4.

1. način

```
In[1]:= tA = {2, -2};
In[2]:= tB = {1, 3};
In[3]:= tC = {1, 1};
In[4]:= tD = {-7, 11};
In[5]:= AB = tB - tA
Out[5]= {-1, 5}
In[6]:= AC = tC - tA
Out[6]= {-1, 3}
In[7]:= AD = tD - tA
Out[7]= {-9, 13}
In[8]:= Dabc = Det[{AB, AC}]
Out[8]= 2
In[9]:= Dabd = Det[{AB, AD}]
Out[9]= 32
```

Kako su Dabc i Dabd istog znaka, tj. trougao ABC i trougao ABD istih orjentacija, tačke C i D se nalaze sa iste strane prave AB (u istoj poluravni).

2. način

Posmatrajmo funkciju f(x,y)=ax+by+c. Ona je jednaka 0 u tačkama prave p: ax+by+c=0, veća od 0 u jednoj poluravni određenoj pravom p, a manja od 0 u drugoj. Dakle, tačke X i Y pripadaju istoj poluravni ako su f(X) i f(Y) istog znaka.

```
In[10]:= implicitna[{x0_, y0_}, {p1_, p2_}] := p2 * x - p1 * y - p2 * x0 + p1 * y0
In[11]:= TraditionalForm[implicitna[tA, AB] == 0]
Out[11]//TraditionalForm=
```

$5x + y - 8 = 0$

```
In[12]:= f[{x_, y_}] := 5 x + y - 8
In[13]:= f[tC]
Out[13]= -2
In[14]:= f[tD]
Out[14]= -32
```

f(C) i f(D) su istog znaka, dakle C i D pripadaju istoj poluravni.

6. a)

In[15]:=
$$d = \frac{\text{Abs}\left[2 * 1 - 3 * \left(-3\right) + 1\right]}{\text{Sqrt}\left[2^2 + \left(-3\right)^2\right]}$$

Out[15]=
$$\frac{12}{\sqrt{13}}$$

In[16]:=
```
tM = {1, -3};
```

In[17]:=
```
f[{x_, y_}] := 2 x - 3 y + 1
```

In[18]:=
```
np = {2, -3};
```

In[19]:=
$$d = \frac{\text{Abs}[f[tM]]}{\text{Norm}[np]}$$

Out[19]=
$$\frac{12}{\sqrt{13}}$$

6. b)

In[20]:=
```
vp = {1, -2, 0};
```

In[21]:=
```
tM = {1, -3, 0};
```

In[22]:=
```
tP = {1, 0, 0};
```

In[23]:=
```
PM = tM - tP
```

Out[23]=
$$\{0, -3, 0\}$$

In[24]:=
$$d = \frac{\text{Norm}[\text{Cross}[vp, PM]]}{\text{Norm}[vp]}$$

Out[24]=
$$\frac{3}{\sqrt{5}}$$

7.

In[25]:=
```
tA = {1, 2};
```

In[26]:=
```
tB = {4, 3};
```

In[27]:=
```
tC = {6, 0};
```

In[28]:=
$$tP = \frac{1}{2}\left(tA + tB\right)$$

Out[28]=
$$\left\{\frac{5}{2}, \frac{5}{2}\right\}$$

In[29]:=
$$tQ = \frac{1}{2}\left(tB + tC\right)$$

Out[29]=
$$\left\{5, \frac{3}{2}\right\}$$

In[30]:=
```
AB = tB - tA
```

Out[30]=
$$\{3, 1\}$$

In[31]:=
```
vp = {1, -3};
```

In[32]:=
```
parametarska[{x0_, y0_}, {p1_, p2_}, p_] := {p1 * p + x0, p2 * p + y0}
```

