MATHEMATICS

A PRACTICAL ODYSSEY

SEVENTH EDITION

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FINANCE

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How you deal with money will have a big impact on the quality of your life. There's no question that you will borrow money. You probably already have a credit card loan or a student loan. Sooner or later, you'll borrow money for a car or a house. The question is whether or not you'll know enough that you can be wise about your borrowing.

Not everybody saves money, but your life will be less stressful and more successful if you do. You might now have a savings account at a bank or a money market account. Will you know enough about the mathematics of finance to be wise about your savings?

It's to your advantage to know the mathematics of finance. Right now, you probably know next to nothing about it. You might not even know what a money market account is or what an annuity is. You should know these things.

WHAT WE WILL DO IN THIS CHAPTER

BORROWING:

- Most loans require that you pay simple interest, so we will explore that.
- Many loans, including car loans and home loans, are amortized—that is, they require monthly payments—so we will explore amortized loans.

SAVING:

- Many investments, including savings accounts, certificates of deposit, and money market accounts, pay compound interest, so we will discuss how compound interest works.
- One of the best ways for an average person to save money is through an annuity, so we will explore that.

5.1 Simple Interest

OBJECTIVES

- Make simple interest calculations
- Determine a credit card finance charge
- Find the payment required by an add-on interest loan

Loans and investments are very similar financial transactions. Each involves:

- the flow of money from a source to another party
- the return of the money to its source
- the payment of a fee to the source for the use of the money

When you make a deposit in your savings account, you are making an investment, but the bank views it as a loan; you are lending the bank your money, which they will lend to another customer, perhaps to buy a house. When you borrow money to buy a car, you view the transaction as a loan, but the bank views it as an investment; the bank is investing its money in you in order to make a profit.

When an investor puts money into a savings account or buys a certificate of deposit (CD) or a Treasury bill, the investor expects to make a profit. The amount of money that is invested is called the **principal.** The profit is the **interest.** How much interest will be paid depends on the **interest rate** (usually expressed as a percent per year); the **term**, or length of time that the money is invested; and how the interest is calculated.

In this section, we'll explore simple interest in investments and short-term loans. **Simple interest** means that the amount of interest is calculated as a percent per year of the principal.

SIMPLE INTEREST FORMULA

The **simple interest** I on a principal P at an annual rate of interest r for a term of t years is

$$I = Prt$$

EXAMPLE 1

USING THE SIMPLE INTEREST FORMULA Tom and Betty buy a two-year CD that pays 5.1% simple interest from their bank for $150,000. (Many banks pay simple interest on larger CDs and compound interest on smaller CDs.) They invest $150,000, so the principal is $P = \$150{,}000$. The interest rate is $r = 5.1\% = 0.051$, and the term is $t = 2$ years.

a. Find the interest that the investment earns.
b. Find the value of the CD at the end of its term.

SOLUTION

a. Using the Simple Interest Formula, we have

$$\begin{aligned} I &= Prt && \textbf{the Simple Interest Formula} \\ &= 150{,}000 \cdot 0.051 \cdot 2 && \textbf{substituting for } \boldsymbol{P, r,} \textbf{ and } \boldsymbol{t} \\ &= \$15{,}300 \end{aligned}$$

b. Two years in the future, the bank will pay Tom and Betty

$150,000 principal + $15,300 interest = $165,300

This is called the *future value* of the CD, because it is what the CD will be worth in the future.

Future Value

In Example 1, we found the future value by adding the principal to the interest. The **future value** FV is always the sum of the principal P plus the interest I. If we combine this fact with the Simple Interest Formula, we can get a formula for the future value:

$$\begin{aligned} FV &= P + I \\ &= P + Prt \quad \text{from the Simple Interest Formula} \\ &= P(1 + rt) \quad \text{factoring} \end{aligned}$$

SIMPLE INTEREST FUTURE VALUE FORMULA

The **future value** FV of a principal P at an annual rate of interest r for t years is

$$FV = P(1 + rt)$$

Capital Letters for Money

There is an important distinction between the variables FV, I, P, r, and t: FV, I, and P measure amounts of money, whereas r and t do not. For example, consider the interest rate r and the interest I. Frequently, people confuse these two. However, the interest rate r is a percentage, and the interest I is an amount of money; Tom and Betty's interest rate is $r = 5.1\%$, but their interest is $I = \$15{,}300$. To emphasize this distinction, we will always use capital letters for variables that measure amounts of money and lowercase letters for other variables. We hope this notation will help you avoid substituting $5.1\% = 0.051$ for I when it should be substituted for r.

Finding the Number of Days: "Through" versus "To"

Clearly, there is only one day from January 1 to January 2. If the answer weren't so obvious, we could find it by subtracting:

$$2 - 1 = 1 \text{ day}$$

This count of days includes January 1 but not January 2.

In contrast, there are *two* days from January 1 *through* January 2. The word “through” means to include both the beginning day (January 1) and the ending day (January 2). If the answer weren’t so obvious, we could find it by doing the “January 1 to January 2” calculation above and then adding the one ending day:

$$\underbrace{2 - 1}_{\text{the 1st to the 2nd}} + \underbrace{1}_{\text{adding the one ending day}} = 2$$

The number of days from August 2 to August 30 is

$$30 - 2 = 28 \text{ days}$$

This count includes August 2 but not August 30.

The number of days from August 2 *through* August 30 is

$$30 - 2 + 1 = 29 \text{ days}$$

This count includes August 2 and August 30.

Short Term Loans

One of the more common uses of simple interest is a short-term (such as a year or less) loan that requires a single lump sum payment at the end of the term. Businesses routinely obtain these loans to purchase equipment or inventory, to pay operating expenses, or to pay taxes. A **lump sum** payment is a single payment that pays off an entire loan. Some loans require smaller monthly payments rather than a single lump sum payment. We will investigate that type of loan later in this section and in Section 5.4.

EXAMPLE 2

USING THE SIMPLE INTEREST FUTURE VALUE FORMULA Espree Clothing borrowed \$185,000 at $7\frac{1}{4}\%$ from January 1 to February 28.

a. Find the future value of the loan.
b. Interpret the future value.

SOLUTION

a. • We are given:

$$P = 185{,}000$$
$$r = 7\tfrac{1}{4}\% = 0.0725$$

• Finding t:
  - January has 31 days.
  - February has 28 days.
  - There are $31 + 28 - 1 = 58$ days from January 1 to February 28.
  - $t = 58$ days

$$= 58 \cancel{\text{days}} \cdot \frac{1 \text{ year}}{365 \cancel{\text{days}}} = \frac{58}{365} \text{ years}$$ **using dimensional analysis (see Appendix E)**

• Finding FV:

$$FV = P(1 + rt)$$ **the Simple Interest Future Value Formula**

$$= 185{,}000\left(1 + 0.0725 \cdot \frac{58}{365}\right)$$ **substituting for *P*, *r*, and *t***

$$= 187{,}131.301\ldots$$

$$\approx \$187{,}131.30$$ **rounding to the nearest penny**

185000 × (1 + .0725 × 58 ÷ 365) =

185000 × (1 + .0725 × 58 ÷ 365) ENTER

b. This means that Espree has agreed to make a lump sum payment to their lender $187,168.05 on February 28.

As a rough check, notice that the answer is somewhat higher than the original $185,000. This is as it should be. The future value includes the original $185,000 plus interest.

Notice that in Example 2, we do not use $t = 2$ months $= \frac{2}{12}$ year. If we did, we would get $187,235.42, an inaccurate answer. Instead, we use the number of days, converted to years.

In Example 2, we naturally used 365 days per year, but some institutions traditionally count a year as 360 days and a month as thirty days (especially if that tradition works in their favor). This is a holdover from the days before calculators and computers—the numbers were simply easier to work with. Also, we used normal round-off rules to round $187,131.301 . . . to $187,131.30; some institutions round off some interest calculations in their favor. In this book, we will count a year as 365 days and use normal round-off rules (unless stated otherwise).

The issue of how a financial institution rounds off its interest calculations might seem unimportant. After all, we are talking about a difference of a fraction of a penny. However, consider one classic form of computer crime: the round-down fraud, performed on a computer system that processes a large number of accounts. Frequently, such systems use the normal round-off rules in their calculations and keep track of the difference between the theoretical account balance if no rounding off is done and the actual account balance with rounding off. Whenever that difference reaches or exceeds 1¢, the extra penny is deposited in (or withdrawn from) the account. A fraudulent computer programmer can write the program so that the extra penny is deposited in his or her own account. This fraud is difficult to detect because the accounts appear to be balanced. While each individual gain is small, the total gain can be quite large if a large number of accounts is processed regularly.

A written contract signed by the lender and the borrower is called a **loan agreement** or a **note.** The **maturity value** of the note (or just the **value** of the note) refers to the note's future value. Thus, the value of the note in Example 2 was $187,168.05. This is what the note is worth to the lender in the future—that is, when the note matures.

National Debt

In almost every year since 1931, the U.S. federal budget called for deficit spending, that is, spending more money than is received. In 2008, the total U.S. federal debt was $10,025 billion, and we paid about $214 billion for interest on that debt. In 2009, the debt grew to $11,343 billion, or about $37,000 per person in the United States.

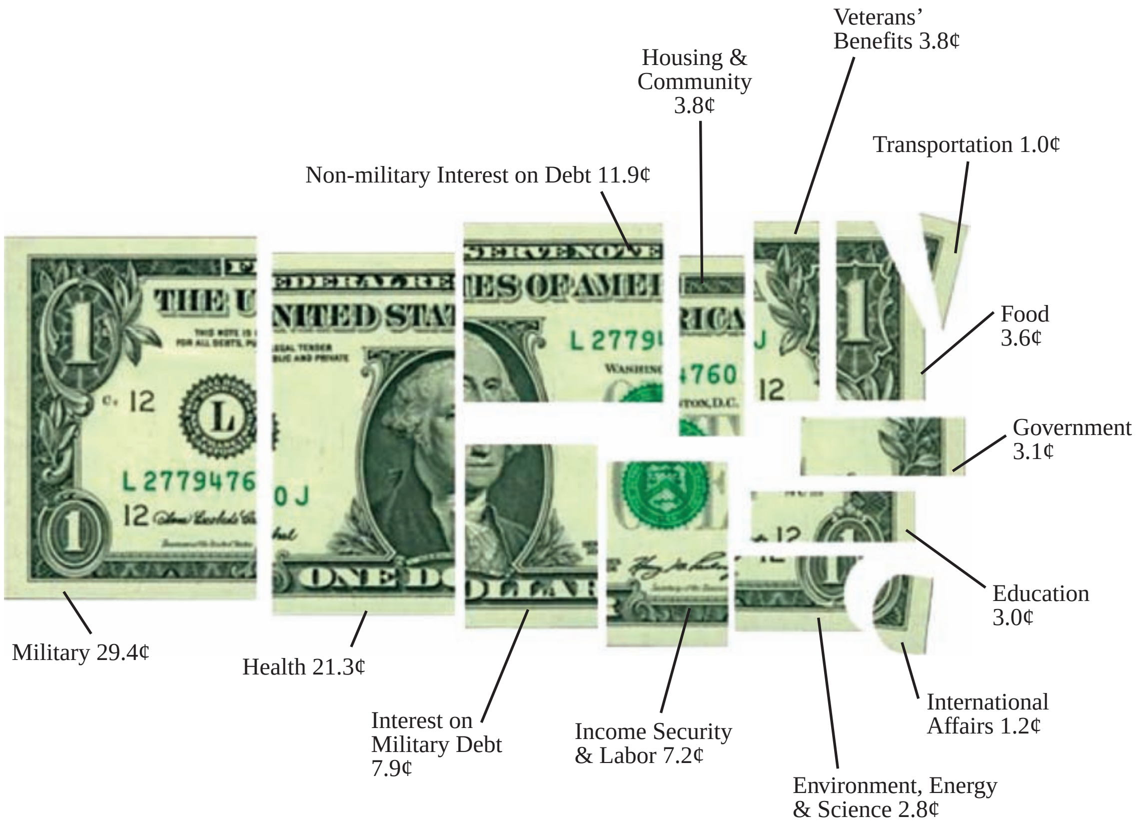

FIGURE 5.1 Where your 2008 federal income tax dollar was spent. *Source:* National Priorities Project.

EXAMPLE 3

THE NATIONAL DEBT Find the simple interest rate that was paid on the 2008 national debt.

SOLUTION

- We are given

$$I = \$214 \text{ billion}$$
$$P = \$10{,}025 \text{ billion}$$
$$t = 1 \text{ year}$$

- Finding *r*:

$$I = Prt \qquad \text{the Simple Interest Formula}$$
$$\$214 \text{ billion} = \$10{,}025 \text{ billion} \cdot r \cdot 1 \qquad \text{substituting}$$
$$r = \$21 \text{ billion}/\$10{,}025 \text{ billion} \cdot r \cdot 1 \qquad \text{solving for } r$$
$$= 0.0213\ldots \approx 2.1\% \qquad \text{rounding}$$

Present Value

EXAMPLE 4

FINDING HOW MUCH TO INVEST NOW Find the amount of money that must be invested now at a $5\frac{3}{4}\%$ simple interest so that it will be worth $1,000 in two years.

SOLUTION

We are asked to find the principal P that will generate a future value of \$1,000.

- We are given

$$FV = \$1{,}000$$
$$r = 5\tfrac{3}{4}\% = 0.0575$$
$$t = 2 \text{ years}$$

- Finding P:

$$FV = P(1 + rt) \quad \textbf{the Simple Interest Future Value Formula}$$
$$1{,}000 = P(1 + 0.0575 \cdot 2) \quad \textbf{substituting}$$
$$P = \frac{1{,}000}{1 + 0.0575 \cdot 2} \quad \textbf{solving for } \boldsymbol{P}$$
$$= 896.86099\ldots \quad \textbf{rounding}$$
$$\approx \$896.86$$

1000 [÷] [(] 1 [+] .0575 [×] 2 [)] [=]

1000 [÷] [(] 1 [+] .0575 [×] 2 [)] [ENTER]

As a rough check, notice that the answer is somewhat smaller than \$1,000. This is as it should be—the principal does not include interest, so it should be smaller than the future value of \$1,000.

In Example 4, the investment is worth \$1,000 two years *in the future;* that is, \$1000 is the investment's *future value.* But the same investment is worth \$896.86 *in the present*. For this reason, we say that \$896.86 is the investment's **present value.** In this case, this is the same thing as the principal; it is just called the *present value* to emphasize that this is its value in the present.

Add-on Interest

An **add-on interest loan** is an older type of loan that was common before calculators and computers, because calculations of such loans can easily be done by hand. With this type of loan, the total amount to be repaid is computed with the Simple Interest Future Value Formula. The payment is found by dividing that total amount by the number of payments.

Add-on interest loans have generally been replaced with the more modern amortized loans (see Section 5.4). However, they are not uncommon at auto lots that appeal to buyers who have poor credit histories.

EXAMPLE 5

AN ADD-ON INTEREST LOAN Chip Douglas's car died, and he must replace it right away. Centerville Auto Sales has a nine-year-old Ford that's "like new" for \$5,988. The sign in their window says, "Bad credit? No problem!" They offered Chip a 5% two-year add-on interest loan if he made a \$600 down payment. Find the monthly payment.

SOLUTION

The loan amount is $P = 5{,}988 - 600 = 5{,}388$. The total amount due is

$$\begin{aligned} FV &= P(1 + rt) && \text{the Simple Interest Future Value Formula} \\ &= 5{,}388(1 + 0.05 \cdot 2) && \text{substituting} \\ &= 5{,}926.80 \end{aligned}$$

The total amount due is spread out over twenty-four monthly payments, so the monthly payment is $5,926.8/24 = $246.95. This means that Chip has to pay $600 when he purchases the car and then $246.95 a month for twenty-four months.

Credit Card Finance Charge

Credit cards have become part of the American way of life. Purchases made with a credit card are subject to a finance charge, but there is frequently a grace period, and no finance charge is assessed if full payment is received by the payment due date. One of the most common methods of calculating credit card interest is the **average daily balance** method. To find the average daily balance, the balance owed on the account is found for each day in the billing period, and the results are averaged. The finance charge consists of simple interest, charged on the result.

EXAMPLE 6

FINDING A CREDIT CARD FINANCE CHARGE The activity on Tom and Betty's Visa account for one billing is shown below. The billing period is October 15 through November 14, the previous balance was $346.57, and the annual in rate is 21%.

October 21	payment	$50.00
October 23	restaurant	$42.14
November 7	clothing	$18.55

a. Find the average daily balance.
b. Find the finance charge.

SOLUTION

a. To find the average daily balance, we have to know the balance for each day in the billing period and the number of days at that balance, as shown in Figure 5.2. The average daily balance is then the weighted average of each daily balance, with each balance weighted to reflect the number of days at that balance.

Time Interval	Days	Daily Balance
October 15–20	20 − 14 = 6	$346.57
October 21–22	22 − 20 = 2	$346.57 − $50 = $296.57
October 23–November 6 (October has 31 days)	31 − 22 = 9 9 + 6 = 15	$296.57 + $42.14 = $338.71
November 7–14	14 − 6 = 8	$338.71 + $18.55 = $357.26

FIGURE 5.2 Preparing to find the average daily balance.

$$\begin{aligned} \text{Average daily balance} &= \frac{6 \cdot 346.57 + 2 \cdot 296.57 + 15 \cdot 338.71 + 8 \cdot 357.26}{6 + 2 + 15 + 8} \\ &= 342.29967\ldots \\ &\approx \$342.30 \end{aligned}$$

(6 × 346.57 + 2 × 296.57 + 15 × 338.71 + 8 × 357.26) ÷ (6 + 2 + 15 + 8) =

With a graphing calculator, type ENTER instead of = .

b. • We are given

$$P = \$342.29967\ldots$$
$$r = 21\% = 0.21$$
$$t = 31 \text{ days} = 31/365 \text{ years}$$

• Finding *I*:

$I = Prt$ **the Simple Interest Formula**

$= 342.29967\ldots \cdot 0.21 \cdot 31/365$ **substituting**

$= 6.1051258\ldots$ **multiplying**

$\approx \$6.11$ **rounding**

CREDIT CARD FINANCE CHARGE

To find the credit card finance charge with the average daily balance method, do the following:

1. Find the balance for each day in the billing period and the number of days at that balance.
2. The average daily balance is the weighted average of these daily balances, weighted to reflect the number of days at that balance.
3. The finance charge is simple interest applied to the average daily balance.

How Many Days?

Financial calculations usually involve computing the term *t* either in a whole number of years or in the number of days converted to years. This requires that you know the number of days in each month. As you can see from Figure 5.3, the months alternate between thirty-one days and thirty days, with two exceptions:

- February has twenty-eight days (twenty-nine in leap years).
- The alternation does not happen from July to August.

January	31 days	May	31 days	September	30 days
February	28 days	June	30 days	October	31 days
March	31 days	July	31 days	November	30 days
April	30 days	August	31 days	December	31 days

FIGURE 5.3 The number of days in a month.

HISTORICAL NOTE

THE HISTORY OF CREDIT CARDS

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Credit cards and charge cards were first used in the United States in 1915, when Western Union issued a metal card to some of its regular customers. Holders of these cards were allowed to defer their payments and were assured of prompt and courteous service. Shortly thereafter, several gasoline companies, hotels, department stores, and railroads issued cards to their preferred customers.

The use of credit and charge cards virtually ceased during World War II, owing to government restrictions on credit. In 1950, a New York lawyer established the Diners' Club after being embarrassed when he lacked sufficient cash to pay a dinner bill. A year later, the club had billed more than $1 million. Carte Blanche and the American Express card soon followed. These "travel and entertainment" cards were and are attractive to the public because they provide a convenient means of paying restaurant, hotel, and airline bills. They eliminate both the possibility of an out-of-town check being refused and the need to carry a large amount of cash.

The first bank card was issued in 1951 by Franklin National Bank in New York. Within a few years, 100 other banks had followed suit. Because these bank cards were issued by individual banks, they were accepted for use only in relatively small geographical areas. In 1965, the California-based Bank of America began licensing other banks (both in the United States and abroad) to issue BankAmericards. Other banks formed similar groups, which allowed customers to use the cards out of state.

In 1970, the Bank of America transferred administration of its bank card program to a new company owned by the banks that issued the card. In 1977, the card was renamed Visa. MasterCard was originally created by several California banks (United California Bank, Wells Fargo, Crocker Bank, and the Bank of California) to compete with BankAmericard. It was then licensed to the First National Bank of Louisville, Kentucky and the Marine Midland Bank of New York.

Credit cards are now a part of the American way of life. In 2004, we started to use credit cards, debit cards and other forms of electronic bill paying more than we use paper checks. Credit card debt has soared, especially among college students who are already in debt with college loans. Credit card issuers have been accused of targeting college students, who tend to make minimum payments and thus incur much more interest. See Exercises 45–48.

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Early Diners' Club, Carte Blanche and American Express cards

5.1 EXERCISES

In Exercises 1–4, find the number of days.

1. **a.** September 1 to October 31 of the same year
   b. September 1 through October 31 of the same year
2. **a.** July 1 to December 31 of the same year
   b. July 1 through December 31 of the same year
3. ▶ **a.** April 1 to July 10 of the same year
   b. April 1 through July 10 of the same year
4. **a.** March 10 to December 20 of the same year
   b. March 10 through December 20 of the same year

In Exercises 5–10, find the simple interest of the given loan amount.

5. ▶ $2,000 borrowed at 8% for three years
6. $35,037 borrowed at 6% for two years
7. ▶ $420 borrowed at $6\frac{3}{4}\%$ for 325 days
8. $8,950 borrowed at $9\frac{1}{2}\%$ for 278 days
9. $1,410 borrowed at $12\frac{1}{4}\%$ from September 1:
   a. to October 31 of the same year
   b. through October 31 of the same year

▶ Selected exercises available online at **www.webassign.net/brookscole**

▶ **10.** \$5,682 borrowed at $11\frac{3}{4}$% from July 1:

a. to December 31 of the same year

b. through December 31 of the same year

In Exercises 11–14, find the future value of the given present value.

▶ **11.** Present value of \$3,670 at $2\frac{3}{4}$% for seven years

12. Present value of \$4,719 at 14.1% for eleven years

▶ **13.** Present value of \$12,430 at $5\frac{7}{8}$% for 660 days

14. Present value of \$172.39 at 6% for 700 days

In Exercises 15–20, find the maturity value of the given loan amount.

▶ **15.** \$1,400 borrowed at $7\frac{1}{8}$% for three years

16. \$3,250 borrowed at $8\frac{1}{2}$% for four years

17. \$5,900 borrowed at $14\frac{1}{2}$% for 112 days

▶ **18.** \$2,720 borrowed at $12\frac{3}{4}$% for 275 days

19. \$16,500 borrowed at $11\frac{7}{8}$% from April 1 through July 10 of the same year

▶ **20.** \$2,234 borrowed at $12\frac{1}{8}$% from March 10 through December 20 of the same year

In Exercises 21–26 find the present value of the given future value.

▶ **21.** Future value \$8,600 at $9\frac{1}{2}$% simple interest for three years

22. Future value \$420 at $5\frac{1}{2}$% simple interest for two years

▶ **23.** Future value \$1,112 at $3\frac{5}{8}$% simple interest for 512 days

▶ **24.** Future value \$5,750 at $4\frac{7}{8}$% simple interest for 630 days

25. Future value \$1,311 at $6\frac{1}{2}$% simple interest from February 10 to October 15 of the same year

26. Future value \$4,200 at $6\frac{3}{4}$% simple interest from April 12 to November 28 of the same year

▶ **27.** If you borrow \$1,000 at 8.5% interest and the loan requires a lump sum payment of \$1,235.84, what is the term of the loan?

▶ **28.** If you borrow \$1,700 at 5.25% interest and the loan requires a lump sum payment of \$1,863.12, what is the term of the loan?

29. The Square Wheel Bicycle store has found that they sell most of their bikes in the spring and early summer. On March 1, they borrowed \$226,500 to buy bicycles. They are confident that they can sell most of these bikes by August 1. Their loan is at $6\frac{7}{8}$% interest. What size lump sum payment would they have to make on August 1 to pay off the loan?

▶ **30.** Ernie Bilko has a business idea. He wants to rent an abandoned gas station for just the months of November and December. He will convert the gas station into a drive-through Christmas wrapping station. Customers will drive in, drop off their gifts, return the next day, and pick up their wrapped gifts. He needs \$338,200 to rent the gas station, purchase wrapping paper, hire workers, and advertise. If he borrows this amount at $6\frac{1}{2}$% interest for those two months, what size lump sum payment will he have to make to pay off the loan?

31. Alice Cohen buys a two-year-old Honda from U-Pay-Less-Cars for \$19,000. She puts \$500 down and finances the rest through the dealer at 13% add-on interest. If she agrees to make thirty-six monthly payments, find the size of each payment.

▶ **32.** Sven Lundgren buys a three-year-old Chevrolet from Skunk Motors for \$14,600. He puts \$300 down and finances the rest through the dealer at 12.5% add-on interest. If he agrees to make twenty-four monthly payments, find the size of each payment.

33. Ray and Teresa Martinez buy a used car from Fowler's Wholesale 2U for \$6,700. They put \$500 down and finance the rest through the car lot at 9.8% add-on interest. If they make thirty-six monthly payments, find the size of each payment.

▶ **34.** Dick Davis buys a five-year-old used Toyota from Pioneer Auto Sales for \$7,999. He puts \$400 down and finances the rest through the car lot at 7.7% add-on interest. He agrees to make thirty-six monthly payments. Find the size of each payment.

In Exercises 35–36, use the following information. When "trading up," it is preferable to sell your old house before buying your new house because that allows you to use the proceeds from selling your old house to buy your new house. When circumstances do not allow this, the homeowner can take out a bridge loan.

