① $L: \mathbb{R}^4 \to \mathbb{R}^3$

$$L(x,y,z,t) = (x+y+z-t,\ 2x-3z+t,\ 2y-z-t)$$

канонска база за $\mathbb{R}^4 \to E = \{e_1=(1,0,0,0),\ e_2=(0,1,0,0),\ e_3=(0,0,1,0),\ e_4=(0,0,0,1)\}$

канi база за $\mathbb{R}^3 \to F = \{f_1=(1,0,0),\ f_2=(0,1,0),\ f_3=(0,0,1)\}$

$$L(e_1) = (1,2,0) = 1\cdot f_1 + 2 f_2 + 0\cdot f_3$$

$$L(e_2) = (1,0,2) = \ldots$$

$$L(e_3) = (1,-3,-1)$$

$$L(e_4) = (-1,1,-1)$$

$$\boxed{[L]_E^F = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 2 & 0 & -3 & 1 \\ 0 & 2 & -1 & -1 \end{bmatrix}}$$

$$\operatorname{Im} L = \mathscr{L}(L(e_1), L(e_2), L(e_3), L(e_4))$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 2 \\ 1 & -3 & -1 \\ -1 & 1 & -1 \end{bmatrix} /\cdot(-1) \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -2 & 2 \\ 0 & -5 & -1 \\ 0 & 3 & -1 \end{bmatrix} \begin{matrix} /\cdot 5 \quad /\cdot 3 \\ /\cdot(-2) \\ /\cdot 2 \end{matrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -2 & 2 \\ 0 & 0 & 12 \\ 0 & 0 & 4 \end{bmatrix} /:3 \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -2 & 2 \\ 0 & 0 & 4 \\ 0 & 0 & 4 \end{bmatrix} /\cdot(-1) \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -2 & 2 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Једна база за } \operatorname{Im} L: \{(1,2,0),(1,0,2),(1,-3,-1)\}}$

$\boxed{\rho(L) = \dim(\operatorname{Im} L) = 3}$

$\operatorname{Ker} L = (x,y,z,t)$

$$\begin{aligned} x+y+z-t &= 0 \quad /\cdot(-2) \\ 2x \quad -3z+t &= 0 \\ 2y-z-t &= 0 \end{aligned}$$

$$\begin{aligned} x+y+z-t &= 0 \\ -2y-5z+3t &= 0 \\ 2y-z-t &= 0 \end{aligned}$$

$$\begin{aligned} x+y+z-t &= 0 \\ -2y-5z+3t &= 0 \\ -6z+2t &= 0 \quad /:2 \end{aligned}$$

$$\begin{aligned} x+y-t+z &= 0 \\ -2y+3t-5z &= 0 \\ t-3z &= 0 \end{aligned}$$

$z = m$

$t = 3m$

$y = \frac{9m-5m}{2} = 2m$

$x = -m+3m-2m$

$x = 0$

$(x,y,z,t) = (0, 2m, m, 3m) = m(0,2,1,3)$

$\boxed{\text{Једна база за } \operatorname{Ker} L: \{(0,2,1,3)\}}$

$\boxed{\delta(L) = \dim(\operatorname{Ker} L) = 1}$

2) a) $L: \mathbb{R}^4 \to \mathbb{R}^4$

$L(x,y,z,t) = (x+4y+2z-t,\ 3x+7y+8z-3t,\ x+2y+3z-t,\ -2x-7y-4z+3t)$

1° адитивност

$u=(u_1,u_2,u_3,u_4)$

$v=(v_1,v_2,v_3,v_4)$

$L(u+v) \stackrel{?}{=} L(u)+L(v)$

$L(u+v) = (u_1+v_1+4u_2+4v_2+2u_3+2v_3-u_4-v_4,$
$3u_1+3v_1+7u_2+7v_2+8u_3+8v_3-3u_4-3v_4,$
$u_1+v_1+2u_2+2v_2+3u_3+3v_3-u_4-v_4,\ -2u_1-2v_1-7u_2-7v_2-4u_3-4v_3+$
$+3u_4+3v_4)$

