\sqrt{15}}{15}, -\frac{\sqrt{15}}{15}, \frac{2\sqrt{15}}{15}, \frac{\sqrt{15}}{5}\right)$$

$$e_3 = \frac{1}{\|\hat{e}_3\|}\hat{e}_3$$

$$= \frac{1}{\sqrt{4+9+1+1}}\cdot(-2,3,-1,1) = \frac{1}{\sqrt{15}}(-2,3,-1,1) = \left(\frac{-2\sqrt{15}}{15}, \frac{\sqrt{15}}{5}, \frac{-\sqrt{15}}{15}, \frac{\sqrt{15}}{15}\right)$$

$\{e_1, e_2, e_3\}$ јесте ортонормирана база за V!

② инверз матрице

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 1 & 0 & 0 & 0 \\ 3 & 5 & 7 & 0 & 0 & 1 & 0 & 0 \\ 2 & 5 & 7 & 3 & 0 & 0 & 1 & 0 \\ 1 & 0 & -2 & -8 & 0 & 0 & 0 & 1 \end{array}\right] /\cdot(-3) + \quad /\cdot(-2) + \quad /\cdot(-1) +$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -2 & -3 & -3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -2 & 0 & 1 & 0 \\ 0 & -2 & -5 & -9 & -1 & 0 & 0 & 1 \end{array}\right] /\cdot(-1)$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 3 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -2 & 0 & 1 & 0 \\ 0 & -2 & -5 & -9 & -1 & 0 & 0 & 1 \end{array}\right] /\cdot(-1) + \quad /\cdot 2 +$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 3 & -1 & 0 & 0 \\ 0 & 0 & -1 & -2 & -5 & 1 & 1 & 0 \\ 0 & 0 & -1 & -3 & 5 & -2 & 0 & 1 \end{array}\right] \cdot(-1)$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 3 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 5 & -1 & -1 & 0 \\ 0 & 0 & -1 & -3 & 5 & -2 & 0 & 1 \end{array}\right] +$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 3 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 5 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 10 & -3 & -1 & 1 \end{array}\right] /\cdot(-1)$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 3 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 5 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -10 & 3 & 1 & -1 \end{array}\right] /\cdot(-2) \quad /\cdot(-3) \quad /\cdot(-1)$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 0 & 11 & -3 & -1 & 1 \\ 0 & 1 & 2 & 0 & 33 & -10 & -3 & 3 \\ 0 & 0 & 1 & 0 & 25 & -7 & -3 & 2 \\ 0 & 0 & 0 & 1 & -10 & 3 & 1 & -1 \end{array}\right] /\cdot(-2) \quad /\cdot(-3)$$

$$\rightarrow \left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & -64 & 18 & 8 & -5 \\ 0 & 1 & 0 & 0 & -17 & 4 & 3 & -1 \\ 0 & 0 & 1 & 0 & 25 & -7 & -3 & 2 \\ 0 & 0 & 0 & 1 & -10 & 3 & 1 & -1 \end{array}\right] / \cdot (-2)$$

$$\rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -30 & 10 & 2 & -3 \\ 0 & 1 & 0 & 0 & -17 & 4 & 3 & -1 \\ 0 & 0 & 1 & 0 & 25 & -7 & -3 & 2 \\ 0 & 0 & 0 & 1 & -10 & 3 & 1 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -30 & 10 & 2 & -3 \\ -17 & 4 & 3 & -1 \\ 25 & -7 & -3 & 2 \\ -10 & 3 & 1 & -1 \end{bmatrix}$$

Ово је инверз матрице A.