35. Dale and Claudia have sold their house, but they will not get the proceeds from the sale for an estimated $2\frac{1}{2}$ months. The owner of the house they want to buy will not hold the house that long. Dale and Claudia have two choices: let their dream house go or take out a bridge loan. The bridge loan would be for \$110,000, at 7.75% simple interest, due in ninety days.

a. How big of a check would they have to write in 90 days?

b. How much interest would they pay for this loan?

▶ **36.** Tina and Mike have sold their house, but they will not get the proceeds from the sale for an estimated 3 months. The owner of the house they want to buy will not hold the house that long. Tina and Mike have two choices: let their dream house go or take out a bridge loan. The bridge loan would be for \$85,000, at 8.5% simple interest, due in 120 days.

a. How big of a check would they have to write in 120 days?

b. How much interest would they pay for this loan?

37. The activity on Stuart Ratner's Visa account for one billing period is shown below. Find the average daily balance and the finance charge if the billing period is

April 11 through May 10, the previous balance was $126.38, and the annual interest rate is 18%.

April 15	payment	$15.00
April 22	DVD store	$25.52
May 1	clothing	$32.18

▶ **38.** The activity on Marny Zell's MasterCard account for one billing period is shown below. Find the average daily balance and the finance charge if the billing period is June 26 through July 25, the previous balance was $396.68, and the annual interest rate is 19.5%.

June 30	payment	$100.00
July 2	gasoline	$36.19
July 10	restaurant	$53.00

▶ **39.** The activity on Denise Hellings' Sears account for one billing period is shown below. Find the average daily balance and the finance charge if the billing period is March 1 through March 31, the previous balance was $157.14, and the annual interest rate is 21%.

March 5	payment	$25.00
March 17	tools	$36.12

▶ **40.** The activity on Charlie Wilson's Visa account for one billing period is shown below. Find the average daily balance and the finance charge if the billing period is November 11 through December 10, the previous balance was $642.38, and the annual interest rate is 20%.

November 15	payment	$150
November 28	office supplies	$23.82
December 1	toy store	$312.58

41. Donovan and Pam Hamilton bought a house from Edward Gurney for $162,500. In lieu of a 10% down payment, Mr. Gurney accepted 5% down at the time of the sale and a promissory note from the Hamiltons for the remaining 5%, due in four years. The Hamiltons also agreed to make monthly interest payments to Mr. Gurney at 10% interest until the note expires. The Hamiltons obtained a loan from their bank for the remaining 90% of the purchase price. The bank in turn paid the sellers the remaining 90% of the purchase price, less a sales commission of 6% of the purchase price, paid to the sellers' and the buyers' real estate agents.

a. Find the Hamiltons' down payment.

b. Find the amount that the Hamiltons borrowed from their bank.

c. Find the amount that the Hamiltons borrowed from Mr. Gurney.

d. Find the Hamilton's monthly interest-only payment to Mr. Gurney.

e. Find Mr. Gurney's total income from all aspects of the down payment (including the down payment, the amount borrowed under the promissory note, and the monthly payments required by the promissory note).

f. Find Mr. Gurney's net income from the Hamiltons' bank.

g. Find Mr. Gurney's total income from all aspects of the sale.

▶ **42.** George and Peggy Fulwider bought a house from Sally Sinclair for $233,500. In lieu of a 10% down payment, Ms. Sinclair accepted 5% down at the time of the sale and a promissory note from the Fulwiders for the remaining 5%, due in four years. The Fulwiders also agreed to make monthly interest payments to Ms. Sinclair at 10% interest until the note expires. The Fulwiders obtained a loan from their bank for the remaining 90% of the purchase price. The bank in turn paid the sellers the remaining 90% of the purchase price, less a sales commission of 6% of the purchase price, paid to the sellers' and the buyers' real estate agents.

a. Find the Fulwiders' down payment.

b. Find the amount that the Fulwiders borrowed from their bank.

c. Find the amount that the Fulwiders borrowed from Ms. Sinclair.

d. Find the Fulwiders' monthly interest-only payment to Ms. Sinclair.

e. Find Ms. Sinclair's total income from all aspects of the down payment (including the down payment, the amount borrowed under the promissory note, and the monthly payments required by the promissory note).

f. Find Ms. Sinclair's net income from the Fulwiders' bank.

g. Find Ms. Sinclair's total income from all aspects of the sale.

▶ **43.** The Obamas bought a house from the Bushes for $389,400. In lieu of a 20% down payment, the Bushes accepted a 10% down payment at the time of the sale and a promissory note from the Obamas for the remaining 10%, due in 4 years. The Obamas also agreed to make monthly interest payments to the Bushes at 11% interest until the note expires. The Obamas obtained a loan for the remaining 80% of the purchase price from their bank. The bank in turn paid the sellers the remaining 80% of the purchase price, less a sales commission of 6% of the sales price paid to the sellers' and the buyers' real estate agents.

a. Find the Obamas' down payment.

b. Find the amount that the Obamas borrowed from their bank.

c. Find the amount that the Obamas borrowed from the Bushes.

d. Find the Obamas' monthly interest-only payment to the Bushes.

e. Find the Bushes' total income from all aspects of the down payment (including the down payment, the amount borrowed under the promissory note, and the monthly payments required by the note).

f. Find the Bushes' income from the Obamas' bank.

g. Find the Bushes' total income from all aspects of the sale.

44. Sam Needham bought a house from Sheri Silva for $238,300. In lieu of a 20% down payment, Ms. Silva accepted a 10% down payment at the time of the sale and a promissory note from Mr. Needham for the remaining 10%, due in four years. Mr. Needham also agreed to make monthly interest payments to Ms. Silva at 9% interest until the note expires. Mr. Needham obtained a loan for the remaining 80% of the purchase price from his bank. The bank in turn paid Ms. Silva the remaining 80% of the purchase price, less a sales commission (of 6% of the sales price) paid to the sellers' and the buyers' real estate agents.

a. Find Mr. Needham's down payment.

b. Find the amount that Mr. Needham borrowed from his bank.

c. Find the amount that Mr. Needham borrowed from the Ms. Silva.

d. Find Mr. Needham's monthly interest-only payment to Ms. Silva.

e. Find Ms. Silva's total income from all aspects of the down payment (including the down payment, the amount borrowed under the promissory note, and the monthly payments required by the note).

f. Find Ms. Silva's income from Mr. Needham's bank.

g. Find Ms. Silva's total income from all aspects of the sale.

45. Your credit card has a balance of $1,000. Its interest rate is 21%. You have stopped using the card, because you don't want to go any deeper into debt. Each month, you make the minimum required payment of $20.

a. During the January 10 through February 9 billing period, you pay the minimum required payment on January 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

b. During the February 10 through March 9 billing period, you pay the minimum required payment on February 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

c. During the March 10 through April 9 billing period, you pay the minimum required payment on March 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

d. Discuss the impact of making the minimum required payment, both on yourself and on the credit card issuer.

▶ **46.** Your credit card has a balance of $1,200. Its interest rate is 20.5%. You have stopped using the card, because you don't want to go any deeper into debt. Each month, you make the minimum required payment of $24.

a. During the September 10 through October 9 billing period, you pay the minimum required payment on September 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

b. During the October 10 through November 9 billing period, you pay the minimum required payment on October 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

c. During the November 10 through December 9 billing period, you pay the minimum required payment on November 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

d. Discuss the impact of making the minimum required payment, both on yourself and on the credit card issuer.

▶ **47.** Your credit card has a balance of $1,000. Its interest rate is 21%. You have stopped using the card, because you don't want to go any deeper into debt. Each month, you make the minimum required payment. Your credit card issuer recently changed their minimum required payment policy, in response to the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005. As a result, your minimum required payment is $40.

a. During the January 10 through February 9 billing period, you pay the minimum required payment on January 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

b. During the February 10 through March 9 billing period, you pay the minimum required payment on February 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

c. During the March 10 through April 9 billing period, you pay the minimum required payment on March 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

d. Compare the results of parts (a) through (c) with those of Exercise 45. Discuss the impact of the credit card issuer's change in their minimum required payment policy.

▶ **48.** Your credit card has a balance of $1,200. Its interest rate is 20.5%. You have stopped using the card, because you don't want to go any deeper into debt. Each month, you make the minimum required payment. Your credit card issuer recently changed their minimum required payment policy, in response to the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005. As a result, your minimum required payment is $48.

a. During the September 10 through October 9 billing period, you pay the minimum required payment on September 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

b. During the October 10 through November 9 billing period, you pay the minimum required payment on October 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

c. During the November 10 through December 9 billing period, you pay the minimum required payment on November 25th. Find the average daily balance, the finance charge and the new balance. (The new balance includes the finance charge.)

d. Compare the results of parts (a) through (c) with those of Exercise 46. Discuss the impact of the credit card issuer's change in their minimum required payment policy.

Answer the following questions using complete sentences and your own words.

• CONCEPT QUESTIONS

49. Could Exercises 5–21 all be done with the Simple Interest Formula? If so, how? Could Exercises 5–21 all be done with the Simple Interest Future Value Formula? If so, how? Why do we have both formulas?

50. Which is always higher: future value or principal? Why?

• HISTORY QUESTIONS

51. Who offered the first credit card?

52. What was the first post–World War II credit card?

53. Who created the first post–World War II credit card?

54. What event prompted the creation of the first post–World War II credit card?

55. What was the first interstate bank card?

5.2 Compound Interest

OBJECTIVES

- Understand the difference between simple interest and compound interest
- Use the Compound Interest Formula
- Understand and compute the annual yield

Many forms of investment, including savings accounts, earn **compound interest,** in which interest is periodically paid on both the original principal and previous interest payments. This results in earnings that are significantly higher over a longer period of time. It is important that you understand this difference in order to make wise financial decisions. We'll explore compound interest in this section.

Compound Interest as Simple Interest, Repeated

EXAMPLE 1

UNDERSTANDING COMPOUND INTEREST Tom and Betty deposit \$1,000 into their new bank account. The account pays 8% interest compounded quarterly. This means that interest is computed and deposited every quarter of a year. Find the account balance after six months, using the Simple Interest Future Value formula to compute the balance at the end of each quarter.

SOLUTION

At the end of the first quarter, $P = \$1,000$, $r = 8\% = 0.08$, and $t =$ one quarter or $\frac{1}{4}$ year.

$$\begin{aligned} FV &= P(1 + rt) && \textbf{the Simple Interest Future Value Formula} \\ &= 1{,}000(1 + 0.08 \cdot \tfrac{1}{4}) && \textbf{substituting} \\ &= 1{,}000(1 + 0.02) \\ &= \$1{,}020 \end{aligned}$$

This means that there is $1,020 in Tom and Betty's account at the end of the first quarter. It also means that the second quarter's interest will be paid on this new principal. So at the end of the second quarter, $P = \$1{,}020$ and $r = 0.08$. Note that $t = \frac{1}{4}$, not $\frac{2}{4}$, because we are computing interest for one quarter.

$$\begin{aligned} FV &= P(1 + rt) && \textbf{the Simple Interest Future Value Formula} \\ &= 1{,}020(1 + 0.08 \cdot \tfrac{1}{4}) && \textbf{substituting} \\ &= 1{,}020(1 + 0.02) \\ &= \$1{,}040.40 \end{aligned}$$

At the end of six months, the account balance is $1,040.40.

As a rough check, notice that the future value is slightly higher than the principal, as it should be.

The Compound Interest Formula

This process would become tedious if we were computing the balance after twenty years. Because of this, compound interest problems are usually solved with their own formula.

The **compounding period** is the time period over which any one interest payment is calculated. In Example 1, the compounding period was a quarter of a year. For each quarter's calculation, we multiplied the annual rate of 8% (0.08) by the time 1 quarter ($\frac{1}{4}$ year) and got 2% (0.02). This 2% is the **quarterly rate** (or more generally, the *periodic rate*). A **periodic rate** is any rate that is prorated in this manner from an annual rate.

If i is the periodic interest rate, then the future value at the end of the first period is

$$FV = \mathbf{P(1 + i)}$$

Because this is the account balance at the beginning of the second period, it becomes the new principal. The account balance at the end of the second period is

$$\begin{aligned} FV &= \mathbf{P(1 + i)} \cdot (1 + i) && \textbf{substituting } \boldsymbol{P(1 + i)} \textbf{ for } \boldsymbol{P} \\ &= P(1 + i)^2 \end{aligned}$$

This means that $P(1 + i)^2$ is the account balance at the beginning of the third period, and the future value at the end of the third period is

$$\begin{aligned} FV &= [P(1 + i)^2] \cdot (1 + i) && \textbf{substituting } \boldsymbol{P(1 + i)^2} \textbf{ for } \boldsymbol{P} \\ &= P(1 + i)^3 \end{aligned}$$

If we generalize these results, we get the Compound Interest Formula.

COMPOUND INTEREST FORMULA

If initial principal P earns compound interest at a periodic interest rate i for n periods, the future value is

$$FV = P(1 + i)^n$$

Notice that we have maintained our variables tradition: i and n do not measure amounts of money, so they are not capital letters. We now have three interest-related variables:

- r, the annual interest rate (not an amount of money)
- i, the periodic interest rate (not an amount of money)
- I, the interest itself (an amount of money)

EXAMPLE 2

USING THE COMPOUND INTEREST FORMULA Use the Compound Interest Formula to recompute Tom and Betty's account balance from Example 1.

SOLUTION

Their 8% interest is compounded quarterly, so each quarter they earn a quarter of $8\% = \frac{1}{4} \cdot 8\% = 2\% = 0.02$. Also, n counts the number of quarters, so $n = 2$.

$$FV = P(1 + i)^n \quad \text{the Compound Interest Formula}$$
$$= 1{,}000(1 + 0.02)^2 = \$1{,}040.40 \quad \text{substituting}$$

1000 [×] [(] 1 [+] .02 [)] [y^x] 2 [=]

1000 [(] 1 [+] .02 [)] [^] 2 [ENTER]

Compound Interest Compared with Simple Interest

As we saw at the beginning of this section, compound interest is just simple interest, repeated. However, there can be profound differences in their results.

EXAMPLE 3

COMPOUND INTEREST OVER A LONG PERIOD OF TIME In 1777, it looked as though the Revolutionary War was about to be lost. George Washington's troops were camped at Valley Forge. They had minimal supplies, and the winter was brutal. According to a 1990 class action suit, Jacob DeHaven, a wealthy Pennsylvania merchant, saved Washington's troops and the revolutionary cause by loaning Washington \$450,000. The suit, filed by DeHaven's descendants, asked the government to repay the still-outstanding loan plus compound interest at the then-prevailing rate of 6%. (*Source: New York Times*, May 27, 1990.) How much did the government owe on the 1990 anniversary of the loan if the interest is compounded monthly?

SOLUTION

The principal is $P = \$450{,}000$. If 6% interest is paid each year, then 1/12th of 6% is paid each month, and $i = \frac{1}{12} \cdot 6\% = 0.06/12$. The term is

$$n = 213 \text{ years}$$
$$= 213 \cancel{\text{years}} \cdot \frac{12 \text{ months}}{1 \cancel{\text{year}}} \quad \text{using dimensional analysis}$$
$$= 2{,}556 \text{ months}$$

Using the Compound Interest Formula, we get

$$FV = P(1 + i)^n \quad \textbf{the Compound Interest Formula}$$
$$= 450{,}000(1 + 0.06/12)^{2556} \quad \textbf{substituting}$$
$$= 1.547627234\ldots \times 10^{11}$$
$$\approx \$154{,}762{,}723{,}400 \quad \textbf{rounding}$$

450000 [×] [(] 1 [+] .06 [÷] 12 [)] [y^x] 2556 [=]

450000 [(] 1 [+] .06 [÷] 12 [)] [^] 2556 [ENTER]

The DeHavens sued for this amount, but they also stated that they were willing to accept a more reasonable payment.

As a rough check, notice that the future value is much higher than the principal, as it should be because of the long time period.

Some sources have questioned the DeHavens' claim: "There is also no evidence to support the claim of the DeHaven family that their ancestor Jacob DeHaven lent George Washington $450,000 in cash and supplies while the army was encamped at Valley Forge. This tradition first appeared in print in a history of the DeHaven family penned by Howard DeHaven Ross. Periodically, the descendants of Jacob DeHaven make attempts to get the "loan" repaid with interest. . . . This remarkably persistent tradition has been thoroughly debunked by Judith A. Meier, of the Historical Society of Montgomery County, whose genealogical research revealed that there were no DeHavens living in the immediate area until after 1790 and that Jacob DeHaven had never been rich enough to make such a fabulous loan." (*Source:* Lorett Treese, *Valley Forge: Making and Remaking a National Symbol*, University Park, PA: Pennsylvania University Press, 1995.)

EXAMPLE 4

COMPARING SIMPLE INTEREST WITH COMPOUND INTEREST OVER A LONG PERIOD OF TIME How much would the government have owed the DeHavens if the interest was simple interest?

SOLUTION

With simple interest, we use r and t, which are *annual* figures, rather than i and n, which are *periodic* figures. So $r = 6\% = 0.06$, and $t = 213$ years.

$$FV = P(1 + rt) \quad \textbf{the Simple Interest Future Value Formula}$$
$$= 450000(1 + 0.06 \cdot 213) \quad \textbf{substituting}$$
$$= \$6{,}201{,}000$$

450000 [×] [(] 1 [+] .06 [×] 213 [)] [=]

450000 [(] 1 [+] .06 [×] 213 [)] [ENTER]

Simple interest would have required a payment of only $6 million. This is a lot, but not in comparison with the $155 billion payment required by compound interest.

In Examples 3 and 4, the future value with compound interest is almost 25,000 times the future value with simple interest. Over longer periods of time, compound interest is immensely more profitable to the investor than simple interest, because compound interest gives interest on interest. Similarly, compounding more frequently (daily rather than quarterly, for example) is more profitable to the investor.

The effects of the size of the time interval and the compounding period can be seen in Figure 5.4.

Type of Interest	Future Value of $1000 at 10% Interest		
	After 1 Year	After 10 Years	After 30 Years
Simple interest	$1100.00	$2000.00	$ 4,000.00
Compounded annually	$1100.00	$2593.74	$17,449.40
Compounded quarterly	$1103.81	$2685.06	$19,358.15
Compounded monthly	$1104.71	$2707.04	$19,837.40
Compounded daily	$1105.16	$2717.91	$20,077.29

FIGURE 5.4 Comparing simple interest and compound interest.

Finding the Interest and the Present Value

EXAMPLE 5

FINDING THE AMOUNT OF INTEREST EARNED Betty's boss paid her an unexpected bonus of $2,500. Betty and her husband decided to save the money for their daughter's education. They deposited it in account that pays 10.3% interest compounded daily. Find the amount of interest that they would earn in fifteen years by finding the future value and subtracting the principal.

SOLUTION

Compounding daily, we have $P = 2500$, $i = 1/365\text{th of } 10.3\% = 0.103/365$, and $n = 15 \text{ years} = 15 \text{ years} \cdot 365 \text{ days/year} = 5{,}475 \text{ days}$.

$FV = P(1 + i)^n$ **the Compound Interest Formula**

$= 2{,}500(1 + 0.103/365)^{5475}$ **substituting**

$= 11{,}717.374\ldots$

$\approx \$11{,}717.37$

2500 [×] [(] 1 [+] .103 [÷] 365 [)] [y^x] 5475 [=]

2500 [(] 1 [+] .103 [÷] 365 [)] [^] 5475 [ENTER]

Warning: If you compute $\frac{0.103}{365}$ separately, you will get a long decimal. Do not round off that decimal, because the resulting answer will be inaccurate. Doing the calculation all at once (as shown above) avoids this difficulty.

The principal is $2,500, and the total of principal and interest is $11,717.37. Thus, the interest is $11,717.37 − $2500 = $9,217.37.

EXAMPLE 6

FINDING THE PRESENT VALUE Find the amount of money that must be invested now at $7\frac{3}{4}\%$ interest compounded annually so that it will be worth \$2,000 in 3 years.

SOLUTION

The question actually asks us to find the present value, or principal P, that will generate a future value of \$2,000. We have $FV = 2{,}000$, $i = 7\frac{3}{4}\% = 0.0775$, and $n = 3$.

$$\begin{aligned} FV &= P(1+i)^n && \text{the Compound Interest Formula} \\ 2{,}000 &= P(1+0.0775)^3 && \text{substituting} \\ P &= \frac{2{,}000}{1.0775^3} && \text{solving for } P \\ &= 1{,}598.7412 && \text{rounding} \\ &\approx \$1{,}598.74 \end{aligned}$$

As a rough check, notice that the principal is somewhat lower than the future value, as it should be, because of the short time period.

Annual Yield

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FIGURE 5.5 Most banks advertise their yields as well as their rates.

Which investment is more profitable: one that pays 5.8% compounded daily or one that pays 5.9% compounded quarterly? It is difficult to tell. Certainly, 5.9% is a better rate than 5.8%, but compounding daily is better than compounding quarterly. The two rates cannot be directly compared because of their different compounding frequencies. The way to tell which is the better investment is to find the annual yield of each.

The **annual yield** (also called the **annual percentage yield** or **APY**) of a compound interest deposit is the *simple interest rate* that has the same future value as the compound rate would have in one year. The annual yields of two different investments can be compared, because they are both simple interest rates. Annual yield provides the consumer with a uniform basis for comparison and banks display both their interest rates and their annual yields, as shown in Figure 5.5. The annual yield should be slightly higher than the compound rate, because compound interest is slightly more profitable than simple interest over a short period of time. The compound rate is sometimes called the **nominal rate** to distinguish it from the yield (here, *nominal* means "named" or "stated").

To find the **annual yield** r of a given compound interest rate, you find the simple interest rate that makes the future value under simple interest the same as the future value under compound interest in one year.

$$FV(\text{simple interest}) = FV(\text{compound interest})$$
$$P(1 + rt) = P(1 + i)^n$$

EXAMPLE 7

FINDING THE ANNUAL YIELD Find the annual yield of $2,500 deposited in an account in which it earns 10.3% interest compounded daily for 15 years.

SOLUTION

Simple interest	**Compound interest**
$P = 2{,}500$	$P = 2{,}500$
$r =$ unknown annual yield	$i = \frac{1}{365}$ th of $10.3\% = \frac{0.103}{365}$
$t = 1$ year	$n = 1$ year $= 365$ days

(one year, not fifteen years—its *annual* yield)

$$\begin{aligned} FV\,(\text{simple interest}) &= FV\,(\text{compounded monthly}) \\ P(1 + rt) &= P(1 + i)^n \\ 2{,}500(1 + r \cdot 1) &= 2{,}500(1 + 0.103/365)^{365} && \textbf{substituting} \\ (1 + r \cdot 1) &= (1 + 0.103/365)^{365} && \textbf{dividing by 2,500} \\ r &= (1 + 0.103/365)^{365} - 1 && \textbf{solving for } r \\ r &= 0.10847\ldots \approx 10.85\% && \textbf{rounding} \end{aligned}$$

> As a rough check, notice that the annual yield is slightly higher than the compound rate, as it should be.

By tradition, we round this to the nearest hundredth of a percent, so the annual yield is 10.85%. This means that in one year's time, 10.3% compounded daily has the same effect as 10.85% simple interest. For any period of time longer than a year, 10.3% compounded daily will yield *more* interest than 10.85% simple interest would, because compound interest gives interest on interest.

Notice that in Example 7, the principal of $2,500 canceled out. If the principal were two dollars or two million dollars, it would still cancel out, and the annual yield of 10.3% compounded daily would still be 10.85%. The principal does not matter in computing the annual yield. Also notice that the fifteen years did not enter into the calculation—*annual* yield is always based on a *one-year* period.

EXAMPLE 8

FINDING THE ANNUAL YIELD Find the annual yield corresponding to a nominal rate of 8.4% compounded monthly.

SOLUTION

We are told neither the principal nor the time, but (as discussed above) these variables do not affect the annual yield.

Compounding monthly, we have $i = 1/12$ of $8.4\% = \frac{0.084}{12}$, $n = 1$ year $= 12$ months, and $t = 1$ year.

$$\begin{aligned} FV\,(\text{simple interest}) &= FV(\text{compounded monthly}) \\ P(1 + rt) &= P(1 + i)^n \\ (1 + rt) &= (1 + i)^n && \textbf{dividing by } P \\ (1 + r \cdot 1) &= (1 + .084/12)^{12} && \textbf{annual, so } t = 1 \textbf{ year and } n = 12 \textbf{ months} \\ r &= (1 + .084/12)^{12} - 1 && \textbf{solving for } r \\ r &= 0.08731\ldots \approx 8.73\% && \textbf{rounding} \end{aligned}$$

> As a rough check, notice that the annual yield is slightly higher than the compound rate, as it should be.

TOPIC X BENJAMIN FRANKLIN'S GIFT
COMPOUND INTEREST IN THE REAL WORLD

In 1789, when Benjamin Franklin was 83, he added a codicil to his will. That codicil was meant to aid young people, in Boston (where Franklin grew up), and Philadelphia (where he had been President of the state of Pennsylvania) over a time span of two hundred years.

In his codicil, Franklin wrote of his experience as an apprentice printer, of the friends who had loaned him the money to set up his own printing business, and of the importance of craftsmen to a city. He then went on to say, "To this End I devote Two thousand Pounds Sterling, which I give, one thousand thereof to the Inhabitants of the Town of Boston, in Massachusetts, and the other thousand to the Inhabitants of the City of Philadelphia, in Trust to and for the Uses, Interests and Purposes hereinafter mentioned and declared." At that time, £1,000 was the equivalent of about $4,500.

Stock Montage/Stock Montage/Getty Images

Franklin's plan called for each town to use the money as a loan fund for 100 years. The money would be loaned out to young tradesmen to help them start their own businesses. He calculated that at the end of 100 years, each fund would have grown to £131,000 (or about $582,000 in 1892 dollars). At that point, one-fourth of the money would continue to be used for a loan fund. The remainder would be used for public works.

He calculated that at the end of 200 years, each of the two city's loan funds would have grown to £4,061,000 (or about $7,000,000 in 1992 dollars). At that point, he called for the money to be given to the two cities and states.