$L(u) = (u_1+4u_2+2u_3-u_4,\ 3u_1+7u_2+8u_3-3u_4,$
$u_1+2u_2+3u_3-u_4,\ -2u_1-7u_2-4u_3+3u_4)$

$L(v) = (v_1+4v_2+2v_3-v_4,\ 3v_1+7v_2+8v_3-3v_4,$
$v_1+2v_2+3v_3-v_4,\ -2v_1-7v_2-4v_3+3v_4)$

$L(u)+L(v) = (u_1+4u_2+2u_3-u_4+v_1+4v_2+2v_3-v_4,$
$3u_1+7u_2+8u_3-3u_4+3v_1+7v_2+8v_3-3v_4,$
$u_1+2u_2+3u_3-u_4+v_1+2v_2+3v_3-v_4,$
$-2u_1-7u_2-4u_3+3u_4-2v_1-7v_2-4v_3+3v_4)$ ✓

$= L(u+v)$

2° хомогеност

$L(\alpha u) \stackrel{?}{=} \alpha \cdot L(u)$

$L(\alpha u) = (\alpha u_1+4\alpha u_2+2\alpha u_3-\alpha u_4,\ 3\alpha u_1+7\alpha u_2+8\alpha u_3-3\alpha u_4,$
$\alpha u_1+2\alpha u_2+3\alpha u_3-\alpha u_4,\ -2\alpha u_1-7\alpha u_2-4\alpha u_3+3\alpha u_4)$

$\alpha \cdot L(u) = \alpha \cdot (u_1+4u_2+2u_3-u_4,\ 3u_1+7u_2+8u_3-3u_4,$
$u_1+2u_2+3u_3-u_4,\ -2u_1-7u_2-4u_3+3u_4) =$

$= (\alpha u_1+4\alpha u_2+2\alpha u_3-\alpha u_4,\ 3\alpha u_1+7\alpha u_2+8\alpha u_3-3\alpha u_4,$
$\alpha u_1+2\alpha u_2+3\alpha u_3-\alpha u_4,\ -2\alpha u_1-7\alpha u_2-4\alpha u_3+3\alpha u_4) =$ ✓

$= L(\alpha u)$

1° и 2° $\Rightarrow$ L је линеарни оператор векторског простора $\mathbb{R}^4$

б) $\operatorname{Ker} L = (x, y, z, t)$

$$\begin{aligned} x+4y+2z-t&=0 \quad /\cdot(-3) \ /\cdot(-1) \ /\cdot 2 \\ 3x+7y+8z-3t&=0 \\ x+2y+3z-t&=0 \\ -2x-7y-4z+3t&=0 \end{aligned}$$

$\overset{?}{\Leftrightarrow} (x, y, z, t) = (0,0,0,0)$

$$\begin{aligned} x+4y+2z-t&=0 \\ -5y+2z&=0 \\ -2y+z&=0 \\ y+t&=0 \end{aligned}$$

$$\begin{aligned} x+4y+2z-t&=0 \\ y+t&=0 \quad /\cdot 2 \ /\cdot 5 \\ -2y+z&=0 \\ -5y+2z&=0 \end{aligned}$$

$$\begin{aligned} x+4y+2z-t&=0 \\ y+t&=0 \\ z+2t&=0 \quad /\cdot(-2) \\ 2z+5t&=0 \end{aligned}$$

$$\begin{aligned} x+4y+2z-t&=0 \\ y+t&=0 \\ z+2t&=0 \\ t&=0 \end{aligned}$$

$z=0$

$y=0$

$x=0$

$(x, y, z, t) = (0,0,0,0) \Rightarrow \boxed{\exists L^{-1} \text{ (} L \text{ је инвертибилан)}}$