In[33]:=
```
parametarskaIspis[{x_, y_}] := Print["x = ", x, "\ny = ", y]
```

In[34]:=
```
jnaP = parametarska[tP, vp, t]
```

Out[34]= $\left\{\frac{5}{2}+t,\ \frac{5}{2}-3\,t\right\}$

In[35]:=
```
parametarskaIspis[jnaP]
```

```
x = 5/2 + t
y = 5/2 - 3 t
```

In[36]:=
```
BC = tC - tB
```

Out[36]= $\{2,\ -3\}$

In[37]:=
```
vq = {-3, -2};
```

In[38]:=
```
PQ = tQ - tP
```

Out[38]= $\left\{\frac{5}{2},\ -1\right\}$

In[39]:=
```
jnaQ = parametarska[tQ, vq, s]
```

Out[39]= $\left\{5-3\,s,\ \frac{3}{2}-2\,s\right\}$

In[40]:=
```
parametarskaIspis[jnaQ]
```

```
x = 5 - 3 s
y = 3/2 - 2 s
```

In[41]:=
```
s = Det[{PQ, vp}] / Det[{vp, vq}]
```

Out[41]= $\frac{13}{22}$

In[42]:=
```
tO = jnaQ /. {s → %}
```

Out[42]= $\left\{\frac{71}{22},\ \frac{7}{22}\right\}$

In[43]:=
```
CO = tO - tC
```

Out[43]= $\left\{-\frac{61}{22},\ \frac{7}{22}\right\}$

In[44]:=
```
r = Norm[CO]
```

Out[44]= $\frac{\sqrt{\frac{1885}{2}}}{11}$

In[45]:=
```
tT = 1/3 (tA + tB + tC)
```

Out[45]= $\left\{\frac{11}{3},\ \frac{5}{3}\right\}$

$$\to O\left(\frac{71}{22}, \frac{7}{22}\right),\ r = \frac{\sqrt{\frac{1885}{2}}}{11},\ T\left(\frac{11}{3}, \frac{5}{3}\right)$$

8.

In[46]:=
```
tA = {-1, 4};
```

In[47]:=
```
tB = {2, 3};
```

In[48]:=
```
tC = {1, 2};
```

In[49]:=
```
tT = 1/3 (tA + tB + tC)
```

Out[49]= $\left\{\frac{2}{3}, 3\right\}$

T($\frac{2}{3}$, 3)

Baricentrične koord. tačke M u odnosu na tačke A, B i C su: M (m1 : m2 : m3) = (Pbcm : Pcam : Pabm)

Baricentrične koord. tačke T ?

In[50]:=
```
Dbct = Det[{tC - tB, tT - tB}]
```

Out[50]= $-\frac{4}{3}$

In[51]:=
```
Dcat = Det[{tA - tC, tT - tC}]
```

Out[51]= $-\frac{4}{3}$

In[52]:=
```
Dabt = Det[{tB - tA, tT - tA}]
```

Out[52]= $-\frac{4}{3}$

T($-\frac{4}{3}:-\frac{4}{3}:-\frac{4}{3}$) tj. T(1:1:1)

In[53]:=
```
BC = tC - tB
```

Out[53]= {-1, -1}

In[54]:=
```
vp = {-1, 1};
```

In[55]:=
```
jnap = parametarska[tA, vp, t]
```

Out[55]= {-1 - t, 4 + t}

In[56]:=
```
parametarskaIspis[jnap]
```

```
x = -1 - t
y = 4 + t
```

In[57]:=
```
AB = tB - tA
```

Out[57]= {3, -1}

In[58]:=
```
vq = {-1, -3};
```

In[59]:=
```
Clear[s]
```

In[60]:= **jnaq = parametarska[tC, vq, s]**

Out[60]= $\{1 - s, 2 - 3\,s\}$

In[61]:= **parametarskaIspis[jnaq]**

$x = 1 - s$
$y = 2 - 3\,s$

In[62]:= **AC = tC – tA**

Out[62]= $\{2, -2\}$

In[63]:= $\frac{\text{Det[\{AC, vq\}]}}{\text{Det[\{vp, vq\}]}}$

Out[63]= -2

In[64]:= **tH = jnap /. {t → %}**

Out[64]= $\{1, 2\}$

In[65]:= **Dbch = Det[{BC, tH – tB}]**

Out[65]= 0

In[66]:= **Dcah = Det[{– AC, tH – tC}]**

Out[66]= 0

In[67]:= **Dabh = Det[{AB, tH – tA}]**

Out[67]= -4

H(1,2), H(0:0:-4) tj. H(0:0:1)

Primetimo da je H(1,2) = C, dakle trougao ABC je pravougli sa pravim uglom kod temena C → O je središte hipotenuze.