Twenty-two tradesmen, including bricklayers, hairdressers, jewelers, and tanners, applied for loans from Franklin's fund in the first month after Franklin's death. The fund remained popular until the onset of the Industrial Revolution in the early 1800s, when young people stopped becoming tradesmen with their own shops. Instead, most went to work as mechanics in factories.

When Boston's fund reached its hundredth anniversary in 1891, its value was approximately $391,000. One-fourth of that money continued to be used for a loan fund, as Franklin wished. The city of Boston decided to use the remainder to build a trade school, because it fit with Franklin's goal of helping young people.

Legal problems delayed the school's founding, but the Benjamin Franklin Institute of Technology was opened in Boston in 1908. At that point, the trade school part of the Boston fund had risen to $432,367, and the loan fund part had grown to $163,971.

The Institute continues to operate today. It has almost 400 students, 90% of whom receive financial aid. It awards Bachelor of Science degrees, Associate in Engineering degrees, and Associate in Science degrees. Ninety-eight percent of its graduates find work in their fields within six months of graduation.

Pennsylvania used its share of the money to fund the Franklin Institute of Philadelphia. Originally, the institute promoted the "mechanical arts." Now it houses a planetarium, an IMAX theater, and a science museum.

Exercises 57–62 explore some aspects of Franklin's bequest.

Source: The Benjamin Franklin Institute of Technology and the Franklin Institute of Philadelphia.

In Example 8, we found that 8.4% compounded monthly generates an annual yield of 8.73%. This means that 8.4% compounded monthly has the same effect as does 8.73% simple interest in one year's time. Furthermore, as Figure 5.6 indicates, 8.4% compounded monthly has the same effect as does 8.73% compounded annually *for any time period*.

For a Principal of $1,000	After 1 Year	After 10 Years
FV at 8.4% compounded monthly	$1,087.31	$2,309.60
FV at 8.73% simple interest	$1,087.30	$1,873.00
FV at 8.73% compounded annually	$1,087.30	$2,309.37

FIGURE 5.6 What annual yield means.

Notice that all three rates have the same future value after one year (the 1¢ difference is due to rounding off the annual yield to 8.73%). However, after ten years, the simple interest has fallen way behind, while the 8.4% compounded monthly and 8.73% compounded annually remain the same (except for the round-off error). This always happens. The annual yield is the simple interest rate that has the same future value that the compound rate would have in one year. It is also the annually compounded rate that has the same future value that the nominal rate would have after any amount of time.

An annual yield formula does exist, but the annual yield can be calculated efficiently without it, as was shown above. The formula is developed in the exercises. See Exercise 51.

5.2 Exercises

In Exercises 1–6, find the periodic rate that corresponds to the given compound rate, if the rate is compounded (a) quarterly, (b) monthly, (c) daily, (d) biweekly (every two weeks), and (e) semimonthly (twice a month). Do not round off the periodic rate.

▶ **1.** 12% **2.** 6%
3. 3.1% **4.** 6.8%
5. 9.7% ▶ **6.** 10.1%

In Exercises 7–10, find the number of periods that corresponds to the given time span, if a period is (a) a quarter of a year, (b) a month, and (c) a day. (Ignore leap years.)

7. $8\frac{1}{2}$ years ▶ **8.** $9\frac{3}{4}$ years
▶ **9.** 30 years **10.** 45 years

In Exercises 11–16, (a) find and (b) interpret the future value of the given amount.

▶ **11.** $3,000 at 6% compounded annually for 15 years
12. $7,300 at 7% compounded annually for 13 years
▶ **13.** $5,200 at $6\frac{3}{4}$% compounded quarterly for $8\frac{1}{2}$ years
14. $36,820 at $7\frac{7}{8}$% compounded quarterly for 4 years
15. $1,960 at $4\frac{1}{8}$% compounded daily for 17 years (ignore leap years)
▶ **16.** $12,350 at 6% compounded daily for 10 years and 182 days (ignore leap years)

In Exercises 17–20, (a) find and (b) interpret the annual yield corresponding to the given nominal rate.

▶ **17.** 8% compounded monthly
18. $5\frac{1}{2}$% compounded quarterly
19. $4\frac{1}{4}$% compounded daily
▶ **20.** $12\frac{5}{8}$% compounded daily

In Exercises 21 and 22, find and interpret the annual yield corresponding to the given nominal rate.

▶ **21.** 10% compounded (a) quarterly, (b) monthly, and (c) daily
22. $12\frac{1}{2}$% compounded (a) quarterly, (b) monthly, and (c) daily

In Exercises 23–26, (a) find and (b) interpret the present value that will generate the given future value.

▶ **23.** $1,000 at 8% compounded annually for 7 years
24. $9,280 at $9\frac{3}{4}$% compounded monthly for 2 years and 3 months
▶ **25.** $3,758 at $11\frac{7}{8}$% compounded monthly for 17 years and 7 months
▶ **26.** $4,459 at $10\frac{3}{4}$% compounded quarterly for 4 years

27. $10,000 is deposited in an account in which it earns 10% interest compounded monthly. No principal or interest is withdrawn from the account. Instead, both continue to earn interest over time. Find the account balance after six months:
a. using the simple interest future value formula to compute the balance at the end of each compounding period.
b. using the compound interest formula.

▶ **28.** $20,000 is deposited in an account in which it earns 9.5% interest compounded quarterly. No principal or interest is withdrawn from the account. Instead, both continue to earn interest over time. Find the account balance after one year:
a. using the simple interest future value formula to compute the balance at the end of each compounding period.
b. using the compound interest formula.

29. $15,000 is deposited in an account in which it earns 6% interest compounded annually. No principal or interest is withdrawn from the account. Instead, both continue to earn interest over time. Find the account balance after three years:
a. using the simple interest future value formula to compute the balance at the end of each compounding period.
b. using the compound interest formula.

▶ Selected exercises available online at **www.webassign.net/brookscole**

30. $30,000 is deposited in an account in which it earns 10% interest compounded annually. No principal or interest is withdrawn from the account. Instead, both continue to earn interest over time. Find the account balance after four years:

a. using the simple interest future value formula to compute the balance at the end of each compounding period.

b. using the compound interest formula.

▶ **31.** Donald Trumptobe decided to build his own dynasty. He is considering specifying in his will that at his death, $10,000 would be deposited into a special account that would earn a guaranteed 6% interest compounded daily. This money could not be touched for 100 years, at which point it would be divided among his heirs. Find the future value.

▶ **32.** How much would Donald Trumptobe in Exercise 31 have to have deposited so that his heirs would have $50,000,000 or more in 100 years if his money earns 7% compounded monthly?

▶ **33.** Donald Trumptobe in Exercise 31 predicts that in 100 years, he will have four generations of offspring (that is, children, grandchildren, great-grandchildren, and great-great-grandchildren). He estimates that each person will have two children. How much will he have to have deposited so that each of his great-great-grandchildren would have $1,000,000 or more in 100 years if his money earns 7.5% compounded monthly?

34. Donald Trumptobe in Exercise 31 predicts that in 100 years, he will have four generations of offspring (that is, children, grandchildren, great-grandchildren, and great-great-grandchildren). He estimates that each person will have two children. How much will he have to have deposited so that each of his great-great-grandchildren would have $1,000,000 or more in 100 years if his money earns 9.25% compounded daily?

▶ **35.** When Jason Levy was born, his grandparents deposited $3,000 into a special account for Jason's college education. The account earned $6\frac{1}{2}$% interest compounded daily.

a. How much will be in the account when Jason is 18?

b. If, on turning 18, Jason arranges for the monthly interest to be sent to him, how much will he receive each thirty-day month?

c. How much would be in the account when Jason turns 18 if his grandparents started Jason's savings account on his tenth birthday?

36. When Alana Cooper was born, her grandparents deposited $5,000 into a special account for Alana's college education. The account earned $7\frac{1}{4}$% interest compounded daily,

a. How much will be in the account when Alana is 18?

b. If, on turning 18, Alana arranges for the monthly interest to be sent to her, how much will she receive each thirty-day month?

c. How much would be in the account when Alana turns 18 if his grandparents started Alana's savings account on her tenth birthday?

For Exercises 37–40, note the following information: An ***Individual Retirement Account (IRA)*** *is an account in which the saver does not pay income tax on the amount deposited but is not allowed to withdraw the money until retirement. (The saver pays income tax at that point, but his or her tax bracket is much lower then.)*

37. At age 27, Lauren Johnson deposited $1,000 into an IRA, in which it earns $7\frac{7}{8}$% compounded monthly.

a. What will it be worth when she retires at 65?

b. How much would the IRA be worth if Lauren didn't set it up until she was 35?

▶ **38.** At age 36, Dick Shoemaker deposited $2,000 into an IRA, in which it earns $8\frac{1}{8}$% compounded semiannually.

a. What will it be worth when he retires at 65?

b. How much would the IRA be worth if Dick didn't set it up until he was 48?

▶ **39.** Marlene Silva wishes to have an IRA that will be worth $100,000 when she retires at age 65.

a. How much must she deposit at age 35 at $8\frac{3}{8}$% compounded daily?

b. If, at age 65, she arranges for the monthly interest to be sent to her, how much will she receive each thirty-day month?

40. David Murtha wishes to have an IRA that will be worth $150,000 when he retires at age 65.

a. How much must he deposit at age 26 at $6\frac{1}{8}$% compounded daily?

b. If, at age 65, he arranges for the monthly interest to be sent to him, how much will he receive each thirty-day month?

For Exercises 41–46, note the following information: A certificate of deposit (CD) is an agreement between a bank and a saver in which the bank guarantees an interest rate and the saver commits to leaving his or her deposit in the account for an agreed-upon period of time.

▶ **41.** First National Bank offers two-year CDs at 9.12% compounded daily, and Citywide Savings offers two-year CDs at 9.13% compounded quarterly. Compute the annual yield for each institution and determine which is more advantageous for the consumer.

▶ **42.** National Trust Savings offers five-year CDs at 8.25% compounded daily, and Bank of the Future offers five-year CDs at 8.28% compounded annually. Compute the annual yield for each institution, and determine which is more advantageous for the consumer.

43. Verify the annual yield for the five-year certificate quoted in the bank sign in Figure 5.7 on page 352, using interest that is compounded daily and:

a. 365-day years

b. 360-day years

c. Various combinations of 360-day and 365-day years

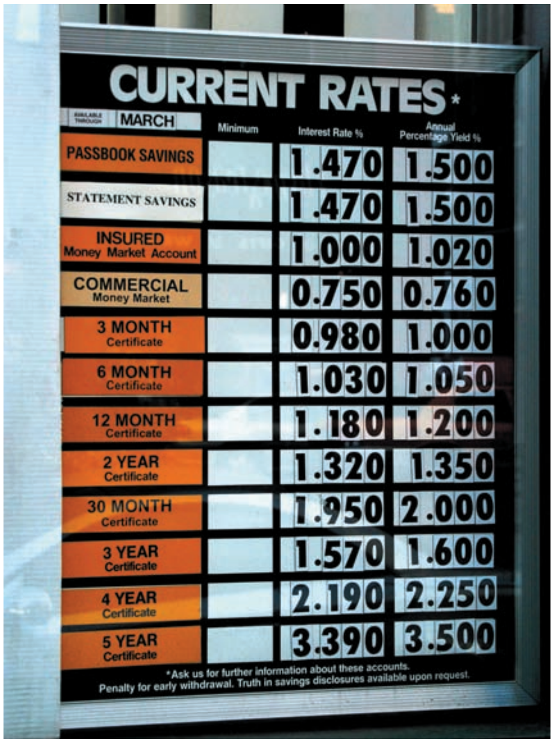

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FIGURE 5.7 A bank sign for Exercise 43.

▶ **44.** Verify the yield for the one-year CDs quoted in Figure 5.8.

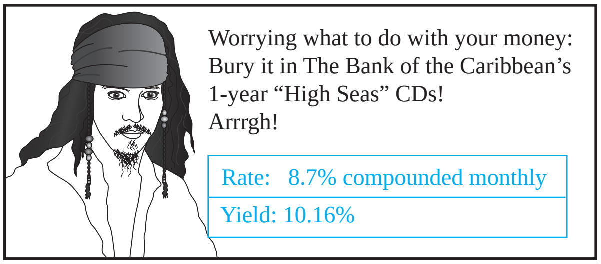


FIGURE 5.8 A savings bank advertisement for Exercise 44.

45. Verify the yield for the two-year CDs quoted in Figure 5.9.

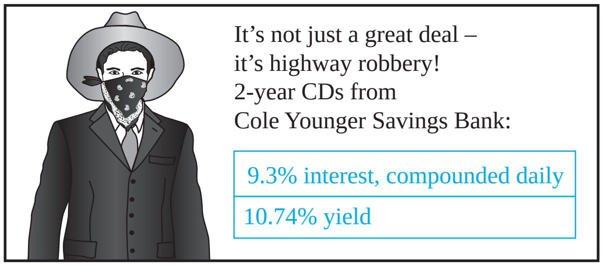


FIGURE 5.9 A savings bank advertisement for Exercise 45.

46. Verify the yield for the six-month CDs quoted in the savings bank advertisement on page 347.

▶ **47.** Recently, Bank of the West offered six-month CDs at 5.0% compounded monthly.

a. Find the annual yield of one of these CDs.
b. How much would a \$1,000 CD be worth at maturity?
c. How much interest would you earn?
d. What percent of the original \$1,000 is this interest?
e. The answer to part (d) is not the same as that of part (a). Why?
f. The answer to part (d) is close to, but not exactly half, that of part (a). Why?

(*Source:* Bank of the West.)

▶ **48.** Recently, Bank of the West offered six-month CDs at 5.0% interest compounded monthly and one-year CDs at 5.20% interest compounded monthly. Maria Ruiz bought a six-month \$2,000 CD, even though she knew she would not need the money for at least a year, because it was predicted that interest rates would rise.

a. Find the future value of Maria's CD.
b. Six months later, Maria's CD has come to term, and in the intervening time, interest rates have risen. She reinvests the principal and interest from her first CD in a second six-month CD that pays 5.31% interest compounded monthly. Find the future value of Maria's second CD.
c. Would Maria have been better off if she had bought a one-year CD instead of two six-month CDs?
d. If Maria's second CD pays 5.46% interest compounded monthly, rather than 5.31%, would she be better off with the two six-month CDs or the one-year CD?

(*Source:* Bank of the West.)

▶ **49.** CD interest rates vary significantly with time. They hit a historical high in 1981, when the average rate was 16.7%. In 2000, it was 8.1%. In 2009, it was 2%. Find the interest earned by a \$1000 two-year CD with interest compounded quarterly:

a. in 1981 **b.** in 2000 **c.** in 2009

50. Use the data in Exercise 49 to find the yield of a \$10,000 ten-year CD with interest compounded daily:

a. in 1981 **b.** in 2000 **c.** in 2009

51. Develop a formula for the annual yield of a compound interest rate.

HINT: Follow the procedure given in Example 8, but use the letters i and n in the place of numbers.)

In Exercises 52–56, use the formula found in Exercise 51 to compute the annual yield corresponding to the given nominal rate.

▶ **52.** $9\frac{1}{2}$% compounded monthly

53. $7\frac{1}{4}$% compounded quarterly

54. $12\frac{3}{8}$% compounded daily

55. $5\frac{5}{8}\%$ compounded (a) semiannually, (b) quarterly, (c) monthly, (d) daily, (e) biweekly, and (f) semimonthly.

56. $10\frac{1}{2}\%$ compounded (a) semiannually, (b) quarterly, (c) monthly, (d) daily, (e) biweekly, and (f) semimonthly.

Exercises 57–62 refer to Benjamin Franklin's gift, discussed on page 349.

▶ **57.** Would Benjamin Franklin have used simple or compound interest in projecting the 100-year future value of his bequest? Why? What interest rate did he use in calculating the 100-year future value? Use pounds, not dollars, in your calculation.

▶ **58.** Would Benjamin Franklin have used simple or compound interest in projecting the 200-year future value of his bequest? Why? What interest rate did he use in calculating the 200-year future value? Use pounds, not dollars, in your calculation.

59. What interest rate did Franklin's Boston bequest actually earn in the first hundred years? Use dollars, not pounds, in your calculation.

▶ **60.** What interest rate did Franklin's Boston bequest actually earn from 1891 to 1908? Use dollars, not pounds, in your calculation.

61. After 100 years, Franklin's total bequest to Boston was worth much more than his original bequest. The future value was what percentage of the original bequest? Use dollars, not pounds, in your calculation.

▶ **62.** In 1908, Franklin's total bequest to Boston was worth much more than his original bequest. The future value was what percentage of the original bequest? Use dollars, not pounds, in your calculation.

Answer the following questions using complete sentences and your own words.

• CONCEPT QUESTIONS

63. Explain how compound interest is based on simple interest.

64. Why is there no work involved in finding the annual yield of a given simple interest rate?

65. Why is there no work involved in finding the annual yield of a given compound interest rate when that rate is compounded annually?

66. Which should be higher: the annual yield of a given rate compounded quarterly or compounded monthly? Explain why, *without* performing any calculations or referring to any formulas.

67. Why should the annual yield of a given compound interest rate be higher than the compound rate? Why should it be only slightly higher? Explain why, *without* performing any calculations or referring to any formulas.

68. Explain the difference between simple interest and compound interest.

69. If money earns compound interest, why must the future value be *slightly* higher than the principal, after a short amount of time? Why must the future value be *much* higher than the principal after a long amount of time?

70. *Money* magazine and other financial publications regularly list the top-paying money-market funds, the top-paying bond funds, and the top-paying CDs and their yields. Why do they list yields rather than interest rates and compounding periods?

71. Equal amounts are invested in two different accounts. One account pays simple interest, and the other pays compound interest at the same rate. When will the future values of the two accounts be the same?

72. Suppose you invest some money in a new account that pays 5% interest compounded annually, and you do not make any further deposits into or withdrawals from that account. Which of the following must be true?

- The account grows by the same dollar amount in the second year as it did in the first year.
- The account grows by a larger dollar amount in the second year as it did in the first year.
- The account grows by a smaller dollar amount in the second year as it did in the first year. Why?

WEB PROJECT

73. Go to the web sites of four different banks.

a. For each bank, try to determine the following:

- The interest rate
- The compounding frequency
- The annual yield (also called the *annual percentage yield*, or A.P.Y.)

for CDs for two different terms. (Use the same terms for each bank.) Some banks might not give all of the above information, but all will give the annual yield, as required by federal law.

b. If a bank omits either the interest rate or the compounding frequency, calculate the omitted information.

c. If a bank omits both the interest rate and the compounding frequency, assume that the compounding frequency is daily and calculate the interest rate.

d. If a bank omits none of the information, verify the annual yield.

e. Which bank offers the best deal? Why?

Some useful links for this web project are listed on the text web site: **www.cengage.com/math/johnson**

• PROJECT

74. Suppose you have $1,000 invested at 5% annual interest and you do not make any further deposits into that account. Let x measure years after you invested the money, and let y measure the future value of the account. Draw a graph that shows the relationship between x and y, for $0 \le x \le 5$, if the interest rate is

a. simple interest

b. compounded annually

c. compounded daily

d. Discuss the differences between the three graphs. In your discussion, address the following:

- The difference in their shapes
- Where they coincide
- Which graph is above the others
- Which is below the others

Doubling Time with a TI's TVM Application

Simple interest is a very straightforward concept. If an account earns 5% simple interest, then 5% of the principal is paid for each year that principal is in the account. In one year, the account earns 5% interest; in two years, it earns 10% interest; in three years, it earns 15% interest, and so on.

It is not nearly so easy to get an intuitive grasp of compound interest. If an account earns 5% interest compounded daily, then it does not earn only 5% interest in one year, and it does not earn only 10% interest in two years.

Annual yield is one way of gaining an intuitive grasp of compound interest. If an account earns 5% interest compounded daily, then it will earn 5.13% interest in 1 year (because the annual yield is 5.13%), but it does not earn merely $2 \cdot 5.13\% = 10.26\%$ interest in two years.

Doubling time is another way of gaining an intuitive grasp of compound interest. **Doubling time** is the amount of time it takes for an account to double in value; that is, it's the amount of time it takes for the future value to become twice the principal. To find the doubling time for an account that earns 5% interest compounded daily, substitute $2P$ for the future value and solve the resulting equation.

$$FV = P(1 + i)^n \quad \textbf{Compound Interest Formula}$$

$$2P = P\left(1 + \frac{0.05}{365}\right)^n \quad \textbf{substituting}$$

$$2 = \left(1 + \frac{0.05}{365}\right)^n \quad \textbf{dividing by } P$$

Solving this equation for n involves mathematics that will be covered in Section 10.0B. For now, we will use the TI-83/84's "Time Value of Money" (TVM) application.

EXAMPLE 9

FINDING DOUBLING TIME Use a TI–83/84's TVM application to find the doubling time for an account that earns 5% interest compounded daily.

SOLUTION

1. Press APPS, select option 1: "Finance", and press ENTER.
2. Select option 1: "TVM Solver", and press ENTER.
3. Enter appropriate values for the variables:
   - N is the number of compounding periods. This is the number we're trying to find. We temporarily enter 0. Later, we'll solve for the actual value of N.
   - I% is the *annual* interest rate (not a periodic rate), as a percent, so enter 5 for I%. Note that we do not convert to a decimal or a periodic rate.
   - PV is the present value. The size of the present value doesn't matter, so we'll make it \$1. However, it's an outgoing amount of money (since we give it to the bank), so we

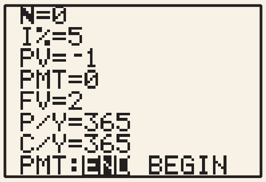


FIGURE 5.10
Preparing the TVM screen.

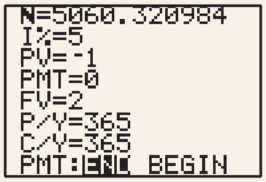


FIGURE 5.11
Solving for N.

enter –1 for PV. You must enter a negative number for any outgoing amount of money.

- PMT is the payment. There are no payments here, so we enter 0.
- FV is the future value. We're looking for the amount of time that it takes the present value to double, so enter 2. This is an incoming amount of money (since the bank gives it to us), so we use 2 rather than –2.
- P/Y is the number of periods per year. The interest is compounded daily, so there are 365 periods per year. Enter 365 for P/Y.
- C/Y is automatically made to be the same as P/Y. In this text, we will never encounter a situation in which C/Y is different from P/Y.

See Figure 5.10.

4. To solve for N, use the arrow buttons to highlight the 0 that we entered for N. Then press ALPHA SOLVE. (Pressing ALPHA makes the ENTER button becomes the SOLVE button.) As a result, we find that N is 5060.320984. See Figure 5.11.

This is the number of *days* for the money to double (a period is a day, because of our P/Y entry). This means that it takes about $5060.320984/365 = 13.86\ldots \approx 13.9$ years for money invested at 5% interest compounded daily to double.

EXERCISES

▶ **75.** If $1,000 is deposited into an account that earns 5% interest compounded daily, the doubling time is approximately 5,061 days.

a. Find the amount in the account after 5,061 days.

b. Find the amount in the account after $2 \cdot 5{,}061$ days.

c. Find the amount in the account after $3 \cdot 5{,}061$ days.

d. Find the amount in the account after $4 \cdot 5{,}061$ days.

e. What conclusion can you make?

▶ **76.** Do the following. (Give the number of periods and the number of years, rounded to the nearest hundredth.)

a. Find the doubling time corresponding to 5% interest compounded annually.

b. Find the doubling time corresponding to 5% interest compounded quarterly.

c. Find the doubling time corresponding to 5% interest compounded monthly.

d. Find the doubling time corresponding to 5% interest compounded daily.

e. Discuss the effect of the compounding period on doubling time.

77. Do the following. (Give the number of periods and the number of years, rounded to the nearest hundredth.)

a. Find the doubling time corresponding to 6% interest compounded annually.

b. Find the doubling time corresponding to 7% interest compounded annually.

c. Find the doubling time corresponding to 10% interest compounded annually.

d. Discuss the effect of the interest rate on doubling time.

▶ **78.** If you invest $10,000 at 8.125% interest compounded daily, how long will it take for you to accumulate $15,000? How long will it take for you to accumulate $100,000? (Give the number of periods and the number of years, rounded to the nearest hundredth.)

79. If you invest $15,000 at $9\frac{3}{8}$% interest compounded daily, how long will it take for you to accumulate $25,000? How long will it take for you to accumulate $100,000? (Give the number of periods and the number of years, rounded to the nearest hundredth.)

80. If you invest $20,000 at $6\frac{1}{4}$% interest compounded daily, how long will it take for you to accumulate $30,000? How long will it take for you to accumulate $100,000? (Give the number of periods and the number of years, rounded to the nearest hundredth.)

5.3 Annuities

OBJECTIVES

- Understand what an annuity is
- Use the Annuity Formulas
- Determine how to use an annuity to save for retirement.

Many people have long-term financial goals and limited means with which to accomplish them. Your goal might be to save $3,000 over the next four years for your college education, to save $10,000 over the next ten years for the down payment on a home, to save $30,000 over the next eighteen years to finance your new baby's college education, or to save $300,000 over the next forty years for your retirement. It seems incredible, but each of these goals can be achieved by saving only $50 a month (if interest rates are favorable)! All you need to do is start an annuity. We'll explore annuities in this section.

An **annuity** is simply a sequence of equal, regular payments into an account in which each payment receives compound interest. Because most annuities involve relatively small periodic payments, they are affordable for the average person. Over longer periods of time, the payments themselves start to amount to a significant sum, but it is really the power of compound interest that makes annuities so amazing. If you pay $50 a month into an annuity for the next forty years, then your total payment is

$$\frac{\$50}{\cancel{\text{month}}} \cdot \frac{12\ \cancel{\text{months}}}{\cancel{\text{year}}} \cdot 40\ \cancel{\text{years}} = \$24{,}000$$

A **Christmas Club** is an annuity that is set up to save for Christmas shopping. A Christmas Club participant makes regular equal deposits, and the deposits and the resulting interest are released to the participant in December when the money is needed. Christmas Clubs are different from other annuities in that they span a short amount of time—a year at most—and thus earn only a small amount of interest. (People set them up to be sure that they are putting money aside rather than to generate interest.) Our first few examples will deal with Christmas Clubs, because their short time span makes it possible to see how an annuity actually works.