$$e = \{\overset{e_1}{(1,0,0,0)}, \overset{e_2}{(0,1,0,0)}, \overset{e_3}{(0,0,1,0)}, \overset{e_4}{(0,0,0,1)}\}$$

$$L(e_1) = (1, 3, 1, -2) = 1\cdot e_1 + 3\cdot e_2 + 1\cdot e_3 - 2e_4$$

$$L(e_2) = (4, 7, 2, -7) = 4\cdot e_1 + 7e_2 + \cdot \;—$$

$$L(e_3) = (2, 8, 3, -4)$$

$$L(e_4) = (-1, -3, -1, 3)$$

$$[L]_e = \begin{bmatrix} 1 & 4 & 2 & -1 \\ 3 & 7 & 8 & -3 \\ 1 & 2 & 3 & -1 \\ -2 & -7 & -4 & 3 \end{bmatrix}$$

$$[L^{-1}]_e = [L]_e^{-1}$$

$$\left[\begin{array}{cccc|cccc} 1 & 4 & 2 & -1 & 1 & 0 & 0 & 0 \\ 3 & 7 & 8 & -3 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & -1 & 0 & 0 & 1 & 0 \\ -2 & -7 & -4 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} /\cdot(-3) \quad /\cdot(-1) \quad /\cdot 2 \\ + \\ + \\ + \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 4 & 2 & -1 & 1 & 0 & 0 & 0 \\ 0 & \boxed{-5} & 2 & 0 & -3 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} /\cdot 5 \; + \\ /\cdot(-2) \quad /\cdot 1 \quad /\cdot 4 \\ /\cdot 5 \; + \\ /\cdot 5 \; + \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 5 & 0 & 18 & -5 & -7 & 4 & 0 & 0 \\ 0 & -5 & 2 & 0 & -3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -2 & 5 & 0 \\ 0 & 0 & 2 & 5 & 7 & 1 & 0 & 5 \end{array}\right] \begin{array}{l} + \\ + \\ /\cdot(-2) \quad /\cdot(-18) \\ + \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 5 & 0 & 0 & -5 & -25 & 40 & -90 & 0 \\ 0 & -5 & 0 & 0 & -5 & 5 & -10 & 0 \\ 0 & 0 & 1 & 0 & 1 & -2 & 5 & 0 \\ 0 & 0 & 0 & \boxed{5} & 5 & 5 & -10 & 5 \end{array}\right] \begin{array}{l} + \\ \\ \\ \end{array} \sim$$

$$\sim \left[\begin{array}{cccc|cccc} 5 & 0 & 0 & 0 & -20 & 45 & -100 & 5 \\ 0 & -5 & 0 & 0 & -5 & 5 & -10 & 0 \\ 0 & 0 & 1 & 0 & 1 & -2 & 5 & 0 \\ 0 & 0 & 0 & 5 & 5 & 5 & -10 & 5 \end{array}\right] \begin{array}{l} /:5 \\ /:(-5) \\ \\ /:5 \end{array} \sim$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -4 & 9 & -20 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 & -2 & 5 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -2 & 1 \end{array}\right] \quad [L^{-1}]_e$$

B) $\boxed{\operatorname{Im} L = \mathcal{L}(L(e_1), L(e_2), L(e_3), L(e_4))} = \mathbb{R}^4$

$L: \mathbb{R}^4 \to \mathbb{R}^4$

$\operatorname{Im} L = \{ v \in \mathbb{R}^4 \mid L(u) = v,\ u \in \mathbb{R}^4 \}$