In[68]:= $\text{tO} = \frac{1}{2}\left(\text{tA} + \text{tB}\right)$

Out[68]= $\left\{\frac{1}{2}, \frac{7}{2}\right\}$

In[69]:= **Dbco = Det[{BC, tO – tB}]**

Out[69]= -2

In[70]:= **Dcao = Det[{– AC, tO – tC}]**

Out[70]= -2

In[71]:= **Dabo = Det[{AB, tO – tA}]**

Out[71]= 0

O $(\frac{1}{2}, \frac{7}{2})$, O(-2:-2:0) tj O(1:1:0)

In[72]:= $\text{vp} = \frac{\text{AC}}{\text{Norm[AC]}} + \frac{\text{AB}}{\text{Norm[AB]}}$

Out[72]= $\left\{\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{10}}\right\}$

In[73]:= **CB = tB - tC**

Out[73]= $\{1, 1\}$

In[74]:= **CA = -AC**

Out[74]= $\{-2, 2\}$

In[75]:= **vq = CA / Norm[-AC] + CB / Norm[CB]**

Out[75]= $\left\{0, \sqrt{2}\right\}$

In[76]:= **jnaq = parametarska[tC, vq, s]**

Out[76]= $\left\{1, 2 + \sqrt{2}\ s\right\}$

In[77]:= **parametarskaIspis[jnaq]**

$x = 1$
$y = 2 + \sqrt{2}\ s$

In[78]:= **Det[{AC, vp}] / Det[{vp, vq}]**

Out[78]= $\dfrac{2\sqrt{\frac{2}{5}}}{1 + \frac{3}{\sqrt{5}}}$

In[79]:= **Simplify[%]**

Out[79]= $\dfrac{2\sqrt{10}}{5 + 3\sqrt{5}}$

In[80]:= **tS = jnaq /. {s → %}**

Out[80]= $\left\{1, 2 + \dfrac{4\sqrt{5}}{5 + 3\sqrt{5}}\right\}$

In[81]:= **Simplify[%]**

Out[81]= $\left\{1, 5 - \sqrt{5}\right\}$

In[82]:= **a = Norm[BC]**

Out[82]= $\sqrt{2}$

In[83]:= **b = Norm[AC]**

Out[83]= $2\sqrt{2}$

In[84]:= **c = Norm[AB]**

Out[84]= $\sqrt{10}$

$S\left(1, 5 - \sqrt{5}\right),\ S\left(\sqrt{2} : 2\sqrt{2} : \sqrt{10}\right)$ tj. $S\left(1 : 2 : \sqrt{5}\right)$

$\to T\left(\dfrac{2}{3}, 3\right),\ T(1 : 1 : 1)$

$\to H(1, 2),\ H(0 : 0 : 1)$

$\to O\left(\dfrac{1}{2}, \dfrac{7}{2}\right),\ O(1 : 1 : 0)$

→ S $\left(1,\ 5-\sqrt{5}\right)$, S $\left(1:2:\sqrt{5}\right)$

9.

In[85]:=
```
tP = {3, 1};
```

In[86]:=
```
tQ = {2, 3};
```

In[87]:=
```
PQ = tQ - tP
```

Out[87]=
```
{-1, 2}
```

9.a)

In[88]:=
```
vp = {1, 0};
```

In[89]:=
```
vq = {1, 1};
```

In[90]:=
```
Det[{vp, vq}]
```

Out[90]=
```
1
```

In[91]:=
```
jnap = parametarska[tP, vp, t]
```

Out[91]=
```
{3 + t, 1}
```

In[92]:=
```
parametarskaIspis[jnap]
```

```
x = 3 + t
y = 1
```

In[93]:=
$$\frac{\text{Det}[\{PQ,\ vq\}]}{\text{Det}[\{vp,\ vq\}]}$$

Out[93]=
```
-3
```

In[94]:=
```
tM = jnap /. {t → -3}
```

Out[94]=
```
{0, 1}
```

$p \cap q = \{M\}$

M (0, 1)

9.b)

In[95]:=
```
vp = {1, 0};
```

In[96]:=
```
vq = {-2, 0};
```

In[97]:=
```
Det[{vp, vq}]
```

Out[97]=
```
0
```

In[98]:=
```
Det[{PQ, vp}]
```

Out[98]=
```
-2
```

Dakle vp i vq su kolinearni tj. prave p i q imaju isti pravac. A kako tačka Q ne pripada pravoj p to su prave p i q paralelne.

$p \cap q = \emptyset$

9.c)

In[99]:=
```
vp = {1, -2};
```

In[100]:=
```
vq = {-2, 4};
```

In[101]:=
```
Det[{vp, vq}]
```

Out[101]=
```
0
```

In[102]:=
```
Det[{PQ, vp}]
```

Out[102]=
```
0
```

Dakle vp i vq su kolinearni tj. prave p i q imaju isti pravac. A kako tačka Q pripada pravoj p to se prave p i q poklapaju.