Annuities as Compound Interest, Repeated

EXAMPLE 1

UNDERSTANDING ANNUITIES On August 12, Patty Leitner joined a Christmas Club through her bank. For the next three months, she would deposit $200 at the beginning of each month. The money would earn $8\frac{3}{4}\%$ interest compounded monthly, and on December 1, she could withdraw her money for shopping. Use the compound interest formula to find the future value of the account.

SOLUTION

We are given $P = 200$ and $r = \frac{1}{12}$th of $8\frac{3}{4}\% = \frac{0.0875}{12}$.

First, calculate the future value of the first payment (made on September 1). Use $n = 3$ because it will receive interest during September, October, and November.

$$FV = P(1 + i)^n \quad \text{the Compound Interest Formula}$$
$$= 200\left(1 + \frac{0.0875}{12}\right)^3 \quad \text{substituting}$$
$$= 204.40698 \approx \$204.41 \quad \text{rounding}$$

Next, calculate the future value of the second payment (made on October 1). Use $n = 2$ because it will receive interest during October and November.

$$FV = P(1 + i)^n \quad \text{the Compound Interest Formula}$$
$$= 200\left(1 + \frac{0.0875}{12}\right)^2 \quad \text{substituting}$$
$$= 202.9273 \approx \$202.93 \quad \text{rounding}$$

Next, calculate the future value of the third payment (made on November 1). Use $n = 1$ because it will receive interest during November.

$$FV = P(1 + i)^n \quad \text{the Compound Interest Formula}$$
$$= 200\left(1 + \frac{0.0875}{12}\right)^1 \quad \text{substituting}$$
$$= 201.45833 \approx \$201.46 \quad \text{rounding}$$

The payment schedule and interest earned are illustrated in Figure 5.12.

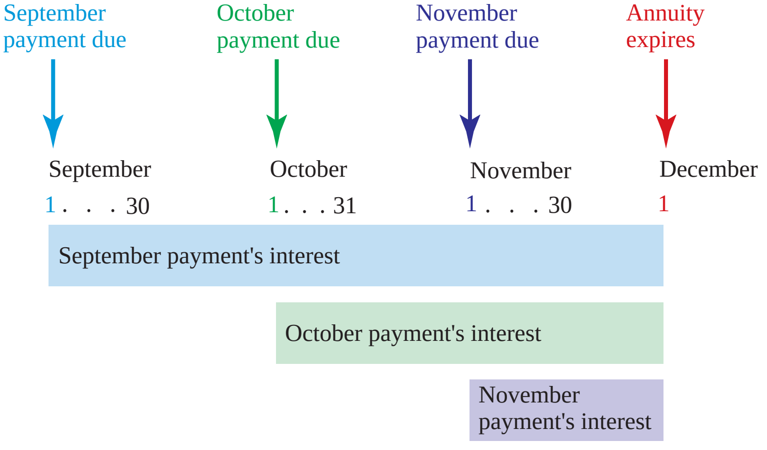


FIGURE 5.12 Patty's payment schedule and interest earning periods.

The future value of Patty's annuity is the sum of the future values of each payment:

$$FV \approx \$204.41 + \$202.93 + \$201.46$$
$$= \$608.80$$

Patty's deposits will total \$600.00. She will earn \$8.80 interest on her deposits.

The **payment period** of an annuity is the time between payments; in Example 1, the payment period was one month. The **term** is the time from the beginning of the first payment period to the end of the last; the term of Patty's Christmas Club was three months. When an annuity has **expired** (that is, when its term is over), the entire account or any portion of it may be withdrawn. Most annuities are **simple,** that is, their compounding period is the same as their payment period (for example, if payments are made monthly, then interest is compounded monthly). In this book, we will work only with simple annuities.

Ordinary Annuities and Annuities Due

An **annuity due** is one in which each payment is due at the beginning of its time period. Patty's annuity in Example 1 was an annuity due, because the payments were due at the *beginning* of each month. An **ordinary annuity** is an annuity for which each payment is due at the end of its time period. As the name implies, this form of annuity is more typical. As we will see in the next example, the difference is one of timing.

EXAMPLE **2**

UNDERSTANDING THE DIFFERENCE BETWEEN AN ORDINARY ANNUITY AND AN ANNUITY DUE Dan Bach also joined a Christmas Club through his bank. His was just like Patty's except that his payments were due at the end of each month, and his first payment was due September 30. Use the Compound Interest Formula to find the future value of the account.

SOLUTION

This is an *ordinary* annuity because payments are due at the *end* of each month. Interest is compounded monthly. From Example 1, we know that $P = 200$, $i = \frac{1}{12}$ of $8\frac{3}{4}\% = \frac{0.0875}{12}$.

To calculate the future value of the first payment (made on September 30), use $n = 2$. This payment will receive interest during October and November.

$$FV = P(1 + i)^n \quad \text{the Compound Interest Formula}$$
$$= 200\left(1 + \frac{0.0875}{12}\right)^2 \quad \text{substituting}$$
$$= 202.9273 \approx \$202.93 \quad \text{rounding}$$

To calculate the future value of the second payment (made on October 31), use $n = 1$. This payment will receive interest during November.

$$FV = P(1 + i)^n \quad \text{the Compound Interest Formula}$$
$$= 200\left(1 + \frac{0.0875}{12}\right)^1 \quad \text{substituting}$$
$$= 201.45833 \approx \$201.46 \quad \text{rounding}$$

To calculate the future value of the second payment (made on November 30), note that no interest is earned, because the payment is due November 30 and the annuity expires December 1. Therefore,

$$FV = \$200$$

Dan's payment schedule and interest payments are illustrated in Figure 5.13.

The future value of Dan's annuity is the sum of the future values of each payment:

$$FV \approx \$200 + \$201.46 + \$202.93$$
$$= \$604.39$$

Dan earned \$4.39 interest on his deposits.

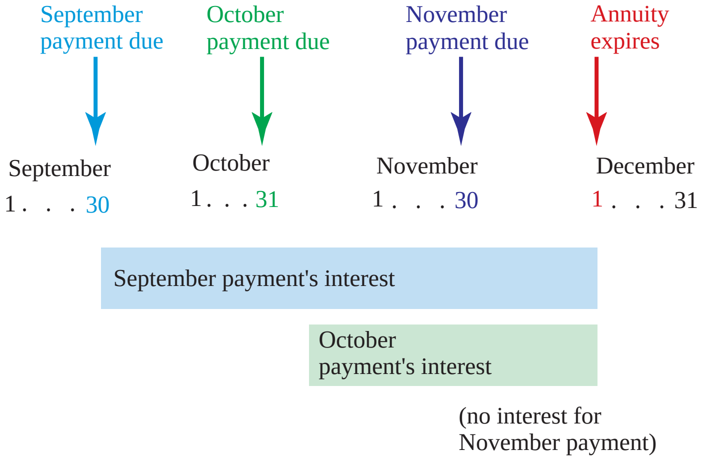


FIGURE 5.13 Dan's payment schedule and interest earning periods.

In Examples 1 and 2, why did Patty earn more interest than Dan? The reason is that each of her payments was made a month earlier and therefore received an extra month's interest. In fact, we could find the future value of Patty's account by giving Dan's future value one more month's interest.

$$\text{Patty's } FV = \text{Dan's } FV \cdot (1 + i)^1$$

$$\$608.80 = \$604.39 \cdot \left(1 + \frac{0.0875}{12}\right)^1$$

More generally, we can find the future value of an annuity due by giving an ordinary annuity's future value one more month's interest.

$$FV(\text{due}) = FV(\text{ordinary}) \cdot (1 + i)^1$$

The difference between an ordinary annuity and an annuity due is strictly a timing difference, because any ordinary annuity in effect will become an annuity due if you leave all funds in the account for one extra period.

The Annuity Formulas

The procedure followed in Examples 1 and 2 reflects what actually happens with annuities, and it works fine for a small number of payments. However, most annuities are long-term, and the procedure would become tedious if we were computing the future value after forty years. Because of this, long-term annuities are calculated with their own formula.

For an *ordinary* annuity with payment *pymt*, a periodic rate *i*, and a term of *n* payments, the first payment receives interest for $n - 1$ periods. The payment is made at the end of the first period, so it received no interest for that one period. Its future value is

$$FV(\text{first } pymt) = pymt(1 + i)^{n-1}$$

The last payment receives no interest (under the annuity), because it is due at the end of the last period and it expires the next day. Its future value is

$$FV(\text{last } pymt) = pymt$$

The next-to-last payment receives one period's interest, so its future value is

$$FV(\text{next-to-last } pymt) = pymt(1 + i)^1$$

The future value of the annuity is the sum of all of these future values of individual payments:

$$FV = pymt + pymt(1 + i)^1 + pymt(1 + i)^2 + \cdots + pymt(1 + i)^{n-1}$$

To get a short-cut formula from all this, we will multiply each side of this equation by $(1 + i)$ and then subtract the original equation from the result. This leads to a lot of cancelling.

$$FV(1 + i) = \cancel{pymt(1 + i)} + \cancel{pymt(1 + i)^2} + \cdots + \cancel{pymt(1 + i)^{n-1}} + pymt(1 + i)^n$$

minus: $$FV = pymt + \cancel{pymt(1 + i)^1} + \cancel{pymt(1 + i)^2} + \cdots + \cancel{pymt(1 + i)^{n-1}}$$

equals: $$FV(1 + i) - FV = pymt(1 + i)^n - pymt \quad \textbf{subtracting}$$

$$FV(1 + i - 1) = pymt[(1 + i)^n - 1] \quad \textbf{factoring}$$

$$FV(i) = pymt[(1 + i)^n - 1]$$

$$FV = pymt\,\frac{(1 + i)^n - 1}{i} \quad \textbf{dividing}$$

This is the future value of the ordinary annuity.

ORDINARY ANNUITY FORMULA

The future value FV of an ordinary annuity with payment size $pymt$, a periodic rate i, and a term of n payments is

$$FV(\text{ord}) = pymt\frac{(1 + i)^n - 1}{i}$$

As we saw in Examples 1 and 2, the future value of an annuity due is the future value of an ordinary annuity plus one more period's interest.

$$FV(\text{due}) = FV(\text{ord}) \cdot (1 + i)$$

This gives us a formula for the future value of an annuity due.

ANNUITY DUE FORMULA

The future value FV of an annuity due with payment size $pymt$, a periodic rate i, and a term of n payments is

$$\begin{aligned} FV(\text{due}) &= FV(\text{ord}) \cdot (1 + i) \\ &= pymt\frac{(1 + i)^n - 1}{i}(1 + i) \end{aligned}$$

Tax-Deferred Annuities

A **tax-deferred annuity (TDA)** is an annuity that is set up to save for retirement. Money is automatically deducted from the participant's paychecks until retirement, and the federal (and perhaps state) tax deduction is computed *after* the annuity payment has been deducted, resulting in significant tax savings. In some cases, the employer also makes a regular contribution to the annuity.

The following example involves a long-term annuity. Usually, the interest rate of a long-term annuity varies somewhat from year to year. In this case, calculations must be viewed as predictions, not guarantees.

EXAMPLE 3

USING AN ANNUITY TO SAVE FOR RETIREMENT Tom and Betty decided that they should start saving for retirement, so they set up a tax-deferred annuity. They arranged to have \$200 taken out of each of Tom's monthly checks, which will earn $8\frac{3}{4}\%$ interest. Because of the tax-deferring effect of the TDA, Tom's take-home pay went down by only \$115. Tom just had his thirtieth birthday, and his ordinary annuity will come to term when he is 65.

a. Find the future value of the annuity.
b. Find Tom's contribution and the interest portion.

SOLUTION

a. This is an ordinary annuity, with $pymt = 200$, $i = \frac{1}{12}$th of $8\frac{3}{4}\% = 0.0875/12$, and $n = 35$ years $= 35$ years $\cdot$ 12 months/year $= 420$ monthly payments.

$$FV(\text{ord}) = pymt\frac{(1+i)^n - 1}{i} \quad \text{the Ordinary Annuity Formula}$$

$$= 200\frac{(1+0.0875/12)^{420} - 1}{0.0875/12} \quad \text{substituting}$$

$$\approx \$552{,}539.96 \quad \text{rounding}$$

Because 0.0875/12 occurs twice in the calculation, compute it first and put it into your calculator's memory. Then type

(1 + RCL) y^x 420 − 1 = ÷ RCL × 200 =

In the above, "RCL" refers to recalling the stored number. The way to do this varies with different calculators.

Because 0.0875/12 occurs twice in the calculation, compute it and store it by typing

.0875 ÷ 12 STO➤ ALPHA I

Then type

200 × ((1 + ALPHA I) ^ 420 − 1) ÷ ALPHA I ENTER

b. The principal part of this \$552,539.96 is Tom's contribution, and the rest is interest.

- *Tom's contribution* is 420 payments of \$200 each $= 420 \cdot \$200 = \$84{,}000$.
- The *interest portion* is then $\$552{,}539.96 - \$84{,}000 = \$468{,}539.96$.

In Example 3, the interest portion is almost six times as large as Tom's contribution! The magnitude of the earnings illustrates the amazing power of annuities and the effect of compound interest over a long period of time.

Sinking Funds

A **sinking fund** is an annuity in which the future value is a specific amount of money that will be used for a certain purpose, such as a child's education or the down payment on a home.

EXAMPLE 4

USING AN ANNUITY TO SAVE A SPECIFIC AMOUNT Tom and Betty have a new baby. They agreed that they would need \$30,000 in eighteen years for the baby's college education. They decided to set up a sinking fund and have money deducted from each of Betty's biweekly paychecks. That money will earn $9\frac{1}{4}\%$ interest in Betty's ordinary annuity. Find their monthly payment.

SOLUTION

This is an ordinary annuity, with $i = \frac{1}{12}$th of $9\frac{1}{4}\% = 0.0925/26$, and $n = 18$ years $\cdot$ 26 periods/year $= 468$ periods, and $FV = \$30{,}000$.

$$FV(\text{ord}) = pymt\frac{(1+i)^n - 1}{i} \qquad \textbf{the Ordinary Annuity Formula}$$

$$\$30{,}000 = pymt\frac{(1+0.0925/26)^{468} - 1}{0.0925/26} \qquad \textbf{substituting}$$

To find *pymt*, we must divide 30,000 by the fraction on the right side of the equation. Because the fraction is so complicated, it is best to first calculate the fraction and then multiply its reciprocal by 30,000.

First calculate 0.0925/26 and put it into your calculator's memory. Then type

(1 + RCL) y^x 468 − 1 = ÷ RCL = 1/x × 30000 =

calculating the fraction (from "(" through "RCL") **reciprocal** (1/x)

Because 0.0925/26 occurs twice in the calculation, compute it and store it first, by typing

.0925 ÷ 26 STO➤ ALPHA I ENTER

Then type

((1 + ALPHA I) ^ 468 − 1) ÷ ALPHA I ENTER

calculating the fraction

x^{-1} × 30000 ENTER

reciprocal (x^{-1})

This gives $pymt = 24.995038\ldots$. Betty would need to have only \$25 taken out of each of her biweekly paychecks to save \$30,000 in eighteen years. Notice that she will not have exactly \$30,000 saved, because she cannot have exactly \$24.995048 . . . deducted from each paycheck.

Present Value of an Annuity

The **present value of an annuity** is the lump sum that can be deposited at the beginning of the annuity's term, at the same interest rate and with the same compounding period, that would yield the same amount as the annuity. This value can help the saver to understand his or her options; it refers to an alternative way of saving the same amount of money in the same time. It is called the *present value* because it refers to the single action that the saver can take *in the present* (i.e., at the beginning of the annuity's term) that would have the same effect as would the annuity.

EXAMPLE 5

FINDING THE PRESENT VALUE Find the present value of Tom and Betty's annuity.

SOLUTION

$$FV = P(1 + i)^n \quad \textbf{the Compound Interest Formula}$$

$$30{,}005.95588 = P\left(1 + \frac{0.0925}{26}\right)^{468} \quad \textbf{substituting}$$

$$P = \frac{30{,}005.95588}{\left(1 + \frac{0.0925}{26}\right)^{468}} \quad \textbf{solving for } \boldsymbol{P}$$

$$= 5{,}693.6451\ldots \approx \$5693.65 \quad \textbf{rounding}$$

This means that Tom and Betty would have to deposit $5,693.55 as a lump sum to save as much money as the annuity would yield. They chose an annuity over a lump sum deposit because they could not afford to tie up $5,700 for eighteen years, but they could afford to deduct $25 out of each paycheck.

EXAMPLE 6

FINDING THE PRESENT VALUE Find the present value of an ordinary annuity that has $200 monthly payments for twenty-five years, where the account receives $10\frac{1}{2}\%$ interest.

SOLUTION

We could find the future value of the annuity and then find the lump sum deposit whose future value matches it, as we did in Example 5. However, it is simpler to do the calculation all at once. The key is to realize that the future value of the lump sum must equal the future value of the annuity:

Future value of lump sum = future value of annuity

$$P(1 + i)^n = pymt\frac{(1 + i)^n - 1}{i}$$

For both the lump sum and the annuity, $i = \frac{1}{12}$ of $10\frac{1}{2}\% = \frac{0.105}{12}$ and $n = 25$ years $=$ 300 months. The annuity's payment is $pymt = \$200$.

$$P(1 + i)^n = pymt\frac{(1 + i)^n - 1}{i}$$

$$P(1 + 0.105/12)^{300} = 200\frac{(1 + 0.105/12)^{300} - 1}{0.105/12} \quad \textbf{substituting}$$

First, calculate the right side, as with any annuity calculation. Then divide by $(1 + 0.105/12)^{300}$ to find P.

$P = 21182.363\ldots$
$P \approx \$21{,}182.36$

This means that one would have to make a lump sum deposit of more than $21,000 to have as much money after twenty-five years as with monthly $200 annuity payments.

PRESENT VALUE OF ANNUITY FORMULA

$$FV(\text{lump sum}) = FV(\text{annuity})$$

$$P(1 + i)^n = pymt\frac{(1 + i)^n - 1}{i}$$

The present value is the lump sum P.

5.3 Exercises

In Exercises 1–14, find the future value of the given annuity.

► **1.** ordinary annuity, \$120 monthly payment, $5\frac{3}{4}\%$ interest, one year

► **2.** ordinary annuity, \$175 monthly payment, $6\frac{1}{8}\%$ interest, eleven years

3. annuity due, \$100 monthly payment, $5\frac{7}{8}\%$ interest, four years

► **4.** annuity due, \$150 monthly payment, $6\frac{1}{4}\%$ interest, thirteen years

► **5.** On September 8, Bert Sarkis joined a Christmas Club. His bank will automatically deduct \$75 from his checking account at the end of each month and deposit it into his Christmas Club account, where it will earn 7% interest. The account comes to term on December 1. Find the following:

a. The future value of the account, using an annuity formula

b. The future value of the account, using the compound interest formula

c. Bert's total contribution to the account

d. The total interest

6. On August 19, Rachael Westlake joined a Christmas Club. Her bank will automatically deduct \$110 from her checking account at the end of each month and deposit it into her Christmas Club account, where it will earn $6\frac{7}{8}\%$ interest. The account comes to term on December 1. Find the following:

a. The future value of the account, using an annuity formula

b. The future value of the account, using the compound interest formula

c. Rachael's total contribution to the account

d. The total interest

7. On August 23, Ginny Deus joined a Christmas Club. Her bank will automatically deduct \$150 from her checking account at the beginning of each month and deposit it into her Christmas Club account, where it will earn $7\frac{1}{4}\%$ interest. The account comes to term on December 1. Find the following:

a. The future value of the account, using an annuity formula

b. The future value of the account, using the compound interest formula

c. Ginny's total contribution to the account

d. The total interest

► **8.** On September 19, Lynn Knight joined a Christmas Club. Her bank will automatically deduct \$100 from her checking account at the beginning of each month and deposit it into her Christmas Club account, where it will earn 6% interest. The account comes to term on December 1. Find the following:

a. The future value of the account, using an annuity formula

b. The future value of the account, using the compound interest formula

c. Lynn's total contribution to the account

d. The total interest

9. Pat Gilbert recently set up a TDA to save for her retirement. She arranged to have \$175 taken out of each of her monthly checks; it will earn $10\frac{1}{2}\%$ interest. She just had her thirty-ninth birthday, and her ordinary annuity comes to term when she is 65. Find the following:

a. The future value of the account

b. Pat's total contribution to the account

c. The total interest

► **10.** Dick Eckel recently set up a TDA to save for his retirement. He arranged to have \$110 taken out of each of his biweekly checks; it will earn $9\frac{7}{8}\%$ interest. He just had his twenty-ninth birthday, and his ordinary annuity comes to term when he is 65. Find the following:

a. The future value of the account

b. Dick's total contribution to the account

c. The total interest

► Selected exercises available online at **www.webassign.net/brookscole**

11. Sam Whitney recently set up a TDA to save for his retirement. He arranged to have \$290 taken out of each of his monthly checks; it will earn 11% interest. He just had his forty-fifth birthday, and his ordinary annuity comes to term when he is 65. Find the following:

a. The future value of the account

b. Sam's total contribution to the account

c. The total interest

▶ **12.** Art Dull recently set up a TDA to save for his retirement. He arranged to have \$50 taken out of each of his biweekly checks; it will earn it will earn $9\frac{1}{8}$% interest. He just had his 30th birthday, and his ordinary annuity comes to term when he is 65. Find the following:

a. The future value of the account

b. Art's total contribution to the account

c. The total interest

In Exercises 13–18, (a) find and (b) interpret the present value of the given annuity.

13. The annuity in Exercise 1

14. The annuity in Exercise 2

15. The annuity in Exercise 5

▶ **16.** The annuity in Exercise 6

17. The annuity in Exercise 9

▶ **18.** The annuity in Exercise 10

In Exercises 19–24, find the monthly payment that will yield the given future value.

▶ **19.** \$100,000 at $9\frac{1}{4}$% interest for thirty years; ordinary annuity

20. \$45,000 at $8\frac{7}{8}$% interest for twenty years; ordinary annuity

▶ **21.** \$250,000 at $10\frac{1}{2}$% interest for forty years, ordinary annuity

22. \$183,000 at $8\frac{1}{4}$% interest for twenty-five years, ordinary annuity

23. \$250,000 at $10\frac{1}{2}$% interest for forty years, annuity due

24. \$183,000 at $8\frac{1}{4}$% interest for twenty-five years, annuity due

▶ **25.** Mr. and Mrs. Gonzales set up a TDA to save for their retirement. They agreed to have \$100 deducted from each of Mrs. Gonzales's biweekly paychecks, which will earn $8\frac{1}{8}$% interest.

a. Find the future value of their ordinary annuity if it comes to term after they retire in $35\frac{1}{2}$ years.

b. After retiring, the Gonzales family convert their annuity to a savings account, which earns 6.1% interest compounded monthly. At the end of each month, they withdraw \$650 for living expenses. Complete the chart in Figure 5.14 for their postretirement account.

Month Number	Account Balance at Beginning of the Month	Interest for the Month	With-drawal	Account Balance at End of the Month
1				
2				
3				
4				
5				

FIGURE 5.14 Chart for Exercise 25.

26. Mr. and Mrs. Jackson set up a TDA to save for their retirement. They agreed to have \$125 deducted from each of Mrs. Jackson's biweekly paychecks, which will earn $7\frac{5}{8}$% interest.

a. Find the future value of their ordinary annuity, if it comes to term after they retire in $32\frac{1}{2}$ years.

b. After retiring, the Jacksons convert their annuity to a savings account, which earns 6.3% interest compounded monthly. At the end of each month, they withdraw \$700 for living expenses. Complete the chart in Figure 5.15 for their postretirement account.

Month Number	Account Balance at Beginning of the Month	Interest for the Month	With-drawal	Account Balance at End of the Month
1				
2				
3				
4				
5				

FIGURE 5.15 Chart for Exercise 26.

▶ **27.** Jeanne and Harold Kimura want to set up a TDA that will generate sufficient interest at maturity to meet their living expenses, which they project to be \$950 per month.

a. Find the amount needed at maturity to generate \$950 per month interest if they can get $6\frac{1}{2}$% interest compounded monthly.

b. Find the monthly payment that they would have to put into an ordinary annuity to obtain the future value found in part (a) if their money earns $8\frac{1}{4}$% and the term is thirty years.

▶ **28.** Susan and Bill Stamp want to set up a TDA that will generate sufficient interest at maturity to meet their living expenses, which they project to be \$1,200 per month.

a. Find the amount needed at maturity to generate $1,200 per month interest, if they can get $7\frac{1}{4}\%$ interest compounded monthly.

b. Find the monthly payment that they would have to make into an ordinary annuity to obtain the future value found in part (a) if their money earns $9\frac{3}{4}\%$ and the term is twenty-five years.

▶ **29.** In June 2004, Susan set up a TDA to save for retirement. She agreed to have $200 deducted from each of her monthly paychecks. The annuity's interest rate was allowed to change once each year.

a. In 2004, the interest rate was 1%. Find the account balance in June 2005.

b. In 2005, the interest rate was 2.25%. Find the account balance in June 2006. To do this, think of the June 2005 account balance as a lump sum that earns compound interest.

c. In 2006, the interest rate was 4.5%. Find the account balance in June 2007.

Interest rate source: Mortgagex.com.

30. In June 2007, Manuel set up a TDA to save for retirement. He agreed to have $175 deducted from each of his monthly paychecks. The annuity's interest rate was allowed to change once each year.

a. In 2007, the interest rate was 5.25%. Find the account balance in June 2008.

b. In 2008, the interest rate was 3.8%. Find the account balance in June 2009. To do this, think of the June 2008 account balance as a lump sum that earns compound interest.

c. In 2009, the interest rate was 2.2%. Find the account balance in June 2010.