$\operatorname{Ker} L = \{ v \in \mathbb{R}^4 \mid L(v) = (0,0,0,0) \}$

$x + 4y + 12z - t = a$

$3x + 7y + 18z - 3t = b$

$x + 2y + 3z - t = c$

$-2x - 7y - 4z + 3t = d$

$(x, y, z, t) = (0,0,0,0)$

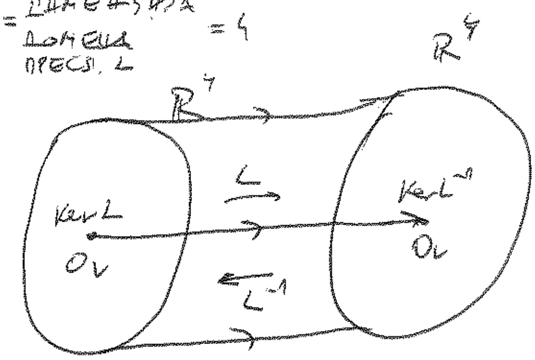
$\Rightarrow \quad a = b = c = d = 0$

$\Rightarrow \boxed{\operatorname{Ker}(L^{-1}) = (0,0,0,0)}$

II начин

- $\operatorname{Ker} L = \{0_v\} \Rightarrow \delta(L) = 0 \Rightarrow \underbrace{\rho(L) = 4}_{\text{(T) } \rho(L) + \delta(L) = \frac{\text{димензија}}{\text{домен на пресл. } L} = 4} \Rightarrow \underline{\operatorname{Im} L = \mathbb{R}^4}$ $\quad \underbrace{\operatorname{Im} L \leq \mathbb{R}^4}$

- L-инвертибилна
  $\Rightarrow L^{-1}$-инвертибилна
  $\Rightarrow \underline{\underline{\operatorname{Ker}(L^{-1}) = \{0_v\}}}$



(3)

$$\begin{vmatrix} 2 & -1 & 3 & 0 \\ \boxed{1} & 3 & 4 & -1 \\ 3 & 1 & 0 & 1 \\ 2 & 7 & 15 & -4 \end{vmatrix} \begin{matrix} \\ /\cdot(-2) \quad /\cdot(-3) \\ \\ \\ \end{matrix} \sim$$

$$\sim \begin{vmatrix} 0 & -7 & -5 & 2 \\ \boxed{1} & 3 & 4 & -1 \\ 0 & -8 & -12 & 4 \\ 0 & 1 & 7 & -2 \end{vmatrix} = -\begin{vmatrix} -7 & -5 & 2 \\ -8 & -12 & 4 \\ 1 & 7 & -2 \end{vmatrix} =$$

$$= -\Big((-7)\cdot(-12)\cdot(-2) + (-5)\cdot 4\cdot 1 + (-8)\cdot 7\cdot 2 -$$

$$- (-12)\cdot 2\cdot 1 - (-7)\cdot 7\cdot 4 - (-8)(-5)(-2) \Big) =$$

$$= -\big(24\cdot(-7) - 20 - 112 + 24 + 49\cdot 4 + 80 \big) =$$

$$= -\big(-168 - 20 - 112 + 24 + 196 + 80 \big) = \boxed{0}$$

(4)

$$A = \begin{bmatrix} 8 & -3 & -3 \\ -9 & 2 & 3 \\ 27 & -9 & -10 \end{bmatrix}$$

$$\chi_A(\lambda) = \det(A - \lambda E) = \begin{vmatrix} 8-\lambda & -3 & -3 \\ -9 & 2-\lambda & 3 \\ 27 & -9 & -10-\lambda \end{vmatrix} =$$

$$= -\begin{vmatrix} \lambda-8 & 3 & 3 \\ 9 & \lambda-2 & -3 \\ -27 & 9 & \lambda+10 \end{vmatrix} = -\Big((\lambda-8)(\lambda-2)(\lambda+10) +$$

$$+ 9\cdot 27 + 9\cdot 27 + 3\cdot 27(\lambda-2) + 27(\lambda-8) - 27(\lambda+10) \Big) =$$