Interest rate source: Mortgagex.com.

In Exercises 31–34, use the following information. An Individual Retirement Account (IRA) is an annuity that is set up to save for retirement. IRAs differ from TDAs in that an IRA allows the participant to contribute money whenever he or she wants, whereas a TDA requires the participant to have a specific amount deducted from each of his or her paychecks.

▶ **31.** When Shannon Pegnim was 14, she got an after-school job at a local pet shop. Her parents told her that if she put some of her earnings into an IRA, they would contribute an equal amount to her IRA. That year and every year thereafter, she deposited $1,000 into her IRA. When she became 25 years old, her parents stopped contributing, but Shannon increased her annual deposit to $2,000 and continued depositing that amount annually until she retired at age 65. Her IRA paid 8.5% interest. Find the following:

a. The future value of the account

b. Shannon's and her parents' total contributions to the account

c. The total interest

d. The future value of the account if Shannon waited until she was 19 before she started her IRA

e. The future value of the account if Shannon waited until she was 24 before she started her IRA

▶ **32.** When Bo McSwine was 16, he got an after-school job at his parents' barbecue restaurant. His parents told him that if he put some of his earnings into an IRA, they would contribute an equal amount to his IRA. That year and every year thereafter, he deposited $900 into his IRA. When he became 21 years old, his parents stopped contributing, but Bo increased his annual deposit to $1,800 and continued depositing that amount annually until he retired at age 65. His IRA paid 7.75% interest. Find the following:

a. The future value of the account

b. Bo's and his parents' total contributions to the account

c. The total interest

d. The future value of the account if Bo waited until he was 18 before he started his IRA

e. The future value of the account if Bo waited until he was 25 before he started his IRA

▶ **33.** If Shannon Pegnim from Exercise 31 started her IRA at age 35 rather than age 14, how big of an annual contribution would she have had to have made to have the same amount saved at age 65?

▶ **34.** If Bo McSwine from Exercise 32 started his IRA at age 35 rather than age 16, how big of an annual contribution would he have had to have made to have the same amount saved at age 65?

▶ **35.** Toni Torres wants to save $1,200 in the next two years to use as a down payment on a new car. If her bank offers her 9% interest, what monthly payment would she need to make into an ordinary annuity to reach her goal?

▶ **36.** Fred and Melissa Furth's daughter Sally will be a freshman in college in six years. To help cover their extra expenses, the Furths decide to set up a sinking fund of $12,000. If the account pays 7.2% interest and they wish to make quarterly payments, find the size of each payment.

▶ **37.** Anne Geyer buys some land in Utah. She agrees to pay the seller a lump sum of $65,000 in five years. Until then, she will make monthly simple interest payments to the seller at 11% interest.

a. Find the amount of each interest payment.

b. Anne sets up a sinking fund to save the $65,000. Find the size of her semiannual payments if her payments are due at the end of every six-month period and her money earns $8\frac{3}{8}\%$ interest.

c. Prepare a table showing the amount in the sinking fund after each deposit.

▶ **38.** Chrissy Fields buys some land in Oregon. She agrees to pay the seller a lump sum of $120,000 in six years.

Until then, she will make monthly simple interest payments to the seller at 12% interest.

a. Find the amount of each interest payment.

b. Chrissy sets up a sinking fund to save the $120,000. Find the size of her semiannual payments if her money earns $10\frac{3}{4}\%$ interest.

c. Prepare a table showing the amount in the sinking fund after each deposit.

39. Develop a new formula for the present value of an ordinary annuity by solving the Present Value of Annuity Formula for P and simplifying.

40. Use the formula developed in Exercise 39 to find the present value of the annuity in Exercise 2.

41. Use the formula developed in Exercise 39 to find the present value of the annuity in Exercise 1.

42. Use the formula developed in Exercise 39 to find the present value of the annuity in Exercise 6.

43. Use the formula developed in Exercise 39 to find the present value of the annuity in Exercise 5.

Answer the following questions using complete sentences and your own words.

• CONCEPT QUESTIONS

44. Explain the difference between compound interest and an annuity.

45. Explain how an annuity is based on compound interest.

46. Describe the difference between an ordinary annuity and an annuity due.

47. Compare and contrast an annuity with a lump sum investment that receives compound interest. Be sure to discuss both the similarity and the difference between these two concepts, as well as the advantages and disadvantages of each.

48. Which is always greater: the present value of an annuity or the future value? Why?

49. If you want to retire on $2,000 a month for twenty-five years, do you need to save $2,000 a month for twenty-five years before retiring? Why or why not?

50. *For those who have completed Section 1.1:* Is the logic used in deriving the Ordinary Annuity Formula inductive or deductive? Why? Is the logic used in deriving the relationship

$$FV(\text{due}) = FV(\text{ordinary}) \cdot (1 + i)$$

inductive or deductive? Why?

WEB PROJECTS

51. Think of something that you would like to be able to afford but cannot—perhaps a car, boat, or motorcycle. In this exercise, you will explore how to make that unaffordable dream a realistic goal.

a. Just what is it that you would like to be able to afford?

b. Determine an appropriate but realistic date for making the purchase. Justify your date.

c. Do some research and determine how much your goal would cost currently. Cite your sources.

d. Do some web research and determine the current rate of inflation. Inflation rate information is readily available on the web. Cite your sources.

e. Use the results of parts (b), (c), and (d), as well as either the simple or compound interest formula, to predict how much your goal will cost in the future.

f. Discuss why you chose to use the simple or compound interest formula in part (e).

g. Go to the web sites of four different banks and determine the current interest rate available for an annuity of an appropriate term.

h. Determine the necessary annuity payment that will allow you to meet your goal. If necessary, alter the date from part (b).

52. Suppose you had a baby in 2005 and you decided that it would be wise to start saving for his or her college education. In this exercise, you will explore how to go about doing that.

a. According to the College Board, four-year public schools cost an average of $5,491 per year in 2005–2006. Four-year private schools cost an average of $21,235 per year. These costs include tuition and fees, room and board, books and supplies, and personal expenses. Furthermore, four-year public schools increased 7.1% from 2004–2005, and four-year private schools increased 5.9% from 2004–2005. Use this information, as well as either the simple or compound interest formula, to predict the cost of your child's college education in his or her freshman, sophomore, junior, and senior years for both public and private institutions. Discuss your assumptions and justify your work.
(*Source:* **www.collegeboard.com/pay**.)

b. Will you save for a public or a private institution? Why?

c. Discuss why you chose to use the simple or compound interest formula in part (a).

d. Go to the web sites of four different banks and determine the current interest rate available for an annuity of an appropriate term.

e. Determine the necessary annuity payment that will allow you to save the total amount from part (a) in time to meet your goal.

Some useful links for these web projects are listed on the text web site:

www.cengage.com/math/johnson

Annuities with a TI's TVM Application

EXAMPLE 7

FINDING HOW LONG IT TAKES Use a TI-83/84's TVM application to find the how long it takes for an ordinary annuity to have a balance of one million dollars if the monthly payments of $600 earn 6% interest.

SOLUTION

1. Press APPS, select option 1: "Finance", and press ENTER.
2. Select option 1: "TVM Solver", and press ENTER.
3. Enter appropriate values for the variables:
   - N is the number of compounding periods. This is the number we're trying to find, so we temporarily enter 0.
   - I% is the *annual* interest rate (not a periodic rate), as a percent, so enter 6 for I%. Note that we do not convert to a decimal or a periodic rate.
   - PV is the present value. There is no present value here, so we enter 0.
   - PMT is the payment. The payment is $600, but it's an outgoing amount of money (since we give it to the bank), so we enter −600 for PMT. You must enter a negative number for any outgoing amount of money.
   - FV is the future value. We're looking for the amount of time that it takes to have $1,000,000, so enter 1000000. This is an incoming amount of money (since the bank gives it to us), so we use 1000000 rather than −1000000.
   - P/Y is the number of periods per year. We make monthly payments, so there are twelve periods per year. Enter 12 for P/Y.
   - C/Y is automatically made to be the same as P/Y. In this text, we will never encounter a situation in which C/Y is different from P/Y. See Figure 5.16.
   - An ordinary annuity's payments are due at the end of each period so highlight "End"

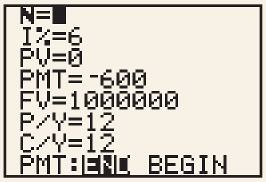


FIGURE 5.16
Preparing the TVM screen.

4. To solve for N, use the arrow buttons to highlight the 0 that we entered for N. Then press ALPHA SOLVE. (Pressing ALPHA makes the ENTER button becomes the SOLVE button.) As a result, we find that N is 447.8343121.

This is the number of *months* it takes (a period is a month, because of our P/Y entry). This means that it takes about $447.8343121/12 = 37.31\ldots \approx 37.3$ years to have a million dollars.

Exercises

53. Redo Example 7 for an annuity due.

54. Find how long it takes for an ordinary annuity to have a balance of two million dollars if the monthly payments of $1100 earn:

a. 2% interest
b. 4% interest
c. 6% interest
d. 10% interest

5.4 Amortized Loans

OBJECTIVES

- Understand what an amortized loan is
- Learn how to do home loan and car loan computations
- Learn how to make and use an amortization schedule

When you buy a house, a car, or a boat, chances are you will finance it with a simple interest amortized loan. A **simple interest amortized loan** is a loan in which each payment consists of principal and interest, and the interest is simple interest computed on the outstanding principal. We'll explore these loans in this section.

An **amortized loan** is a loan for which the loan amount, plus interest, is paid off with a series of regular equal payments. An add-on interest loan (discussed in Section 5.1) is an amortized loan. A simple interest amortized loan is also an amortized loan, but it is calculated differently from an add-on interest loan. You should be aware that the payments are smaller with a simple interest amortized loan than they are with an add-on interest loan (assuming, naturally, that the interest rates, loan amounts, and number of payments are the same).

A simple interest amortized loan is calculated as an ordinary annuity whose future value is the same as the loan amount's future value, under compound interest. Realize, though, that a simple interest amortized loan's payments are used to pay off a loan, whereas an annuity's payments are used to generate savings.

SIMPLE INTEREST AMORTIZED LOAN FORMULA

Future value of annuity = future value of loan amount

$$pymt\frac{(1+i)^n - 1}{i} = P(1+i)^n$$

where *pymt* is the loan payment, *i* is the periodic interest rate, *n* is the number of periods, and *P* is the present value or loan amount.

Algebraically, this formula could be used to determine any one of the four unknowns (*pymt*, *i*, *n*, and *P*) if the other three are known. We will use it to find the payment when the annual interest rate, number of periods, and loan amount are known.

EXAMPLE 1

BUYING A CAR Tom and Betty decide that they need a more dependable car, now that they have a baby. They buy one for $13,518.77. They make a $1,000 down payment and finance the balance through a four-year simple interest amortized loan from their bank. They are charged 12% interest.

a. Find their monthly payment.
b. Find the total interest they will pay over the life of the loan.

SOLUTION

a. Use the simple interest amortized loan formula, with a loan amount of $P = \$13{,}518.77 - \$1{,}000 = \$12{,}518.77$, a monthly interest rate of $12\%/12 = 1\% = 0.01$, and a term of $n = 4$ years $= 48$ months.

$$pymt\frac{(1+i)^n - 1}{i} = P(1+i)^n$$ **the Simple Interest Amortized Loan Formula**

$$pymt\frac{(1+0.01)^{48} - 1}{0.01} = 12{,}518.77(1+0.01)^{48}$$ **substituting**

To find *pymt*, we need to divide the right side by the fraction on the left side or, equivalently, multiply by its reciprocal. First, find the fraction on the left side, as with any annuity calculation. Then multiply the right side by the fraction's reciprocal.

(1 + .01) y^x 48 − 1 = ÷ .01 =

fraction on the left side

1/x × 12518.77 × (1 + .01) y^x 48 =

reciprocal **fraction on the right side**

(((1 + .01) ^ 48 − 1) ÷ .01)

fraction on the left side

x^{-1} × 12518.77 (1 + .01) ^ 48 ENTER

reciprocal **fraction on the right side**

We get $pymt = 329.667\ldots \approx \$329.67$.

> As a rough check, note that Tom and Betty borrowed \$12,518.77. If they paid no interest, each payment would be $\$12{,}518.77/48 \approx \$260.81$. Since they must pay interest, each payment must be larger than \$260.81. Our calculation checks, because $\$329.67 > \$260.81$.

b. The total of their payments is $48 \cdot \$329.67 = \$15{,}824.16$. This includes both principal and interest. Of this, \$12,518.77 is principal, so $\$15{,}824.16 - \$12{,}518.77 = \$3{,}305.39$ is interest. Over the life of the loan, Tom and Betty are paying \$3,305.39 in interest.

Amortization Schedules

The simple interest amortized loan formula is an equation with the ordinary annuity formula on one side of the equal symbol and the compound interest formula on the other side. However, with such a loan, you do not pay compound interest. Instead, the interest portion of each payment is simple interest on the outstanding principal.

An **amortization schedule** is a list of several periods of payments, the principal and interest portions of those payments, and the **outstanding principal** (or **balance**) after each of those payments is made.

The data on the amortization schedule are important to the borrower for two reasons. The borrower needs to know the total interest paid, for tax purposes. Interest paid on a home loan is usually deductible from the borrower's income tax, and interest paid on a loan by a business is usually deductible. The borrower would also need the data on an amortization schedule if he or she is considering paying off the loan early. Such prepayment could save money because an advance payment would be all principal and would not include any interest; however, the lending institution may charge a **prepayment penalty** which would absorb some of the interest savings.

EXAMPLE 2

PREPARING AN AMORTIZATION SCHEDULE Prepare an amortization schedule for the first two months of Tom and Betty's loan.

SOLUTION

For any simple interest loan, the interest portion of each payment is simple interest on the outstanding principal, so use the Simple Interest Formula, $I = Prt$, to compute the interest. Recall that r and t are annual figures. For each payment, $r = 12\% = 0.12$ and $t = 1 \text{ month} = \frac{1}{12}$ year. For payment number 1, $P = \$12{,}418.77$ (the amount borrowed). The interest portion of payment number 1 is

$$I = Prt \quad \text{the Simple Interest Formula}$$
$$= 12{,}518.77 \cdot 0.12 \cdot \frac{1}{12} \quad \text{substituting}$$
$$\approx \$125.19 \quad \text{rounding}$$

The principal portion of payment number 1 is

$$\text{Payment} - \text{interest portion}$$
$$= \$329.67 - \$125.19$$
$$= \$204.48$$

The outstanding principal or balance is

$$\text{Previous principal} - \text{principal portion}$$
$$= \$12{,}518.77 - \$204.48$$
$$= \$12{,}314.29$$

All of the above information goes into the amortization schedule. See Figure 5.17.

Payment number	Principal portion	Interest portion	Total payment	Balance
0	—	—	—	$12,518.77
1	$204.48	$125.19	$329.67	$12,314.29
2	$206.53	$123.14	$329.67	$12,107.76

FIGURE 5.17 An amortization schedule for the first two months of Tom and Betty's loan.

When it is time to make payment number 2, less money is owed. The outstanding principal is $12,314.29, so this is the new value of P. The interest portion of payment number 2 is

$$I = Prt \quad \text{the Simple Interest Formula}$$
$$= 12{,}314.29 \cdot 0.12 \cdot \frac{1}{12} \quad \text{substituting}$$
$$\approx \$123.14 \quad \text{rounding}$$

The principal portion is

$$\begin{aligned}&\text{Payment} - \text{interest portion}\\&= \$329.67 - \$123.14\\&= \$206.53\end{aligned}$$

The outstanding principal or balance is

$$\begin{aligned}&\text{Previous principal} - \text{principal portion}\\&= \$12{,}314.29 - \$206.53\\&= \$12{,}107.76\end{aligned}$$

All of the above information is shown in the amortization schedule in Figure 5.17.

Notice how in Example 2, the principal portion increases and the interest portion decreases. This continues throughout the life of the loan, and the final payment is mostly principal. This happens because after each payment, the amount due is somewhat smaller, so the interest on the amount due is somewhat smaller also.

EXAMPLE 3

AMORTIZATION SCHEDULES AND WHY THE LAST PAYMENT IS DIFFERENT Comp-U-Rent needs to borrow $60,000 to increase their inventory of rental computers. The company is confident that their expanded inventory will generate sufficient extra income to allow them to pay off the loan in a short amount of time, so they wish to borrow the money for only three months. First National Bank offers them a simple interest amortized loan at $8\frac{3}{4}$% interest.

a. Find what their monthly payment would be with First National.
b. Prepare an amortization schedule for the entire term of the loan.

SOLUTION

a. $P = \$60{,}000$, $i = \frac{1}{12}$ of $8\frac{3}{4}\% = 0.0875/12$, and $n = 3$ months.

$$\text{Future value of annuity} = \text{future value of loan amount}$$

$$pymt\frac{(1+i)^n - 1}{i} = P(1+i)^n$$

$$pymt\frac{\left(1+\frac{0.0875}{12}\right)^3 - 1}{\frac{0.0875}{12}} = 60000\left(1+\frac{0.0875}{12}\right)^3 \quad \textbf{substituting}$$

First, compute 0.0875/12 and store it in your calculator's memory. They type

[( ] 1 [+] [RCL] [ )] [y^x] 3 [−] 1 [=] [÷] [RCL] [=]

fraction on the left side

[1/x] [×] 60000 [×] [( ] 1 [+] [RCL] [ )] [y^x] 3 [=]

reciprocal **fraction on right side**

First compute and store 0.0875/12 by typing

.0875 [÷] 12 [STO➤] [ALPHA] [I] [ENTER]

Then type

[(] [(] [(] 1 [+] [ALPHA] [I] [)] [^] 3 [−] 1 [)] [÷] [ALPHA] [I] [)]

fraction on the left side

[x^{-1}] [×] 60000 [(] 1 [+] [ALPHA] [I] [)] [^] 3 [ENTER]

reciprocal **fraction on the right side**

See Figure 5.18 on page 374.

We get

$$pymt = 20{,}292.375\ldots \quad \textbf{solving for } \textbf{\textit{pymt}}$$
$$\approx \$20{,}292.38 \quad \textbf{rounding}$$

See the "Total Payment" column in Figure 5.18 on page 374.

As a rough check, notice that Comp-U-Rent borrowed \$60,000, and each payment includes principal and interest, so the payment must be larger than $\frac{\$60{,}000}{3} = \$20{,}000$. Our calculation checks, because $\$20{,}292.38 > \$20{,}000$.

b. For each payment, $r = 8\frac{3}{4}\% = 0.0875$ and $t = 1$ month $= \frac{1}{12}$ year.

For payment number 1, $P = \$60{,}000.00$ (the amount borrowed). The interest portion of payment number 1 is

$$I = Prt \quad \textbf{the Simple Interest Formula}$$
$$= 60{,}000.00 \cdot 0.0875 \cdot \frac{1}{12} \quad \textbf{substituting}$$
$$= \$437.50$$

See the "Interest portion" column in Figure 5.18.

The principal portion of payment number 1 is

Payment − interest portion

$$= \$20{,}292.38 - \$437.50 = \$19{,}854.88$$

See the "Principal portion" column in Figure 5.18.

The outstanding principal or balance is

Previous principal − principal portion

$$= \$60{,}000.00 - \$19{,}854.88 = \$40{,}145.12$$

For payment number 2, $P = \$40{,}145.12$ (the outstanding principal).

The interest portion of payment number 2 is

$$I = Prt \quad \textbf{the Simple Interest Formula}$$
$$= 40{,}145.12 \cdot 0.0875 \cdot \frac{1}{12} \quad \textbf{substituting}$$
$$\approx \$292.72 \quad \textbf{rounding}$$

The Principle portion of payment number 2 is

Payment − interest portion

$$= \$20{,}292.38 - \$292.72 = \$19{,}999.66$$

The outstanding principal or balance is

$$\$40{,}145.12 - \$19{,}999.66 = \$20{,}145.46$$

For payment number 3, $P = \$20{,}145.46$

The interest portion is

$$\begin{aligned} I &= PRT \\ &= \$20{,}145.46 \cdot 0.0875 \cdot \frac{1}{12} \\ &= \$146.89 \end{aligned}$$

The principal portion is

$$\text{Payment} - \text{interest portion} = \$20{,}292.38 - \$146.89 = \$20{,}145.49$$

The outstanding principal or balance is

$$\begin{aligned} &\text{Previous principal} - \text{principal portion} \\ &\quad = \$20{,}145.46 - \$20{,}145.49 = -\$0.03 \end{aligned}$$

Negative three cents cannot be correct. After the final payment is made, the amount due *must be* \$0.00. The discrepancy arises from the fact that we rounded off the payment size from \$20,292.375. . . to \$20,292.38. If there were some way in which the borrower could make a monthly payment that is not rounded off, then the calculation above would have yielded an amount due of \$0.00. To repay the exact amount owed, we must compute the last payment differently.

The principal portion of payment number 3 *must be* \$20,145.46, because that is the balance, and this payment is the only chance to pay it. The payment must also include \$146.89 interest, as calculated above. The last payment is the sum of the principal due and the interest on that principal:

$$\$20{,}145.46 + \$146.89 = \$20{,}292.35$$

The closing balance is then \$20,145.46 − \$20,145.46 = \$0.00, as it should be. The amortization schedule is given in Figure 5.18.

Payment number	Principal portion	Interest portion	Total payment	Balance
0	—	—	—	\$60,000.00
1	\$19,854.88	\$437.50	\$20,292.38	\$40,145.12
2	\$19,999.66	\$292.72	\$20,292.38	\$20,145.46
3	\$20,145.46	\$146.89	\$20,292.35	\$0.00

FIGURE 5.18 The amortization schedule for Comp-U-Rent's loan.

AMORTIZATION SCHEDULE STEPS

For each payment, list the payment number, principal portion, interest portion, total payment, and balance.

For each payment:

1. Find the interest on the balance, using the simple interest formula.

For each payment except the last:

2. The principal portion is the payment minus the interest portion.
3. The new balance is the previous balance minus the principal portion.

For the last payment:

4. The principal portion is the previous balance.
5. The total payment is the sum of the principal portion and interest portion.

Figure 5.19 shows these steps in chart form.

Payment number	Principal portion	Interest portion	Total payment	Balance
0	—	—	—	Loan amount
First through next-to-last	Total payment minus interest portion	Simple interest on previous balance; use $I = Prt$	Use simple interest amortized loan formula	Previous balance minus this payment's principal portion
Last	Previous balance	Simple interest on previous balance; use $I = Prt$	Principal portion plus interest portion	$0.00

FIGURE 5.19 Amortization schedule steps.

In preparing an amortization schedule, the new balance is the previous balance minus the principal portion. This means that only the principal portion of a payment goes toward paying off the loan. The interest portion is the lender's profit.

Prepaying a Loan

Sometimes a borrower needs to pay his or her loan off early, before the loan's term is over. This is called **prepaying a loan.** This often happens when people sell their houses. It also happens when interest rates have fallen since the borrower obtained his or her loan and the borrower decides to refinance the loan at a lower rate.

When you prepay a loan, you pay off the unpaid balance. You can find that unpaid balance by looking at an amortization schedule. If an amortization schedule has not already been prepared, the borrower can easily approximate the **unpaid balance** by subtracting the current value of the annuity from the current value of the loan. This approximation is usually off by at most a few pennies.

UNPAID BALANCE FORMULA

Unpaid balance = current value of loan amount − current value of annuity

$$\approx P(1+i)^n - pymt\,\frac{(1+i)^n - 1}{i}$$

where *pymt* is the loan payment, *i* is the periodic interest rate, *n* is the number of periods *from the beginning of the loan to the present,* and *P* is the loan amount.

A common error in using this formula is to let *n* equal the number of periods in the entire life of the loan rather than the number of periods from the beginning of the loan until the time of prepayment. This results in an answer of 0. After all of the payments have been made, the unpaid balance should be 0.

EXAMPLE 4

PREPAYING A LOAN Ten years ago, Tom and Betty bought a house for \$140,000. They paid the sellers a 20% down payment and obtained a simple interest amortized loan for \$112,000 from their bank at $10\frac{3}{4}$% for thirty years. Their monthly payment is \$1,045.50.

Their old home was fine when there were just two of them. But with their new baby, they need more room. They are considering selling their existing home and buying a new home. This would involve paying off their home loan. They would use the income from the house's sale to do this. Find the cost of paying off the existing loan.

SOLUTION

The loan is a thirty-year loan, so if we were computing the payment, we would use $n = 30 \text{ years} \cdot 12 \text{ months/year} = 360$ months. However, we are computing the unpaid balance, not the payment, so n is not 360. Tom and Betty have made payments on their loan for 10 years = 120 months, so for this calculation, $n = 120$.

Unpaid balance

$$= \text{current value of loan amount} - \text{current value of annuity}$$

$$\approx P(1+i)^n - pymt\frac{(1+i)^n - 1}{i}$$

$$= 112{,}000\left(1 + \frac{0.1075}{12}\right)^{120} - 1{,}045.50\frac{\left(1 + \frac{0.1075}{12}\right)^{120} - 1}{\frac{0.1075}{12}}$$

$$= \$102{,}981.42$$

First, compute 0.1075/12, and store it in the calculator's memory (as ALPHA I with a graphing calculator). Then type

112000 [×] [(] 1 [+] [RCL] [)] [y^x] 120 [−] 1045.50 [×] [(] [(] 1 [+] [RCL] [)] [y^x] 120 [−] 1 [)] [÷] [RCL] [=]

112000 [(] 1 [+] [ALPHA] [I] [)] [^] 120 [−] 1045.50 [(] [(] 1 [+] [ALPHA] [I] [)] [^] 120 [−] 1 [)] [÷] [ALPHA] [I] [ENTER]

As a rough check, note that Tom and Betty borrowed \$112,000, so their unpaid balance must be less. Our calculation checks, because \$102,981.42 < \$112,000.

The result of Example 4 means that after ten years of payments of over \$1000 a month on a loan of \$112,000, Tom and Betty still owe approximately \$102,981.42! They were shocked. It is extremely depressing for first-time home purchasers to discover how little of their beginning payments actually goes toward paying off the loan. At the beginning of the loan, you owe a lot of money, so most of each payment is interest, and very little is principal. Later, you do not owe as much, so most of each payment is principal and very little is interest.

5.4 EXERCISES

In the following exercises, all loans are simple interest amortized loans with monthly payments unless labeled otherwise.

In Exercises 1–6, find (a) the monthly payment and (b) the total interest for the given simple interest amortized loan.

▶ **1.** \$5,000, $9\frac{1}{2}\%$, 4 years

2. \$8,200, $10\frac{1}{4}\%$, 6 years

3. \$10,000, $6\frac{1}{8}\%$, 5 years

4. \$20,000, $7\frac{3}{8}\%$, $5\frac{1}{2}$ years

▶ **5.** \$155,000, $9\frac{1}{2}\%$, 30 years

6. \$289,000, $10\frac{3}{4}\%$, 35 years

▶ **7.** Wade Ellis buys a new car for \$16,113.82. He puts 10% down and obtains a simple interest amortized loan for the rest at $11\frac{1}{2}\%$ interest for four years.

a. Find his monthly payment.

b. Find the total interest.

c. Prepare an amortization schedule for the first two months of the loan.

8. Guy dePrimo buys a new car for \$9,837.91. He puts 10% down and obtains a simple interest amortized loan for the rest at $8\frac{7}{8}\%$ interest for four years.

a. Find his monthly payment.

b. Find the total interest.

c. Prepare an amortization schedule for the first two months of the loan.

9. Chris Burditt bought a house for \$212,500. He put 20% down and obtains a simple interest amortized loan for the rest at $10\frac{7}{8}\%$ for thirty years.

a. Find his monthly payment.

b. Find the total interest.

c. Prepare an amortization schedule for the first two months of the loan.

d. Most lenders will approve a home loan only if the total of all the borrower's monthly payments, including the home loan payment, is no more than 38% of the borrower's monthly income. How much must Chris make to qualify for the loan?

▶ **10.** Shirley Trembley bought a house for \$187,600. She put 20% down and obtains a simple interest amortized loan for the rest at $6\frac{3}{8}\%$ for thirty years.

a. Find her monthly payment.

b. Find the total interest.

c. Prepare an amortization schedule for the first two months of the loan.

d. Most lenders will approve a home loan only if the total of all the borrower's monthly payments, including the home loan payment, is no more than 38% of the borrower's monthly income. How much must Shirley make to qualify for the loan?

▶ **11.** Dennis Lamenti wants to buy a new car that costs \$15,829.32. He has two possible loans in mind. One loan is through the car dealer; it is a four-year add-on interest loan at $7\frac{3}{4}\%$ and requires a down payment of \$1,000. The second is through his bank; it is a four-year simple interest amortized loan at at $7\frac{3}{4}\%$ and requires a down payment of \$1,000.

a. Find the monthly payment for each loan.

b. Find the total interest paid for each loan.

c. Which loan should Dennis choose? Why?

▶ **12.** Barry Wood wants to buy a used car that costs \$4,000. He has two possible loans in mind. One loan is through the car dealer; it is a three-year add-on interest loan at 6% and requires a down payment of \$300. The second is through his credit union; it is a three-year simple interest amortized loan at 9.5% and requires a 10% down payment.

a. Find the monthly payment for each loan.

b. Find the total interest paid for each loan.

c. Which loan should Barry choose? Why?

▶ **13.** Investigate the effect of the term on simple interest amortized auto loans by finding the monthly payment and the total interest for a loan of \$11,000 at $9\frac{7}{8}\%$ interest if the term is

a. three years

b. four years

c. five years

▶ **14.** Investigate the effect of the interest rate on simple interest amortized auto loans by finding the monthly payment and the total interest for a four-year loan of \$12,000 at

a. 8.5% **b.** 8.75%

c. 9% **d.** 10%.

▶ **15.** Investigate the effect of the interest rate on home loans by finding the monthly payment and the total interest for a thirty-year loan of \$100,000 at

a. 6% **b.** 7%

c. 8% **d.** 9%

e. 10% **f.** 11%

▶ **16.** Some lenders offer fifteen-year home loans. Investigate the effect of the term on home loans by finding the monthly payment and total interest for a loan of \$100,000 at 10% if the term is

a. thirty years **b.** fifteen years

▶ **17.** Some lenders offer loans with biweekly payments rather than monthly payments. Investigate the effect of this on home loans by finding the payment and total interest on a thirty-year loan of \$100,000 at 10% interest if payments are made (a) monthly and (b) biweekly.

▶ Selected exercises available online at **www.webassign.net/brookscole**

18. Some lenders are now offering loans with biweekly payments rather than monthly payments. Investigate the effect of this on home loans by finding the payment and total interest on a thirty-year loan of $150,000 at 8% interest if payments are made (a) monthly and (b) biweekly.

19. Verify (a) the monthly payments and (b) the interest savings in the savings and loan advertisement in Figure 5.20.

Courtesy Continental Savings of America

FIGURE 5.20 Loan terms for Exercise 19.

20. The home loan in Exercise 19 presented two options. The thirty-year option required a smaller monthly payment. A consumer who chooses the thirty-year option could take the savings in the monthly payment that this option generates and invest that savings in an annuity. At the end of fifteen years, the annuity might be large enough to pay off the thirty-year loan. Determine whether this is a wise plan if the annuity's interest rate is 7.875%. (Disregard the tax ramifications of this approach.)

▶ **21.** Pool-N-Patio World needs to borrow $75,000 to increase its inventory for the upcoming summer season. The owner is confident that he will sell most, if not all, of the new inventory during the summer, so he wishes to borrow the money for only four months. His bank has offered him a simple interest amortized loan at $7\frac{3}{4}$% interest.

a. Find the size of the monthly bank payment.

b. Prepare an amortization schedule for all four months of the loan.

▶ **22.** Slopes R Us needs to borrow $120,000 to increase its inventory of ski equipment for the upcoming season. The owner is confident that she will sell most, if not all, of the new inventory during the winter, so she wishes to borrow the money for only five months. Her bank has offered her a simple interest amortized loan at $8\frac{7}{8}$% interest.

a. Find the size of the monthly bank payment.

b. Prepare an amortization schedule for all five months of the loan.

23. The owner of Blue Bottle Coffee is opening a second store and needs to borrow $93,000. Her success with her first store has made her confident that she will be able to pay off her loan quickly, so she wishes to borrow the money for only four months. Her bank has offered her a simple interest amortized loan at $9\frac{1}{8}$% interest.

a. Find the size of the monthly bank payment.

b. Prepare an amortization schedule for all four months of the loan.

▶ **24.** The Green Growery Nursery needs to borrow $48,000 to increase its inventory for the upcoming summer season. The owner is confident that he will sell most, if not all, of the new plants during the summer, so he wishes to borrow the money for only four months. His bank has offered him a simple interest amortized loan at $9\frac{1}{4}$% interest.

a. Find the size of the monthly bank payment.

b. Prepare an amortization schedule for all four months of the loan.

For Exercises 25–28, note the following information. A ***line of credit*** *is an agreement between a bank and a borrower by which the borrower can borrow any amount of money (up to a mutually agreed-upon maximum) at any time, simply by writing a check. Typically, monthly simple interest payments are required, and the borrower is free to make principal payments as frequently or infrequently as he or she wants. Usually, a line of credit is secured by the title to the borrower's house, and the interest paid to the bank by the borrower is deductible from the borrower's income taxes.*

25. Kevin and Roxanne Gahagan did not have sufficient cash to pay their income taxes. However, they had previously set up a line of credit with their bank. On April 15, they wrote a check to the Internal Revenue Service on their line of credit for $6,243. The line's interest rate is 5.75%.

a. Find the size of the required monthly interest payment.

b. The Gahagans decided that it would be in their best interests to get this loan paid off in eight months.

Find the size of the monthly principal-plus-interest payment that would accomplish this. (*HINT:* In effect, the Gahagans are converting the loan to an amortized loan.)

c. Prepare an amortization schedule for all eight months of the loan.

d. Find the amount of line of credit interest that the Gahagans could deduct from their taxes next year.

▶ **26.** James and Danna Wright did not have sufficient cash to pay their income taxes. However, they had previously set up a line of credit with their bank. On April 15, they wrote a check to the Internal Revenue Service on their line of credit for $10,288. The line's interest rate is 4.125%.

a. Find the size of the required monthly interest payment.

b. The Wrights decided that it would be in their best interests to get this loan paid off in six months. Find the size of the monthly principal-plus-interest payment that would accomplish this. (*HINT:* In effect, the Wrights are converting the loan to an amortized loan.)

c. Prepare an amortization schedule for all six months of the loan.

d. Find the amount of line of credit interest that the Wrights could deduct from their taxes next year.

▶ **27.** Homer Simpson had his bathroom remodeled. He did not have sufficient cash to pay for it. However, he had previously set up a line of credit with his bank. On July 12, he wrote a check to his contractor on his line of credit for $12,982. The line's interest rate is 8.25%.

a. Find the size of the required monthly interest payment.

b. Homer decided that it would be in his best interests to get this loan paid off in seven months. Find the size of the monthly principal-plus-interest payment that would accomplish this. (*HINT:* In effect, Simpson is converting the loan to an amortized loan.)

c. Prepare an amortization schedule for all seven months of the loan.

d. Find the amount of line of credit interest that Simpson could deduct from his taxes next year.

28. Harry Trask had his kitchen remodeled. He did not have sufficient cash to pay for it. However, he had previously set up a line of credit with his bank. On July 12, he wrote a check to his contractor on his line of credit for $33,519. The line's interest rate is 7.625%.

a. Find the size of the required monthly interest payment.

b. Harry decided that it would be in his best interests to get this loan paid off in seven months. Find the size of the monthly principal-plus-interest payment that would accomplish this. (*HINT:* In effect, Trask is converting the loan to an amortized loan.)

c. Prepare an amortization schedule for all seven months of the loan.

d. Find the amount of line of credit interest that Trask could deduct from his taxes next year.

▶ **29.** Wade Ellis buys a car for $16,113.82. He puts 10% down and obtains a simple interest amortized loan for the balance at $11\frac{1}{2}$% interest for four years. Three years and two months later, he sells his car. Find the unpaid balance on his loan.

30. Guy de Primo buys a car for $9837.91. He puts 10% down and obtains a simple interest amortized loan for the balance at $10\frac{7}{8}$% interest for four years. Two years and six months later, he sells his car. Find the unpaid balance on his loan.

31. Gary Kersting buys a house for $212,500. He puts 20% down and obtains a simple interest amortized loan for the balance at $10\frac{7}{8}$% interest for thirty years. Eight years and two months later, he sells his house. Find the unpaid balance on his loan.

▶ **32.** Shirley Trembley buys a house for $187,600. She puts 20% down and obtains a simple interest amortized loan for the balance at $11\frac{3}{8}$% interest for thirty years. Ten years and six months later, she sells her house. Find the unpaid balance on her loan.

33. Harry and Natalie Wolf have a three-year-old loan with which they purchased their house. Their interest rate is $13\frac{3}{8}$%. Since they obtained this loan, interest rates have dropped, and they can now get a loan for $8\frac{7}{8}$% through their credit union. Because of this, the Wolfs are considering refinancing their home. Each loan is a thirty-year simple interest amortized loan, and neither has a prepayment penalty. The existing loan is for $152,850, and the new loan would be for the current amount due on the old loan.

a. Find their monthly payment with the existing loan.

b. Find the loan amount for their new loan.

c. Find the monthly payment with their new loan.

d. Find the total interest they will pay if they do *not* get a new loan.

e. Find the total interest they will pay if they *do* get a new loan.

f. Should the Wolfs refinance their home? Why or why not?

▶ **34.** Russ and Roz Rosow have a ten-year-old loan with which they purchased their house. Their interest rate is $10\frac{5}{8}$%. Since they obtained this loan, interest rates have dropped, and they can now get a loan for $9\frac{1}{4}$% through their credit union. Because of this, the Rosows are considering refinancing their home. Each loan is a 30-year simple interest amortized loan, and neither has a prepayment penalty. The existing loan is for $112,000, and the new loan would be for the current amount due on the old loan.

a. Find their monthly payment with the existing loan.

b. Find the loan amount for their new loan.

c. Find the monthly payment with their new loan.

d. Find the total interest they will pay if they do *not* get a new loan.

e. Find the total interest they will pay if they *do* get a new loan.

f. Should the Rosows refinance their home? Why or why not?

▶ **35.** Michael and Lynn Sullivan have a ten-year-old loan for \$187,900 with which they purchased their house. They just sold their highly profitable import-export business and are considering paying off their home loan. Their loan is a thirty-year simple interest amortized loan at 10.5% interest and has no prepayment penalty.

a. Find their monthly payment.

b. Find the unpaid balance of the loan.

c. Find the amount of interest they will save by prepaying.

d. The Sullivans decided that if they paid off their loan, they would deposit the equivalent of half their monthly payment into an annuity. If the ordinary annuity pays 9% interest, find its future value after 20 years.

e. The Sullivans decided that if they do not pay off their loan, they would deposit an amount equivalent to their unpaid balance into an account that pays $9\frac{3}{4}$% interest compounded monthly. Find the future value of this account after 20 years.

f. Should the Sullivans prepay their loan? Why or why not?

36. Charlie and Ellen Wilson have a twenty-five-year-old loan for \$47,000 with which they purchased their house. The Wilsons are retired and are living on a fixed income, so they are contemplating paying off their home loan. Their loan is a thirty-year simple interest amortized loan at $4\frac{1}{2}$% and has no prepayment penalty. They also have savings of \$73,000, which they have invested in a certificate of deposit currently paying $8\frac{1}{4}$% interest compounded monthly. Should they pay off their home loan? Why or why not?

37. Ray and Helen Lee bought a house for \$189,500. They put 10% down, borrowed 80% from their bank for thirty years at $11\frac{1}{2}$%, and convinced the owner to take a second mortgage for the remaining 10%. That 10% is due in full in five years (this is called a *balloon payment*), and the Lees agree to make monthly interest-only payments to the seller at 12% simple interest in the interim.

a. Find the Lees' down payment

b. Find the amount that the Lees borrowed from their bank.

c. Find the amount that the Lees borrowed from the seller.

d. Find the Lees' monthly payment to the bank.

e. Find the Lees' monthly interest payment to the seller.

▶ **38.** Jack and Laurie Worthington bought a house for \$163,700. They put 10% down, borrowed 80% from their bank for thirty years at 12% interest, and convinced the owner to take a second mortgage for the remaining 10%. That 10% is due in full in five years, and the Worthingtons agree to make monthly interest-only payments to the seller at 12% simple interest in the interim.

a. Find the Worthingtons' down payment

b. Find the amount that the Worthingtons borrowed from their bank.

c. Find the amount that the Worthingtons borrowed from the seller.

d. Find the Worthingtons' monthly payment to the bank.

e. Find the Worthingtons' monthly interest payment to the seller.

39. a. If the Lees in Exercise 37 save for their balloon payment with a sinking fund, find the size of the necessary monthly payment into that fund if their money earns 6% interest.

b. Find the Lees' total monthly payment for the first five years.

c. Find the Lees' total monthly payment for the last twenty-five years.

▶ **40. a.** If the Worthingtons in Exercise 38 save for their balloon payment with a sinking fund, find the size of the necessary monthly payment into that fund if their money earns 7% interest.

b. Find the Worthingtons' total monthly payment for the first five years.

c. Find the Worthingtons' total monthly payment for the last twenty-five years.

41. In July 2005, Tom and Betty bought a house for \$275,400. They put 20% down and financed the rest with a thirty-year loan at the then-current rate of 6%. In 2007, the real estate market crashed. In August 2009, they had to sell their house. The best they could get was \$142,000. Was this enough to pay off the loan? It so, how much did they profit? It not, how much did they have to pay out of pocket to pay off the loan?

▶ **42.** In August 2004, Bonnie Martin bought a house for \$395,000. She put 20% down and financed the rest with a thirty-year loan at the then-current rate of $5\frac{3}{4}$%. In 2007, the real estate market crashed. In June 2009, she had to sell her house. The best she could get was \$238,000. Was this enough to pay off the loan? If so, how much did she profit? It not, how much did she have to pay out of pocket to pay off the loan?

43. Al-Noor Koorji bought a house for \$189,000. He put 20% down and financed the rest with a thirty-year loan at $5\frac{3}{4}$%.

a. Find his monthly payment.

b. If he paid an extra $100 per month, how early would his loan be paid off?

(*HINT:* This involves some trial-and-error work.)

▶ **44.** The Franklins bought a house for $265,000. They put 20% down and financed the rest with a thirty-year loan at $5\frac{3}{8}\%$.

a. Find their monthly payment.

b. If they paid an extra $85 per month, how early would their loan be paid off?

(*HINT:* This involves some trial-and-error work.)

Answer the following questions using complete sentences and your own words.

• CONCEPT QUESTIONS

45. In Exercise 11, an add-on interest amortized loan required larger payments than did a simple interest amortized loan at the same interest rate. What is there about the structure of an add-on interest loan that makes its payments larger than those of a simple interest amortized loan?

(*HINT:* The interest portion is a percentage of what quantity?)

46. Why are the computations for the last period of an amortization schedule different from those for all preceding periods?

47. Give two different situations in which a borrower would need the information contained in an amortization schedule.

48. A borrower would need to know the unpaid balance of a loan if the borrower were to prepay the loan. Give three different situations in which it might be in a borrower's best interests to prepay a loan.

(*HINT:* See some of the preceding exercises.)

49. If you double the period of an amortized loan, does your monthly payment halve? Why or why not?

50. If you double the loan amount of an amortized loan, does your monthly payment double? Why or why not?

WEB PROJECTS

51. This is an exercise in buying a car. It involves choosing a car and selecting the car's financing. Write a paper describing all of the following points.

a. You might not be in a position to buy a car now. If that is the case, fantasize about your future. What job do you have? How long have you had that job? What is your salary? If you are married, does your spouse work? Do you have a family? What needs will your car fulfill? Make your fantasy realistic. Briefly describe what has happened between the present and your future fantasy. (If you are in a position to buy a car now, discuss these points on a more realistic level.)

b. Go shopping for a car. Look at new cars, used cars, or both. Read newspaper and magazine articles about your choices (see, for example, *Consumer Reports*, *Motor Trend*, and *Road and Track*). Discuss in detail the car you selected and why you did so. How will your selection fulfill your (projected) needs? What do newspapers and magazines say about your selection? Why did you select a new or a used car?

c. Go to the web sites of four different banks. Get all of the information you need about a car loan. Perform all appropriate computations yourself—do not have the lenders tell you the payment size, and do not use web calculators. Summarize the appropriate data (including the down payment, payment size, interest rate, duration of loan, type of loan, and loan fees) in your paper, and discuss which loan you would choose. Explain how you would be able to afford your purchase.

52. This is an exercise in buying a home. It involves choosing a home and selecting the home's financing. Write a paper describing all of the following points.

a. You might not be in a position to buy a home now. If this is the case, fantasize about your future. What job do you have? How long have you had that job? What is your salary? If you are married, does your spouse work? Do you have a family? What needs will your home fulfill? Make your future fantasy realistic. Briefly describe what has happened between the present and your future fantasy. (If you are in a position to buy a home now, discuss these points on a more realistic level.)

b. Go shopping for a home. Look at houses, condominiums, or both. Look at new homes, used homes, or both. Used homes can easily be visited by going to an "open house," where the owners are gone and the real estate agent allows interested parties to inspect the home. Open houses are probably listed in your local newspaper. Read appropriate newspaper and magazine articles (for example, in the real estate section of your local newspaper). Discuss in detail the home you selected and why you did so. How will your selection fulfill your (projected) needs? Why did you select a house or a condominium? Why did you select a new or a used home? Explain your choice of location, house size, and features of the home.

c. Go to the web sites of four different banks. Get all of the information you need about a home loan. Perform all appropriate computations yourself—do not have the lenders tell you the payment size, and do not use web calculators. Summarize the appropriate data in your paper and discuss which loan you would choose. Include in your discussion the down payment, the duration of the loan, the interest rate, the payment size, and other terms of the loan.

d. Also discuss the real estate taxes (your instructor will provide you with information on the local tax rate) and the effect of your purchase on your income taxes (interest paid on a home loan is deductible from the borrower's income taxes).

e. Most lenders will approve a home loan only if the total of all the borrower's monthly payments, including the home loan payment, real estate taxes, credit card payments, and car loan payments, is no more than 38% of the borrower's monthly income. Discuss your ability to qualify for the loan.

Some useful links for the above web projects are listed on the text web site: **www.cengage.com/math/johnson**

• PROJECTS

*For Exercises 53–55, note the following information. An **adjustable-rate mortgage** (or ARM) is, as the name implies, a mortgage in which the interest rate is allowed to change. As a result, the payment changes too. At first, an ARM costs less than a fixed-rate mortgage—its initial interest rate is usually two or three percentage points lower. As time goes by, the rate is adjusted. As a result, it might or might not continue to hold this advantage.*

53. Trustworthy Savings offers a thirty-year adjustable-rate mortgage with an initial rate of 5.375%. The rate and the required payment are adjusted annually. Future rates are set at 2.875 percentage points above the 11th District Federal Home Loan Bank's cost of funds. Currently, that cost of funds is 4.839%. The loan's rate is not allowed to rise more than two percentage points in any one adjustment, nor is it allowed to rise above 11.875%. Trustworthy Savings also offers a thirty-year fixed-rate mortgage with an interest rate of 7.5%.

a. Find the monthly payment for the fixed-rate mortgage on a loan amount of $100,000.

b. Find the monthly payment for the ARM's first year on a loan amount of $100,000.

c. How much would the borrower save in the mortgage's first year by choosing the adjustable rather than the fixed-rate mortgage?

d. Find the unpaid balance at the end of the ARM's first year.

e. Find the interest rate and the value of *n* for the ARM's second year if the 11th District Federal Home Loan Bank's cost of funds does not change during the loan's first year.

f. Find the monthly payment for the ARM's second year if the 11th District Federal Home Loan Bank's cost of funds does not change during the loan's first year.

g. How much would the borrower save in the mortgage's first two years by choosing the adjustable-rate mortgage rather than the fixed-rate mortgage if the cost of funds does not change?

h. Discuss the advantages and disadvantages of an adjustable-rate mortgage.

54. American Dream Savings Bank offers a thirty-year adjustable-rate mortgage with an initial rate of 4.25%. The rate and the required payment are adjusted annually. Future rates are set at three percentage points above the one-year Treasury bill rate, which is currently 5.42%. The loan's rate is not allowed to rise more than two percentage points in any one adjustment, nor is it allowed to rise above 10.25%. American Dream Savings Bank also offers a thirty-year fixed-rate mortgage with an interest rate of 7.5%.

a. Find the monthly payment for the fixed-rate mortgage on a loan amount of $100,000.

b. Find the monthly payment for the ARM's first year on a loan amount of $100,000.

c. How much would the borrower save in the mortgage's first year by choosing the adjustable rather than the fixed-rate mortgage?

d. Find the unpaid balance at the end of the ARM's first year.

e. Find the interest rate and the value of *n* for the ARM's second year if the one-year Treasury bill rate does not change during the loan's first year.

f. Find the monthly payment for the ARM's second year if the Treasury bill rate does not change during the loan's first year.

g. How much would the borrower save in the mortgage's first two years by choosing the adjustable-rate mortgage rather than the fixed-rate mortgage if the Treasury bill rate does not change?

h. Discuss the advantages and disadvantages of an adjustable-rate mortgage.

55. Bank Two offers a thirty-year adjustable-rate mortgage with an initial rate of 3.95%. This initial rate is in effect for the first six months of the loan, after which it is adjusted on a monthly basis. The monthly payment is adjusted annually. Future rates are set at 2.45 percentage points above the 11th District Federal Home Loan Bank's cost of funds. Currently, that cost of funds is 4.839%.

a. Find the monthly payment for the ARM's first year on a loan amount of $100,000.

b. Find the unpaid balance at the end of the ARM's first six months.

c. Find the interest portion of the seventh payment if the cost of funds does not change.

d. Usually, the interest portion is smaller than the monthly payment, and the difference is subtracted from the unpaid balance. However, the interest portion found in part (c) is larger than the monthly payment found in part (a), and the difference is added to the unpaid balance found in part (b). Why would this difference be added to the unpaid balance? What effect will this have on the loan?

e. The situation described in part (d) is called **negative amortization.** Why?

f. What is there about the structure of Bank Two's loan that allows negative amortization?

Amortization Schedules on a Computer

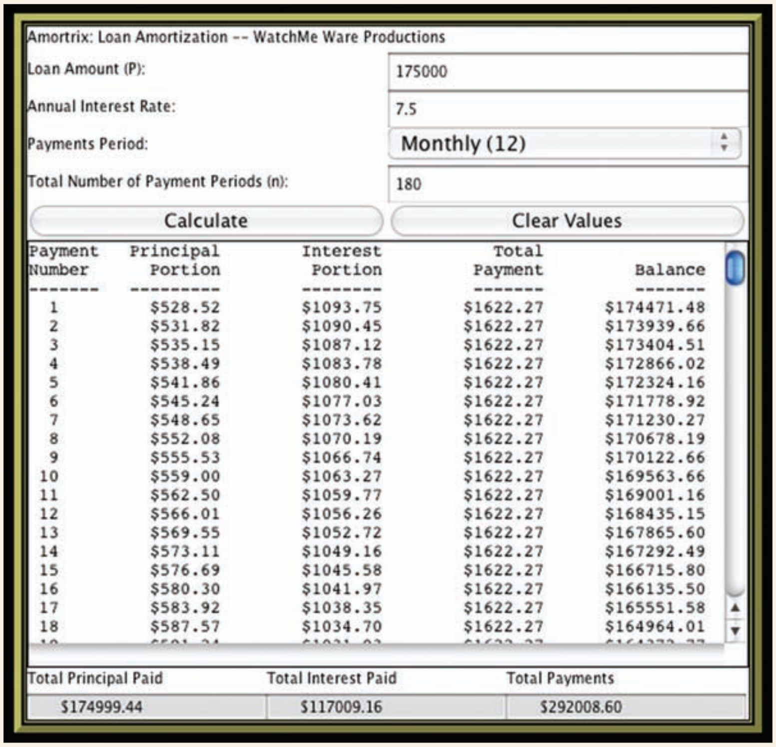
Interest paid on a home loan is deductible from the borrower's income taxes, and interest paid on a loan by a business is usually deductible. A borrower with either of these types of loans needs to know the total interest paid on the loan during the final year. The way to determine the total interest paid during a given year is to prepare an amortization schedule for that year. Typically, the lender provides the borrower with an amortization schedule, but it is not uncommon for this schedule to arrive after taxes are due. In this case, the borrower must either do the calculation personally or pay taxes without the benefit of the mortgage deduction and then file an amended set of tax forms after the amortization schedule has arrived.