$$= -\Big((\lambda^2 - 10\lambda + 16)(\lambda+10) + \cancel{18\cdot 27} + 3\cdot 27\cdot\lambda - 6\cdot 27 + \cancel{27\lambda} - \cancel{8\cdot 27} -$$

$$- \cancel{27\lambda} - \cancel{10\cdot 27} \Big) =$$

$$= -(\lambda^3 + 19\lambda^2 - 10\lambda^2 - 100\lambda + 162\lambda + 160 + 81\lambda - 162) =$$

$$= -(\lambda^3 - 3\lambda - 2) = -(\lambda+1)(\lambda^2 - \lambda - 2) = -(\lambda+1)^2(\lambda-2)$$

	1	0	−3	−2
−1	1	−1	−2	0

$$x_{1,2} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} = \begin{matrix} -1 \\ 2 \end{matrix}$$

$$\boxed{\chi_A(\lambda) = -(\lambda+1)^2(\lambda-2)}$$

Кандидати за $\mu_A(\lambda)$: $(\lambda+1)(\lambda-2)$

$(\lambda+1)^2(\lambda-2)$

$$(A+E)(A-2E) = \begin{bmatrix} 9 & -3 & -3 \\ -9 & 3 & 3 \\ 27 & -9 & -9 \end{bmatrix} \cdot \begin{bmatrix} 6 & -3 & -3 \\ -9 & 0 & 3 \\ 27 & -9 & -12 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \boxed{\mu_A(\lambda) = (\lambda+1)(\lambda-2)}$$

$\boxed{\text{Собствени стойности: } \lambda_1 = -1, \ \lambda_2 = 2}$

$\underline{\lambda_1 = -1:}$

$A \cdot v = -v$

$v \in R^3, \quad v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 8 & -3 & -3 \\ -9 & 2 & 3 \\ 27 & -9 & -10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -x \\ -y \\ -z \end{bmatrix}$$

$8x - 3y - 3z + x = 0$

$-9x + 2y + 3z + y = 0$

$27x - 9y - 10z + z = 0$

$9x - 3y - 3z = 0 \ /:3$

$-9x + 3y + 3z = 0 \ /:3$

$27x - 9y - 9z = 0 \ /:9$

$-y + 3x - z = 0 \ /(-1)$

$y - 3x + z = 0$

$-y + 3x - z = 0$

$-y + 3x - z = 0$

$0 = 0$

$0 = 0$

$z = m, \ x = n$

$y = 3n - m$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} n \\ 3n-m \\ m \end{bmatrix} = n\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + m\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$\boxed{\text{Собствени вектори за } \lambda_1: \ v_1 = (1,3,0) \text{ и } v_2 = (0,-1,1)}$

$\underline{\lambda_2 = 2:}$ $\quad A \cdot v = 2v \quad v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 8 & -3 & -3 \\ -9 & 2 & 3 \\ 27 & -9 & -10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix}$$

$8x - 3y - 3z - 2x = 0$

$-9x + 2y + 3z - 2y = 0$

$27x - 9y - 10z - 2z = 0$

$6x - 3y - 3z = 0 \ /:3$

$-9x + 3z = 0 \ /:3$

$27x - 9y - 12z = 0 \ /:3$

$-y + 2x - z = 0 \ /(-3)$

$-3y + 9x - 4z = 0$

$-3x + z = 0$

$$\begin{aligned} -y+2x-z&=0\\ 3x-z&=0\\ -3x+z&=0 \end{aligned}$$

$$\begin{aligned} -y-z+2x&=0\\ -z+3x&=0\\ 0&=0 \end{aligned}$$

$x = m$

$z = 3m$

$y = 2m - 3m = -m$

$$\begin{bmatrix} x\\ y\\ z \end{bmatrix} = \begin{bmatrix} m\\ -m\\ 3m \end{bmatrix} = m\begin{bmatrix} 1\\ -1\\ 3 \end{bmatrix}$$