Computing a year's amortization schedule is rather tedious, and neither a scientific calculator nor a graphing calculator offers relief. The best tool for the job is a computer, combined either with the Amortrix computer program (available with this book) or with a computerized spreadsheet. Each is discussed below.

Amortization Schedules and Amortrix

Amortrix is one of the features of the text web site (**www.cengage.com/math/johnson**). This software will enable you to quickly and easily compute an amortization schedule for any time period. We'll illustrate this process by using Amortrix to prepare an amortization schedule for a fifteen-year $175,000 loan at 7.5% interest with monthly payments.

When you start Amortrix, a main menu appears. Click on the "Amortization Schedule" option. Once you're at the page labeled "Amortization Schedule," enter "175000" (without commas or dollar signs) for the Loan Amount (P), "7.5" (without a percent sign) for the Annual Interest Rate, and "180" for the Total Number of Payment Periods (n). See the top part of Figure 5.21. Click on "Calculate" and the software will create an amortization schedule. See the bottom part of Figure 5.21.



Amortrix: Loan Amortization -- WatchMe Ware Productions

Loan Amount (P):	175000
Annual Interest Rate:	7.5
Payments Period:	Monthly (12)
Total Number of Payment Periods (n):	180
Calculate	Clear Values

Payment Number	Principal Portion	Interest Portion	Total Payment	Balance
1	$528.52	$1093.75	$1622.27	$174471.48
2	$531.82	$1090.45	$1622.27	$173939.66
3	$535.15	$1087.12	$1622.27	$173404.51
4	$538.49	$1083.78	$1622.27	$172866.02
5	$541.86	$1080.41	$1622.27	$172324.16
6	$545.24	$1077.03	$1622.27	$171778.92
7	$548.65	$1073.62	$1622.27	$171230.27
8	$552.08	$1070.19	$1622.27	$170678.19
9	$555.53	$1066.74	$1622.27	$170122.66
10	$559.00	$1063.27	$1622.27	$169563.66
11	$562.50	$1059.77	$1622.27	$169001.16
12	$566.01	$1056.26	$1622.27	$168435.15
13	$569.55	$1052.72	$1622.27	$167865.60
14	$573.11	$1049.16	$1622.27	$167292.49
15	$576.69	$1045.58	$1622.27	$166715.80
16	$580.30	$1041.97	$1622.27	$166135.50
17	$583.92	$1038.35	$1622.27	$165551.58
18	$587.57	$1034.70	$1622.27	$164964.01

Total Principal Paid	Total Interest Paid	Total Payments
$174999.44	$117009.16	$292008.60

FIGURE 5.21 Part of an amortization schedule prepared with Amortrix.

The software will *not* correctly compute the last loan payment; it will compute the last payment in the same way it computes all other payments rather than in the way shown earlier in this section. You will have to correct this last payment if you use the computer to prepare an amortization schedule for a time period that includes the last payment.

Amortization Schedules and Excel

A **spreadsheet** is a large piece of paper marked off in rows and columns. Accountants use spreadsheets to organize numerical data and perform computations. A **computerized spreadsheet** such as Microsoft Excel is a computer program that mimics the appearance of a paper spreadsheet. It frees the user from performing any computations.

When you start a computerized spreadsheet, you see something that looks like a table waiting to be filled in. The rows are labeled with numbers and the columns are labeled with letters, as shown in Figure 5.22. The individual boxes are called **cells.** The cell in column A, row 1 is called cell A1.

◇	A	B	C	D	E	F	G
1	payment number	principal portion	interest portion	total payment	balance		
2	0				$175,000.00	rate	7.50%
3						years	15
4						payments/yr	12
5							

FIGURE 5.22 The spreadsheet after step 1.

Excel is an ideal tool to use in creating an amortization schedule. We will illustrate this process by preparing the amortization schedule for a fifteen-year $175,000 loan at 7.5% interest with monthly payments.

1. *Set up the spreadsheet*. Start by entering the information shown in Figure 5.22.

- Adjust the columns' widths. To make column A wider, place your cursor on top of the line dividing the "A" and "B" labels. Then use your cursor to move that dividing line.
- Format cell E2 as currency by highlighting that cell and pressing the "$" button on the Excel ribbon at the top of the screen.
- Be certain that you include the % symbol in cell G2.
- Save the spreadsheet.

If you have difficulty creating a spreadsheet that looks like this, you may download it from our web site at **www.cengage.com/math/johnson.** See the file 5.4.xls and select the "step 1" sheet. Other sheets show later steps.

2. *Compute the monthly payment*. While you can do this with your calculator, as we discussed earlier in this section, you can also use the Excel "PMT" function to compute the monthly payment. This allows us to change the rate (in cell G2) or the loan amount (in cell E2) and Excel will automatically compute the new payment and change the entire spreadsheet accordingly. To use the "PMT" function, do the following:

- Click on cell D3, where we will put the total payment.
- Click on the *fx* button at the top of the screen.
- In the resulting "Paste Function" box, select "Financial" under "Function Category", select "PMT" under "Function name", and press "OK." A "PMT" box will appear.

- Cell G2 holds 7.5%, the annual interest rate, and cell G4 holds 12, the number of periods/year. The periodic interest rate is $7.5\%/12 = \text{G2}/\text{G4}$. In the PMT box after "Rate", type "$G$2/$G$4".
- The number of periods is $15 \cdot 12 = \text{G3} \cdot \text{G4}$. In the PMT box after "Nper", type "$G$3*$G$4."
- After "PV", type "$E$2". This is where we have stored the present value of the loan.
- Do not type anything after "Fv" or "type".
- Press "OK" at the bottom of the "PMT" box, and "($1,622.27)" should appear in cell D2. The parentheses mean that the number is negative.
- Save the spreadsheet.

Doing the above creates two problems that we must fix. The payment is negative. We can fix that with an extra minus sign. Also, the payment is not rounded to the nearest penny. We can fix that with the ROUND function.

In cell D2, Excel displays ($1622.27), but it stores −1622.27163. . . , and all calculations involving cell D2 will be done by using −1622.27163. . . . This will make the amortization schedule incorrect. To fix this, click on cell D3 and the long box at the top of the screen will have

=PMT($G$2/$G$4,$G$3*$G$4,$E$2)

in it. Change this to

=ROUND (−PMT($G$2/$G$4,$G$3*$G$4,$E$2),2)

↑ a new minus sign

the round fuction **part of the round function**

The new parts here are "ROUND(−" at the beginning and ",2)" at the end. The "ROUND (,2)" part tells Excel to round to two decimal places, as a bank would. The minus sign after "ROUND(" makes the payment positive. Place your cursor where the new parts go, and add them.

3. *Fill in row 3.*

- We need to add a year each time we go down one row. Type "= A2 + 1" in cell A3.
- Cell C3 should contain instructions on computing simple interest on the previous balance.

$$\begin{aligned} I &= P \cdot r \cdot t \\ &= \text{the previous balance} \cdot 0.075 \cdot 1/12 \\ &= \text{E2*\$G\$2/\$G\$4} \end{aligned}$$

  Move to cell C3 and type in "=ROUND(E2*$G$2/$G$4,2)" and press "return". The "ROUND (,2)" part rounds the interest to two decimal places, as a bank would.

- Cell B3 should contain instructions on computing the payment's principal portion.

$$\begin{aligned} \text{Principal portion} &= \text{payment} - \text{interest portion} \\ &= \text{D3} - \text{C3} \end{aligned}$$

  In cell B3, type "= D3 − C3".

- Cell E3 should contain instructions on computing the new balance.

$$\begin{aligned} \text{New balance} &= \text{previous balance} - \text{principal portion} \\ &= \text{E2} - \text{B3} \end{aligned}$$

  In cell E3, type "= E2 − B3".

See Figure 5.23.

	A	B	C	D	E	F	G
1	payment number	principal portion	Interest portion	total payment	balance		
2	0				$175,000.00	rate	7.50%
3	1	$ 528.52	$1,093.75	$1,622.27	$174,471.48	years	15
4						payments/yr	12
5							

FIGURE 5.23 The spreadsheet after completing step 3.

4. *Fill in rows 4 and beyond*. All of the remaining payments' computations are just like payment 1 computations (except for the last payment), so all we have to do is copy the payment 1 instructions in row 3 and paste them in rows 4 and beyond. After completion of this step, the top and bottom of your spreadsheet should look like that in Figure 5.24.

	A	B	C	D	E	F	G
1	payment number	principal portion	interest portion	total payment	balance		
2	0				$175,000.00	rate	7.50%
3	1	$ 528.52	$1,093.75	$1,622.27	$174,471.48	years	15
4	2	$ 531.82	$1,090.45	$1,622.27	$173,939.66	payments/yr	12
5	3	$ 535.15	$1,087.12	$1,622.27	$173,404.51		
6	4	$ 538.49	$1,083.78	$1,622.27	$172,866.02		

The top of the spreadsheet, and ...

178	176	$ $1,572.51	$49.76	$1,622.27	$ 6,389.50
179	177	$ $1,582.34	$39.93	$1,622.27	$ 4,807.16
180	178	$ $1,592.23	$30.04	$1,622.27	$ 3,214.93
181	179	$ $1,602.18	$20.09	$1,622.27	$ 1,612.75
182	180	$ $1,612.19	$10.08	$1,622.27	$ 0.56
183					

... the bottom of the spreadsheet, after completing step 4

FIGURE 5.24 The final spreadsheet.

5. *Fix the last payment*. Clearly, the ending balance of $0.56 in cell E182 is not correct—we cannot owe $0.56 or any other amount after making our last payment. See Exercise 56.

6. *Find the interest paid*. Typing "SUM(C3:C6)" in a cell will result in the sum of cells C4, C4, C5, and C6 appearing in that cell. Using this function makes it easy to find the total interest paid for any time period.

EXERCISES

In Exercises 56–62, first use Amortrix or Excel to create an amortization schedule for a fifteen-year $175,000 loan at 7.5% interest with monthly payments. (Figure 5.24 shows the first and last lines of this amortization schedule, created with Excel.) You will alter parts of this amortization schedule to answer Exercises 56–62.

56. **a.** The very last line of your schedule is incorrect, because the calculations for the last line of any amortization schedule are done differently from those of all other lines. What should the last line be?

b. If you use Excel, state exactly what you should type in row 182 to fix the last payment.

57. Use Excel or Amortrix to find the total interest paid in
  a. the loan's first year and
  b. the loan's last year if the first month of the loan is January.
  c. If you use Excel, state exactly what you type in parts (a) and (b) to find the total interest paid.

58. Right before you signed your loan papers, the interest rate dropped from 7.5% to 7%.
  a. How does this affect the total payment?
  b. How does this affect the total interest paid in the loan's first year if the first month of the loan is January?
  c. How does this affect the total interest paid in the loan's last year?

59. Right before you signed your loan papers, you convinced the seller to accept a smaller price. As a result, your loan amount dropped from $175,000 to $165,000. (The interest rate remains at 7.5%)
  a. How does this affect the total payment?
  b. How does this affect the total interest paid in the loan's first year if the first month of the loan is January?
  c. How does this affect the total interest paid in the loan's last year?

60. You are considering a twenty-year loan. (The interest rate remains at 7.5% and the loan amount remains at $175,000.)
  a. How does this affect the total payment?
  b. How does this affect the total interest paid in the loan's first year if the first month of the loan is January?
  c. How does this affect the total interest paid in the loan's fifteenth year?
  d. Explain, without performing any calculations, why the total payment should go down and the total interest paid during the first year should go up.

61. You are considering a thirty-year loan. (The interest rate remains at 7.5%, and the loan amount remains at $175,000.)
  a. How does this affect the total payment?
  b. How does this affect the total interest paid in the loan's first year if the first month of the loan is January?
  c. How does this affect the total interest paid in the loan's fifteenth year?
  d. Explain, without performing any calculations, why the total payment should go down and the total interest paid during the first year should go up.

62. You are considering a loan with payments every two weeks. (The interest rate remains at 7.5%, the loan amount remains at $175,000, and the term remains at fifteen years.)
  a. How does this affect the total payment?
  b. How does this affect the total interest paid in the loan's first year if the first month of the loan is January?
  c. How does this affect the total interest paid in the loan's last year?
  d. Explain, without performing any calculations, why the total payment and the total interest paid during the first year should both go down.

In Exercises 63–66:

  a. *Use Amortrix or Excel to prepare an amortization schedule for the given loan.*
  b. *Find the amount that could be deducted from the borrower's taxable income (that is, find the total interest paid) in the loan's first year if the first payment was made in January. (Interest on a car loan is deductible only in some circumstances. For the purpose of these exercises, assume that this interest is deductible.)*
  c. *Find the amount that could be deducted from the borrower's taxable income (that is, find the total interest paid) in the loan's last year.*
  d. *If you sell your home or car after three years, how much will you still owe on the mortgage?*

63. A five-year simple interest amortized car loan for $32,600 at 12.25% interest

64. A four-year simple interest amortized car loan for $26,200 at 5.75% interest

65. A fifteen-year simple interest amortized home loan for $220,000 at 6.25% interest

66. A thirty-year simple interest amortized home loan for $350,000 at 14.5% interest

67. Use Amortrix or Excel to do Exercise 29 on page 379. Is your answer the same as that of Exercise 29? If not, why not? Which answer is more accurate? Why?

68. Use Amortrix or Excel to do Exercise 30 on page 379. Is your answer the same as that of Exercise 30? If not, why not? Which answer is more accurate? Why?

69. Use Amortrix or Excel to do Exercise 31 on page 379. Is your answer the same as that of Exercise 31? If not, why not? Which answer is more accurate? Why?

70. Use Amortrix or Excel to do Exercise 32 on page 379. Is your answer the same as that of Exercise 32? If not, why not? Which answer is more accurate? Why?

71. *For Excel users only:* When interest rates are high, some lenders offer the following type of home loan. You borrow $200,000 for thirty years. For the first five years, the interest rate is 5.75%, and the payments are calculated as if the interest rate were going to remain unchanged for the life of the loan. For the last twenty-five years, the interest rate is 9.25%.
  a. What are the monthly payments for the first five years?
  b. What are the monthly payments for the last twenty-five years?
  c. What is the total interest paid, during the loan's fifth year if the first month of the loan is January?
  d. What is the total interest paid during the loan's sixth year?
  e. Why would a lender offer such a loan?

5.5 Annual Percentage Rate with a TI's TVM Application

Objectives

- Understand what an annual percentage rate (APR) is
- Find an APR
- Use APRs to compare loans

A **simple interest loan** is any loan for which the interest portion of each payment is simple interest on the outstanding principal. A *simple interest amortized loan* fulfills this requirement; in fact, we compute an amortization schedule for a simple interest amortized loan by finding the simple interest on the outstanding principal.

The APR of an Add-on Interest Loan

Whenever a loan is not a simple interest loan, the Truth in Lending Act requires the lender to disclose the annual percentage rate to the borrower. The **annual percentage rate (APR) of an add-on interest loan** is the simple interest rate that makes the dollar amounts the same if the loan is recomputed as a simple interest amortized loan.

We'll explore APRs in this section. They are important because they allow someone who is shopping for a loan to compare loans with different terms and to determine which loan is better.

EXAMPLE 1

FINDING THE APR In Example 5 in Section 5.1, Chip Douglas bought a nine-year-old Ford that's "like new" from Centerville Auto Sales for \$5,988. He financed the purchase with the dealer's two-year, 5% add-on interest loan, which required a \$600 down payment and monthly payments of \$246.95 a month for twenty-four months. Use a TI-83/84's TVM application to find the APR of the loan.