$\boxed{\text{Сопствени вектор за } \lambda_3: \ v_3 = (1,-1,3)}$

$$\begin{bmatrix} 1 & 3 & 0\\ 0 & -1 & 1\\ 1 & -1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0\\ 0 & -1 & 1\\ 0 & -4 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0\\ 0 & -1 & 1\\ 0 & 0 & -1 \end{bmatrix}$$

v_1, v_2 и v_3 су линеарно независни $\Rightarrow$

$\Rightarrow$ $\boxed{\text{A је слична дијагоналној.}}$

$$\boxed{P = \begin{bmatrix} 1 & 0 & 1\\ 3 & -1 & -1\\ 0 & 1 & 3 \end{bmatrix}} \quad \boxed{D = \begin{bmatrix} -1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 2 \end{bmatrix}}$$

Провера:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0\\ 3 & -1 & -1 & 0 & 1 & 0\\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0\\ 0 & -1 & -4 & -3 & 1 & 0\\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0\\ 0 & -1 & -4 & -3 & 1 & 0\\ 0 & 0 & -1 & -3 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 1\\ 0 & -1 & 0 & 9 & -3 & -4\\ 0 & 0 & -1 & -3 & 1 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 1\\ 0 & 1 & 0 & -9 & 3 & 4\\ 0 & 0 & 1 & 3 & -1 & -1 \end{array}\right] = P^{-1}$$

$$PDP^{-1} = \begin{bmatrix} 1 & 0 & 1\\ 3 & -1 & -1\\ 0 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 1\\ -9 & 3 & 4\\ 3 & -1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2\\ -3 & 1 & -2\\ 0 & -1 & 6 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 1\\ -9 & 3 & 4\\ 3 & -1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} 8 & -3 & -3\\ -9 & 2 & 3\\ 27 & -9 & -10 \end{bmatrix} = A$$

5. $V \le \mathbb{R}^4$

$f_1 = (1,1,1,1)$

$f_2 = (4,3,7,2)$

$f_3 = (-1,2,6,1)$

$V = \mathcal{L}(f_1, f_2, f_3)$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & 7 & 2 \\ -1 & 2 & 6 & 1 \end{pmatrix} \cdot(-4) \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 3 & -2 \\ 0 & 3 & 7 & 2 \end{pmatrix} \cdot 3 \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 3 & -2 \\ 0 & 0 & 16 & -4 \end{pmatrix}$$

$\Rightarrow f_1, f_2$ и f_3 са лин. независими

$\hat{e}_1 = f_1 = (1,1,1,1)$

$$\hat{e}_2 = f_2 - \frac{\langle f_2, \hat{e}_1\rangle}{\langle \hat{e}_1, \hat{e}_1\rangle}\cdot \hat{e}_1 = (4,3,7,2) - \frac{(4,3,7,2)\circ(1,1,1,1)}{(1,1,1,1)\circ(1,1,1,1)}\cdot(1,1,1,1) =$$

$$= (4,3,7,2) - \frac{4+3+7+2}{4}\cdot(1,1,1,1) = (4,3,7,2) - (4,4,4,4) = (0,-1,3,-2)$$

$$\hat{e}_3 = f_3 - \left(\frac{\langle f_3, \hat{e}_1\rangle}{\langle \hat{e}_1, \hat{e}_1\rangle}\cdot \hat{e}_1 + \frac{\langle f_3, \hat{e}_2\rangle}{\langle \hat{e}_2, \hat{e}_2\rangle}\cdot \hat{e}_2\right) =$$

$$= (-1,2,6,1) - \left(\frac{(-1,2,6,1)\circ(1,1,1,1)}{4}\cdot(1,1,1,1) + \frac{(-1,2,6,1)\circ(0,-1,3,-2)}{(0,-1,3,-2)\circ(0,-1,3,-2)}\cdot(0,-1,3,-2)\right) =$$

$$= (-1,2,6,1) - \left(\frac{-1+2+6+1}{4}\cdot(1,1,1,1) + \frac{-2+18-2}{1+9+4}\cdot(0,-1,3,-2)\right) =$$

$$= (-1,2,6,1) - \big((2,2,2,2) + (0,-1,3,-2)\big) = (-1,2,6,1) - (2,1,5,0) =$$

$$= (-3,1,1,1)$$

$$\boxed{e_1 = \frac{1}{\|\hat{e}_1\|}\cdot \hat{e}_1 = \frac{1}{\sqrt{\langle \hat{e}_1, \hat{e}_1\rangle}}\cdot \hat{e}_1 = \frac{1}{\sqrt{4}}\cdot(1,1,1,1) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)}$$

$$\boxed{e_2 = \frac{1}{\|\hat{e}_2\|}\cdot \hat{e}_2 = \frac{1}{\sqrt{1+9+4}}\cdot(0,-1,3,-2) = \left(0, -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, -\frac{2}{\sqrt{14}}\right)}$$

$$\boxed{e_3 = \frac{1}{\|\hat{e}_3\|}\cdot \hat{e}_3 = \frac{1}{\sqrt{9+1+1+1}}\cdot(-3,1,1,1) = \left(-\frac{3}{\sqrt{12}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{12}}\right)}$$

ОНБ за V : $\{e_1, e_2, e_3\}$

$$V^\perp = \{ v \mid v \perp f_1,\ v \perp f_2,\ v \perp f_3 \}$$

$$v \perp f_1 \iff \langle v, f_1 \rangle = 0$$

$$v = (v_1, v_2, v_3, v_4)$$

$$\begin{aligned}
v_1 + v_2 + v_3 + v_4 &= 0 \quad /\cdot(-4) \\
4v_1 + 3v_2 + 7v_3 + 2v_4 &= 0 \\
-v_1 + 2v_2 + 6v_3 + v_4 &= 0
\end{aligned}$$

$$\begin{aligned}
v_1 + v_2 + v_3 + v_4 &= 0 \\
-v_2 + 3v_3 - 2v_4 &= 0 \quad /\cdot 3 \\
3v_2 + 7v_3 + 2v_4 &= 0
\end{aligned}$$

$$\begin{aligned}
v_1 + v_2 + v_3 + v_4 &= 0 \\
-v_2 + 3v_3 - 2v_4 &= 0 \\
16v_3 - 4v_4 &= 0 \quad /:4
\end{aligned}$$

$$\begin{aligned}
v_1 + v_2 + v_4 + v_3 &= 0 \\
-v_2 - 2v_4 + 3v_3 &= 0 \\
-v_4 + 4v_3 &= 0
\end{aligned}$$

$$v_3 = m \qquad v_4 = 4m$$

$$v_2 = -8m + 3m = -5m$$

$$v_1 = -m - 4m + 5m = 0$$

$$v = (0, -5m, m, 4m) = m(0, -5, 1, 4)$$

$k_1 = \{(0,-5,1,4)\}$ – једна база за $V^\perp$

$$\hat{e}_1 = k_1 = (0, -5, 1, 4)$$

$$e_1 = \frac{1}{\|\hat{e}_1\|} \cdot \hat{e}_1 = \frac{1}{\sqrt{25+1+16}} \cdot (0,-5,1,4) =$$

$$\begin{array}{r} 26 \\ +\,16 \\ \hline 42 \end{array}$$

$$= \frac{1}{\sqrt{42}} \cdot (0,-5,1,4) = \left(0, -\frac{5}{\sqrt{42}}, \frac{1}{\sqrt{42}}, \frac{4}{\sqrt{42}}\right)$$

$$\boxed{\text{ОНБ за } V^\perp: \left\{ \left(0, -\frac{5}{\sqrt{42}}, \frac{1}{\sqrt{42}}, \frac{4}{\sqrt{42}}\right) \right\}}$$