SOLUTION

1. Press APPS, select option 1: "Finance", and press ENTER.
2. Select option 1: "TVM Solver", and press ENTER.
3. Enter appropriate values for the variables:
   - N is the number of payments. Enter 24 for N.
   - I% is the annual interest rate. This is the number we're trying to find, so temporarily enter 0. Later, we'll solve for the actual value of N.
   - PV is the present value, which is \$5988 − \$600 = \$5388.
   - PMT is the payment. The payment is \$246.95, but it's an outgoing amount of money (since we give it to the bank), so we enter –246.95 for PMT. You must enter a negative number for any outgoing amount of money.
   - FV is the future value. There is no future value here, so we enter 0.
   - P/Y is the number of periods per year. We make monthly payments, so there are twelve periods per year. Enter 12 for P/Y.
   - C/Y is automatically made to be the same as P/Y. In this text, we will never encounter a situation in which C/Y is different from P/Y. See Figure 5.25.

```
N=24
I%=0
PV=5388
PMT=-246.95
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

FIGURE 5.25

Preparing the TVM screen.

4. To solve for I%, use the arrow buttons to highlight the 0 that we entered for it earlier. Then press ALPHA SOLVE. (Pressing ALPHA makes the ENTER button become the

HISTORICAL NOTE

TRUTH IN LENDING ACT

Courtesy of LBJ Library

The **Truth in Lending Act** was signed by President Lyndon Johnson in 1968. Its original intent was to promote credit shopping by requiring lenders to use uniform language and calculations and to make full disclosure of credit charges so that consumers could shop for the most favorable credit terms. The Truth in Lending Act is interpreted by the Federal Reserve Board's Regulation Z.

During the 1970s, Congress amended the act many times, and corresponding changes were made in Regulation Z. These changes resulted in a huge increase in the length and complexity of the law. Its scope now goes beyond disclosure to grant significant legal rights to the borrower.

SOLVE button.) As a result, we find that I% is 9.323544% ≈ 9.32%, and the add-on interest loan has an APR of 9.32%. This means that the 5% add-on interest loan requires the same monthly payment that a 9.32% simple interest amortized loan would have!

The Truth in Lending Act, which requires a lender to divulge its loans' APR, allows a tolerance of one-eighth of 1% (0.125%) in the claimed APR. Thus, the dealership would be legally correct if it stated that the APR was between 9.323544% − 0.125% = 9.198544% and 9.323544% + 0.125% = 9.448544%.

The APR of a Simple Interest Loan

Sometimes **finance charges** other than the interest portion of the monthly payment are associated with a loan; these charges must be paid when the loan agreement is signed. For example, a **point** is a finance charge that is equal to 1% of the loan amount; a **credit report fee** pays for a report on the borrower's credit history, including any late or missing payments and the size of all outstanding debts; an **appraisal fee** pays for the determination of the current market value of the property (the auto, boat, or home) to be purchased with the loan. The Truth in Lending Act requires the lender to inform the borrower of the total finance charge, which includes the interest, the points, and some of the fees (see Figure 5.26). Most of the finance

Costs Included in the Prepaid Finance Charges		***Other Costs Not Included in the Finance Charge***	
2 points	$2,415.32	appraisal fee	$ 70.00
prorated interest	$1,090.10	credit report	$ 60.00
prepaid mortgage insurance	$ 434.50	closing fee	$ 670.00
loan fee	$1,242.00	title insurance	$ 202.50
document preparation fee	$ 80.00	recording fee	$ 20.00
tax service fee	$ 22.50	notary fee	$ 20.00
processing fee	$ 42.75	tax and insurance escrow	$ 631.30
subtotal	$5,327.17	subtotal	$1,673.80

FIGURE 5.26 Sample portion of a federal truth-in-lending disclosure statement (loan amount = $120,765.90).

charges must be paid before the loan is awarded, so in essence the borrower must pay money now in order to get more money later. The law says that this means the lender is not really borrowing as much as he or she thinks. According to the law, the actual amount loaned is the loan amount minus all points and those fees included in the finance charge. The **APR of a simple interest amortized loan** is the rate that reconciles the payment and this actual loan amount.

EXAMPLE 2

FINDING THE APR OF A SIMPLE INTEREST LOAN Glen and Tanya Hansen bought a home for $140,000. They paid the sellers a 20% down payment and obtained a simple interest amortized loan for the balance from their bank at $10\frac{3}{4}\%$ for thirty years. The bank in turn paid the sellers the remaining 80% of the purchase price, less a 6% sales commission paid to the sellers' and the buyers' real estate agents. (The transaction is illustrated in Figure 5.27.) The bank charged the Hansens 2 points plus fees totaling $3,247.60; of these fees, $1,012.00 were included in the finance charge.

a. Find the size of the Hansens' monthly payment.
b. Find the total interest paid.
c. Compute the total finance charge.
d. Find the APR

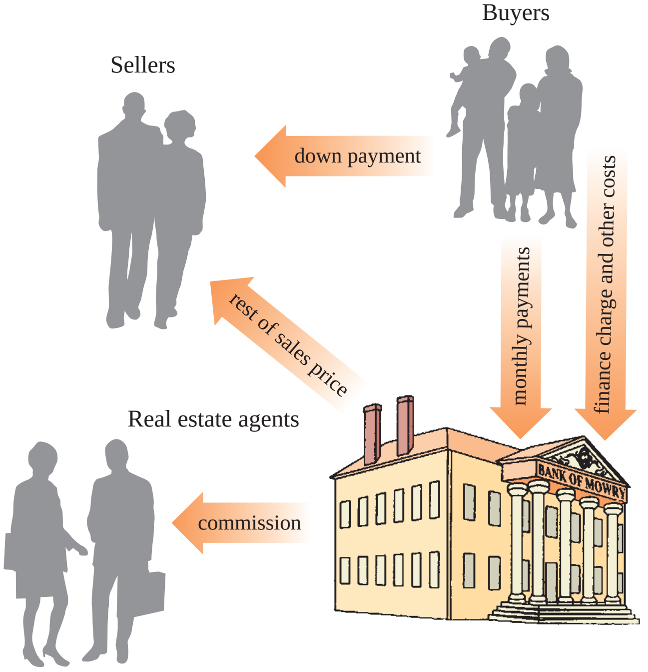


FIGURE 5.27 Buying a home.

SOLUTION

a. *Finding their monthly payment* We are given down payment = 20% of $140,000 = $28,000, P = loan amount = $140,000 − $28,000 = $112,000, $i = \frac{1}{12}$ of $10\frac{3}{4}\% = \frac{0.1075}{12}$, and n = 30 years = 30 years · (12 months)/(1 year) = 360 months.

$$\text{Future value of annuity} = \text{future value of loan amount}$$

$$pymt\frac{(1+i)^n - 1}{i} = P(1+i)^n$$

$$pymt\frac{\left(1+\frac{0.1075}{12}\right)^{360} - 1}{\frac{0.1075}{12}} = 112{,}000\left(1+\frac{0.1075}{12}\right)^{360}$$

Computing the fraction on the left side and multiplying its reciprocal by the right side, we get

$$pymt = 1{,}045.4991 \approx \$1{,}045.50$$

b. *Finding the total interest paid* The total interest paid is the total amount paid minus the amount borrowed. The Hansens agreed to make 360 monthly payments of $1045.50 each, for a total of 360 · $1,045.50 = $376,380.00. Of this, $112,000 is principal; therefore, the total interest is

$$376{,}380 - 112{,}000 = \$264{,}380$$

c. *Computing their total finance charge*

$$\begin{aligned} \text{2 points} = \text{2\% of } \$112{,}000 &= \$\ \ \ 2{,}240 \\ \text{included fees} &= \$\ \ \ 1{,}012 \\ \text{total interest paid} &= \underline{\$264{,}380} \quad \textbf{from part (b)} \\ \text{total finance charge} &= \$267{,}632 \end{aligned}$$

d. *Finding the APR* The APR is the simple interest rate that makes the dollar amounts the same if the loan is recomputed using the legal loan amount (loan amount less points and fees) in place of the actual loan amount. The legal loan amount is

$$P = \$112{,}000 - \$3252 = \$108{,}748$$

We'll find the APR with the TVM application.

1. Press APPS, select option 1: "Finance", and press ENTER.
2. Select option 1: "TVM Solver", and press ENTER.
3. Enter appropriate values for the variables.

- N is the number of payments. Enter 360.
- I% is the annual interest rate. This is the number we're trying to find, so temporarily enter 0. Later, we'll solve for the actual value of N.
- PV is the present value, which is $108,748.
- PMT is the payment. The payment is $1,045.50, but it's an outgoing amount of money (since we give it to the bank), so we enter −1045.5 for PMT. You must enter a negative number for any outgoing amount of money.
- FV is the future value. There is no future value here, so we enter 0.
- P/Y is the number of periods per year. We make monthly payments, so there are twelve periods per year. Enter 12 for P/Y.
- C/Y is automatically made to be the same as P/Y. In this text, we will never encounter a situation in which C/Y is different from P/Y. See Figure 5.28.

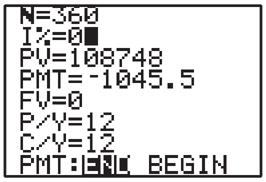


FIGURE 5.28
Preparing the TVM screen.

4. To solve for I%, use the arrow buttons to highlight the 0 that we entered for I%. Then press ALPHA SOLVE. (Pressing ALPHA makes the ENTER button becomes the SOLVE button.) As a result, we find that I% is 11.11993521% ≈ 11.12%, and the loan has an APR of 11.12%. This means the $10\frac{3}{4}$% loan requires the same monthly payment that a 11.12% loan with no points or fees would have. In other words, the points and fees in effect increase the interest rate from $10\frac{3}{4}$% to 11.12%.

If you need to obtain a loan, it is not necessarily true that the lender with the lowest interest rate will give you the least expensive loan. One lender may charge more points or higher fees than does another lender. One lender may offer an add-on interest loan, while another lender offers a simple interest amortized loan. These differences can have a significant impact on the cost of a loan. If two lenders offer loans at the same interest rate but differ in any of these ways, that difference will be reflected in the APR. "Lowest interest rate" does *not* mean "least expensive loan," but "lowest APR," *does* mean "least expensive loan."

Estimating Prepaid Finance Charges

As we have seen, a borrower usually must pay an assortment of fees when obtaining a home loan. These fees can be quite substantial, and they can vary significantly from lender to lender. For example, the total fees in Figure 5.26 were $7,000.97; the Hansens' total fees in Example 2 were $5,487.60. When shopping for a home loan, a borrower should take these fees into consideration.

All fees must be disclosed to the borrower when he or she signs the loan papers. Furthermore, the borrower must be given an estimate of the fees when he or she applies for the loan. Before an application is made, the borrower can use a loan's APR to obtain a reasonable approximation of those fees that are included in the finance charge (for a fixed rate loan). This would allow the borrower to make a more educated decision in selecting a lender.

EXAMPLE 3

ESTIMATING PREPAID FINANCE CHARGES Felipe and Delores Lopez are thinking of buying a home for $152,000. A potential lender advertises an 80%, thirty-year simple interest amortized loan at $7\frac{1}{2}$% interest with an APR of 7.95%.

a. Find the size of the Lopezes' monthly payment.
b. Use the APR to approximate the fees included in the finance charge.

SOLUTION

a. *Finding the monthly payment* The loan amount is 80% of $152,000 = $121,600, $i = \frac{1}{12}$ of 7.5% $= \frac{0.075}{12}$, and $n = 30$ years $= 360$ months.

$$\text{Future value of annuity} = \text{future value of loan amount}$$

$$pymt\frac{(1+i)^n - 1}{i} = P(1+i)^n$$

$$pymt\frac{\left(1+\frac{0.075}{12}\right)^{360} - 1}{\frac{0.075}{12}} = 121{,}600\left(1+\frac{0.075}{12}\right)^{360}$$

Computing the fraction on the left side and multiplying its reciprocal by the right side, we get

$$pymt = 850.2448 \approx \$850.25$$

b. *Approximating the fees included in the finance charge* We approximate the fees by computing the legal loan amount (loan amount less points and fees), using the APR as the interest rate and the payment computed in part (a) as *pymt*. Therefore, $i = \frac{1}{12}$ of 7.95% $= \frac{0.0795}{12}$, $pymt = 850.25$.

$$\text{Future value of annuity} = \text{future value of loan amount}$$

$$pymt\frac{(1+i)^n - 1}{i} = P(1+i)^n$$

$$850.25\frac{\left(1+\frac{0.0795}{12}\right)^{360} - 1}{\frac{0.0795}{12}} = P\left(1+\frac{0.0795}{12}\right)^{360}$$

Computing the left side, and dividing by $(1 + \frac{0.0795}{12})^{360}$, we get

$$P = 116{,}427.6304 \approx \$116{,}427.63$$

This is the legal loan amount, that is, the loan amount less points and fees. The Lopezes were borrowing $121,600, so that leaves $121,600 − $116,427.63 = $5,172.37 in points and fees. This is an estimate of the points and fees included in the finance charge, such as a loan fee, a document preparation fee, and a processing fee; it does not include fees such as an appraisal fee, a credit report fee, and a title insurance fee.

5.5 EXERCISES

▶ **1.** Wade Ellis buys a new car for $16,113.82. He puts 10% down and obtains a simple interest amortized loan for the balance at $11\frac{1}{2}$% interest for four years. If loan fees included in the finance charge total $814.14, find the APR.

▶ **2.** Guy de Primo buys a new car for $9,837.91. He puts 10% down and obtains a simple interest amortized loan for the balance at $10\frac{7}{8}$% interest for four years. If loan fees included in the finance charge total $633.87, find the APR.

▶ **3.** Chris Burditt bought a house for $212,500. He put 20% down and obtained a simple interest amortized loan for the balance at $10\frac{7}{8}$% interest for thirty years. If Chris paid 2 points and $4,728.60 in fees, $1,318.10 of which are included in the finance charge, find the APR.

▶ **4.** Shirley Trembley bought a house for $187,600. She put 20% down and obtained a simple interest amortized loan for the balance at $11\frac{3}{8}$% for thirty years. If Shirley paid 2 points and $3,427.00 in fees, $1,102.70 of which are included in the finance charge, find the APR.

▶ **5.** Jennifer Tonda wants to buy a used car that costs $4,600. The used car dealer has offered her a four-year add-on interest loan that requires no down payment at 8% annual interest, with an APR of $14\frac{1}{4}$%.

a. Find the monthly payment.

b. Verify the APR.

6. Melody Shepherd wants to buy a used car that costs $5,300. The used car dealer has offered her a four-year add-on interest loan that requires a $200 down payment at 7% annual interest, with an APR of 10%.

a. Find the monthly payment.

b. Verify the APR.

▶ **7.** Anne Scanlan is buying a used car that costs $10,340. The used car dealer has offered her a five-year add-on interest loan at 9.5% interest, with an APR of 9.9%. The loan requires a 10% down payment.

a. Find the monthly payment.

b. Verify the APR.

▶ **8.** Stan Loll bought a used car for $9,800. The used car dealer offered him a four-year add-on interest loan at 7.8% interest, with an APR of 8.0%. The loan requires a 10% down payment.

a. Find the monthly payment.

b. Verify the APR.

9. Susan Chin is shopping for a car loan. Her savings and loan offers her a simple interest amortized loan for four years at 9% interest. Her bank offers her a simple interest amortized loan for four years at 9.1% interest. Which is the less expensive loan?

10. Stephen Tamchin is shopping for a car loan. His credit union offers him a simple interest amortized loan for four years at 7.1% interest. His bank offers him a simple interest amortized loan for four years at 7.3% interest. Which is the less expensive loan?

11. Ruben Lopez is shopping for a home loan. Really Friendly Savings and Loan offers him a thirty-year simple interest amortized loan at 9.2% interest, with an APR of 9.87%. The Solid and Dependable Bank offers him a thirty-year simple interest amortized loan at 9.3% interest, with an APR of 9.80%. Which loan would have the lower payments? Which loan would be the least expensive, taking into consideration monthly payments, points, and fees? Justify your answers.

12. Keith Moon is shopping for a home loan. Sincerity Savings offers him a thirty-year simple interest amortized loan at 8.7% interest, with an APR of 9.12%. Pinstripe National Bank offers him a thirty-year simple interest amortized loan at 8.9% interest, with an APR of 8.9%. Which loan would have the lower payments? Which loan would be the least expensive, taking into consideration monthly payments, points, and fees? Justify your answers.

▶ **13.** The Nguyens are thinking of buying a home for $119,000. A potential lender advertises an 80%, thirty-year simple interest amortized loan at $8\frac{1}{4}$% interest, with an APR of 9.23%. Use the APR to approximate the fees included in the finance charge.

▶ **14.** Ellen Taylor is thinking of buying a home for $126,000. A potential lender advertises an 80%, thirty-year simple interest amortized loan at $10\frac{3}{4}$% interest, with an APR of 11.57%. Use the APR to approximate the fees included in the finance charge.

▶ **15.** James Magee is thinking of buying a home for $124,500. Bank of the Future advertises an 80%, thirty-year simple interest amortized loan at $9\frac{1}{4}$% interest, with an APR of 10.23%. R.T.C. Savings and Loan advertises an 80%, 30-year simple interest amortized loan at 9% interest with an APR of 10.16%.

a. Find James's monthly payment if he borrows through Bank of the Future.

b. Find James's monthly payment if he borrows through R.T.C. Savings and Loan.

c. Use the APR to approximate the fees included in the finance charge by Bank of the Future.

d. Use the APR to approximate the fees included in the finance charge by R.T.C. Savings and Loan.

e. Discuss the advantages of each of the two loans. Who would be better off with the Bank of the Future loan? Who would be better off with the R.T.C. loan?

▶ **16.** Holly Kresch is thinking of buying a home for $263,800. State Bank advertises an 80%, thirty-year simple interest amortized loan at $6\frac{1}{4}$% interest, with an APR of 7.13%.

Boonville Savings and Loan advertises an 80%, thirty-year simple interest amortized loan at $6\frac{1}{2}\%$ interest with an APR of 7.27%.

a. Find Holly's monthly payment if she borrows through State Bank.

b. Find Holly's monthly payment if she borrows through Boonville Savings and Loan.

c. Use the APR to approximate the fees included in the finance charge by State Bank.

d. Use the APR to approximate the fees included in the finance charge by Boonville Savings and Loan.

e. Discuss the advantages of the two loans. Who would be better off with the State Bank loan? Who would be better off with the Boonville loan?

Answer the following questions using complete sentences and your own words.

• CONCEPT QUESTIONS

17. If the APR of a simple interest amortized home loan is equal to the loan's interest rate, what conclusions could you make about the loan's required fees and points?

18. Compare and contrast the annual percentage rate of a loan with the annual yield of a compound interest rate. Be sure to discuss both the similarity and the difference between these two concepts.

19. Substitute the dollar amounts from Example 1 into the simple interest amortized loan formulas. Then discuss why the resulting equation can't be solved with algebra.

5.6 Payout Annuities

OBJECTIVES

- Understand the difference between a payout annuity and an ordinary annuity
- Use the payout annuity formulas
- Determine how to use a payout annuity to save for retirement

If you save for your retirement with an IRA or an annuity or both, you'll do a lot better if you start early. This allows you to take advantage of the "magic" effect of compound interest over a long period of time.

You'll also do better if your annuity turns into a *payout annuity* when it comes to term. This allows your money to earn a higher rate of interest *after* you retire and start collecting from it. We'll explore payout annuities in this section.

The annuities that we have discussed are all savings instruments. In Section 5.3, we defined an annuity as a sequence of equal, regular payments into an account in which each payment receives compound interest. A saver who utilizes such an annuity can accumulate a sizable sum.

Annuities can be payout instruments rather than savings instruments. After you retire, you might wish to have part of your savings sent to you each month for living expenses. You might wish to receive equal, regular payments from an account where each payment has earned compound interest. Such an annuity is called a **payout annuity.** Payout annuities are also used to pay for a child's college education.

Calculating Short-Term Payout Annuities

EXAMPLE 1

UNDERSTANDING WHAT A PAYOUT ANNUITY IS On November 1, Debra Landre will make a deposit at her bank that will be used for a payout annuity. For the next three months, commencing on December 1, she will receive a payout of $1,000 per month. Use the Compound Interest Formula to find how much money she must deposit on November 1 if her money earns 10% compounded monthly.

SOLUTION First, calculate the principal necessary to receive \$1,000 on December 1. Use $FV = 1{,}000$ and $n = 1$ (interest is earned for one month).

$$FV = P(1 + i)^n \quad \text{the Compound Interest Formula}$$

$$1{,}000 = P\left(1 + \frac{0.10}{12}\right)^1 \quad \text{substituting}$$

$$P = 1{,}000 \div \left(1 + \frac{0.10}{12}\right)^1 \quad \text{solving for } P$$

$$= 991.7355\ldots \approx \$991.74 \quad \text{rounding}$$

Next, calculate the principal necessary to receive \$1,000 on January 1. Use $n = 2$ (interest is earned for two months).

$$FV = P(1 + i)^n \quad \text{the Compound Interest Formula}$$

$$1{,}000 = P\left(1 + \frac{0.10}{12}\right)^2 \quad \text{substituting}$$

$$P = 1{,}000 \div \left(1 + \frac{0.10}{12}\right)^2 \quad \text{solving for } P$$

$$= 983.5393\ldots \approx \$983.54 \quad \text{rounding}$$

Now calculate the principal necessary to receive \$1,000 on February 1. Use $n = 3$ (interest is earned for three months).

$$FV = P(1 + i)^n \quad \text{the Compound Interest Formula}$$

$$1{,}000 = P\left(1 + \frac{0.10}{12}\right)^3 \quad \text{substituting}$$

$$P = 1{,}000 \div \left(1 + \frac{0.10}{12}\right)^3 \quad \text{solving for } P$$

$$= 975.4109\ldots \approx \$975.41 \quad \text{rounding}$$

Debra must deposit the sum of the above three amounts if she is to receive three monthly payouts of \$1,000 each. Her total principal must be

$$\$991.74 + \$983.54 + \$975.41 = \$2{,}950.69$$

If Debra's principal received no interest, then she would need $3 \cdot \$1{,}000 = \$3{,}000$. Since her principal does receive interest, she needs slightly less than \$3,000.

Comparing Payout Annuities and Savings Annuities

Payout annuities and savings annuities are similar but not identical. It is important to understand their differences before we proceed. The following example is the "savings annuity" version of Example 1; that is, it is the savings annuity most similar to the payout annuity discussed in Example 1.

EXAMPLE 2

UNDERSTANDING THE DIFFERENCE BETWEEN A PAYOUT ANNUITY AND AN ORDINARY ANNUITY On November 1, Debra Landre set up an ordinary annuity with her bank. For the next three months, she will make a payment of $1,000 per month. Each of those payments will receive compound interest. At the end of the three months (i.e., on February 1), she can withdraw her three $1,000 payments plus the interest that they will have earned. Use the Compound Interest Formula to find how much money she can withdraw.

SOLUTION

Her first payment of $1,000 would be due on November 30. It would earn interest for two months (December and January).

$$FV = P(1 + i)^n$$ **the Compound Interest Formula**

$$FV = 1{,}000\left(1 + \frac{0.10}{12}\right)^2 \approx \$1{,}016.74$$ **substituting**

Her second payment of $1,000 would be due on December 31. It would earn interest for one month (January).

$$FV = P(1 + i)^n$$ **the Compound Interest Formula**

$$FV = 1{,}000\left(1 + \frac{0.10}{12}\right)^1 \approx \$1{,}008.33$$ **substituting**

Her third payment would be due on January 31. It would earn no interest (since the annuity expires February 1), so its future value is $1,000. On February 1, Debra could withdraw

$$\$1{,}016.74 + \$1{,}008.33 + \$1{,}000 = \$3{,}025.07$$

Naturally, we could have found the future value of Debra's ordinary annuity more easily with the Ordinary Annuity Formula:

$$FV(\text{ordinary}) = pymt\frac{(1 + i)^n - 1}{i}$$ **the Ordinary Annuity Formula**

$$= 1{,}000\frac{\left(1 + \frac{0.10}{12}\right)^3 - 1}{\frac{0.10}{12}}$$ **substituting**

$$\approx \$3{,}025.07$$ **rounding**

The point of the method shown in Example 2 is to illustrate the differences between a savings annuity and a payout annuity.

Calculating Long-Term Payout Annuities

The procedure used in Example 1 reflects what actually happens with payout annuities, and it works fine for a small number of payments. However, most annuities are long-term. In the case of a savings annuity, we do not need to calculate the future value of each individual payment, as we did in Example 2; instead, we can use the Ordinary Annuity Formula. We need such a formula for payout annuities. We can find a formula if we look more closely at how the payout annuity from Example 1 compares with the savings annuity from Example 2.

In Example 2, we found that the future value of an ordinary annuity with three \$1,000 payments is

$$FV = 1{,}000\left(1 + \frac{0.10}{12}\right)^2 + 1{,}000\left(1 + \frac{0.10}{12}\right)^1 + 1{,}000$$

In Example 1, we found that the total principal necessary to generate three \$1,000 payouts is

$$P = 1{,}000 \div \left(1 + \frac{0.10}{12}\right)^1 + 1{,}000 \div \left(1 + \frac{0.10}{12}\right)^2 + 1{,}000 \div \left(1 + \frac{0.10}{12}\right)^3$$

This can be rewritten, using exponent laws, as

$$P = 1{,}000\left(1 + \frac{0.10}{12}\right)^{-1} + 1{,}000\left(1 + \frac{0.10}{12}\right)^{-2} + 1{,}000\left(1 + \frac{0.10}{12}\right)^{-3}$$

This is quite similar to the future value of the ordinary annuity—the only difference is the exponents. If we multiply each side by $(1 + \frac{0.10}{12})^3$, even the exponents will match.

$$\begin{aligned} P\left(1 + \frac{0.10}{12}\right)^3 &= 1{,}000\left(1 + \frac{0.10}{12}\right)^{-1}\left(1 + \frac{0.10}{12}\right)^3 \\ &\quad + 1{,}000\left(1 + \frac{0.10}{12}\right)^{-2}\left(1 + \frac{0.10}{12}\right)^3 \\ &\quad + 1{,}000\left(1 + \frac{0.10}{12}\right)^{-3}\left(1 + \frac{0.10}{12}\right)^3 \\ P\left(1 + \frac{0.10}{12}\right)^3 &= 1{,}000\left(1 + \frac{0.10}{12}\right)^2 + 1{,}000\left(1 + \frac{0.10}{12}\right)^1 + 1{,}000 \end{aligned}$$

The right side of the above equation is the future value of an ordinary annuity, so we can use the Ordinary Annuity Formula to rewrite it.

$$P\left(1 + \frac{0.10}{12}\right)^3 = 1{,}000\,\frac{\left(1 + \frac{0.10}{12}\right)^3 - 1}{\frac{0.10}{12}}$$

If we generalize by replacing $\frac{0.10}{12}$ with i, 3 with n, and 1,000 with *pymt*, we have our Payout Annuity Formula.

PAYOUT ANNUITY FORMULA

$$P(1 + i)^n = pymt\frac{(1 + i)^n - 1}{i}$$

where P is the total principal necessary to generate n payouts, *pymt* is the size of the payout, and i is the periodic interest rate.

We have seen this formula before. In Section 5.3, we used it to find the present value P of a savings annuity. In Section 5.4, we used it to find the payment of a simple interest amortized loan. In this section, we use it to find the required principal for a payout annuity. This is a versatile formula.

The following example involves a long-term annuity. Usually, the interest rate of a long-term annuity varies somewhat from year to year. In this case, calculations must be viewed as predictions, not guarantees.

EXAMPLE 3

FINDING THE NECESSARY DEPOSIT Fabiola Macias is about to retire, so she is setting up a payout annuity with her bank. She wishes to receive a payout of $1,000 per month for the next twenty-five years. Use the Payout Annuity Formula to find how much money she must deposit if her money earns 10% compounded monthly.

SOLUTION

We are given that $pymt = 1{,}000$, $i = \frac{1}{12}$ of $10\% = \frac{0.10}{12}$, and $n = 25 \cdot 12 = 300$.

$$P(1+i)^n = pymt\frac{(1+i)^n - 1}{i} \quad \text{the Payout Annuity Formula}$$

$$P\left(1+\frac{0.10}{12}\right)^{300} = 1{,}000\frac{\left(1+\frac{0.10}{12}\right)^{300} - 1}{\frac{0.10}{12}} \quad \text{substituting}$$

To find P, we need to calculate the right side and then divide by the $\left(1+\frac{0.10}{12}\right)^{300}$ from the left side.

We get $110{,}047.23005 \approx \$110{,}047.23$. This means that if Fabiola deposits $110,047.23, she will receive monthly payouts of $1,000 each for twenty-five years, or a total of $25 \cdot 12 \cdot 1{,}000 = \$300{,}000$.

> If Fabiola's principal received no interest, then she would need $300 \cdot \$1{,}000 = \$300{,}000$. Since her principal does receive compound interest for a long time, she needs significantly less than $300,000.

If Fabiola Macias in Example 3 were like most people, she would not have $110,047.23 in savings when she retires, so she wouldn't be able to set up a payout annuity for herself. However, if she had set up a savings annuity thirty years before retirement, she could have saved that amount by making monthly payments of only $48.68. This is something that almost anyone can afford, and it's a wonderful deal. Thirty years of monthly payments of $48.68, while you are working, can generate twenty-five years of monthly payments of $1,000 when you are retired. In the exercises, we will explore this combination of a savings annuity and a payout annuity.

Payout Annuities with Inflation

The only trouble with Fabiola's retirement payout annuity in Example 3 is that she is ignoring inflation. In twenty-five years, she will still be receiving $1,000 a month, but her money won't buy as much as it does today. Fabiola would be better off if she allowed herself an annual **cost-of-living adjustment (COLA).**

EXAMPLE 4

UNDERSTANDING WHAT A COLA IS After retiring, Fabiola Macias set up a payout annuity with her bank. For the next twenty-five years, she will receive payouts that start at $1,000 per month and then receive an annual COLA of 3%. Find the size of her monthly payout for

a. the first year
b. the second year
c. the third year
d. the 25th year

SOLUTION

a. During the first year, no adjustment is made, so she will receive $1,000 per month.

b. During the second year, her monthly payout of $1,000 will increase 3%, so her new monthly payout will be

$$1{,}000 \cdot (1 + .03) = \$1{,}030$$

c. During the third year, her monthly payout of $1,030 will increase 3%, so her new monthly payout will be

$$1{,}030 \cdot (1 + .03) = \$1{,}060.90$$

Since the 1,030 in the above calculation came from computing $1{,}000 \cdot (1 + .03)$, we could rewrite this calculation as

$$1000 \cdot (1 + .03)^2 = \$1{,}060.90$$

d. By the twenty-fifth year, she will have received twenty-four 3% increases, so her monthly payout will be

$$1{,}000 \cdot (1 + .03)^{24} = 2{,}032.7941\ldots \approx \$2{,}032.79$$

If a payout annuity is to have automatic annual cost-of-living adjustments, the following formula should be used to find the principal that must be deposited. The COLA is an annual one, so all other figures must also be annual figures; in particular, r is the *annual* interest rate, t is the duration of the annuity in *years*, and we use an *annual* payout.

ANNUAL PAYOUT ANNUITY WITH COLA FORMULA

A payout annuity of t years, where the payouts receive an annual COLA, requires a principal of

$$P = (pymt)\frac{1 - \left(\frac{1 + c}{1 + r}\right)^t}{r - c}$$

where *pymt* is the annual payout for the first year, c is the annual COLA rate, and r is the annual rate at which interest is earned on the principal.

EXAMPLE 5

A PAYOUT ANNUITY WITH A COLA After retiring, Sam Needham set up a payout annuity with his bank. For the next twenty-five years, he will receive annual payouts that start at $12,000 and then receive an annual COLA of 3%. Use the Annual Payout Annuity with COLA Formula to find how much money he must deposit if his money earns 10% interest per year.

SOLUTION

We are given that the annual payout is $pymt = 12{,}000$, $r = 10\% = 0.10$, $c = 3\% = 0.03$, and $t = 25$.

$$P = (pymt)\frac{1 - \left(\frac{1 + c}{1 + r}\right)^t}{r - c} \quad \text{the COLA Formula}$$

$$= (12{,}000)\frac{1 - \left(\frac{1 + 0.03}{1 + 0.10}\right)^{25}}{0.10 - 0.03} \quad \text{substituting}$$

$$= (12{,}000)\frac{1 - \left(\frac{1.03}{1.10}\right)^{25}}{0.10 - 0.03} \quad \text{simplifying}$$

$$= 138{,}300.4587 \approx \$138{,}300.46 \quad \text{rounding}$$

1 [−] [( ] 1.03 [÷] 1.10 [ )] [y^x] 25 [=] [÷] [( ] .10 [−] .03 [ )] [×] 12000 [=]

With a graphing calculator, press [^] rather than [y^x] and [ENTER] rather than [=].

This means that if Sam deposits $138,300.46 now, he will receive

$12,000 in one year
$\$12{,}000 \cdot (1 + .03)^1 = \$12{,}360$ in two years
$\$12{,}000 \cdot (1 + .03)^2 = \$12{,}730.80$ in three years
$\$12{,}000 \cdot (1 + .03)^{24} = \$24{,}393.53$ in twenty-five years

If Sam's principal received no interest, he would need $25 \cdot \$12{,}000 = \$300{,}000$. Since his principal does receive compound interest for a long time, he needs significantly less than $300,000.

Compare Sam's payout annuity in Example 5 with Fabiola's in Example 3. Sam receives annual payouts that start at $12,000 and slowly increase to $24,393. Fabiola receives exactly $1,000 per month (or $12,000 per year) for the same amount of time. Sam was required to deposit $138,300, and Fabiola was required to deposit $110,047.23.

5.6 Exercises

▶ **1.** Cheryl Wilcox is planning for her retirement, so she is setting up a payout annuity with her bank. She wishes to receive a payout of $1,200 per month for twenty years.
- **a.** How much money must she deposit if her money earns 8% interest compounded monthly?
- **b.** Find the total amount that Cheryl will receive from her payout annuity.

2. James Magee is planning for his retirement, so he is setting up a payout annuity with his bank. He wishes to receive a payout of $1,100 per month for twenty-five years.
- **a.** How much money must he deposit if his money earns 9% interest compounded monthly?
- **b.** Find the total amount that James will receive from his payout annuity.

▶ **3.** Dean Gooch is planning for his retirement, so he is setting up a payout annuity with his bank. He wishes to receive a payout of $1,300 per month for twenty-five years.
- **a.** How much money must he deposit if his money earns 7.3% interest compounded monthly?
- **b.** Find the total amount that Dean will receive from his payout annuity.

▶ **4.** Holly Krech is planning for her retirement, so she is setting up a payout annuity with her bank. She wishes to receive a payout of $1,800 per month for twenty years.
- **a.** How much money must she deposit if her money earns 7.8% interest compounded monthly?
- **b.** Find the total amount that Holly will receive from her payout annuity.

▶ **5.**
- **a.** How large a monthly payment must Cheryl Wilcox (from Exercise 1) make if she saves for her payout annuity with an ordinary annuity, which she sets up thirty years before her retirement? (The two annuities pay the same interest rate.)
- **b.** Find the total amount that Cheryl will pay into her ordinary annuity, and compare it with the total

amount that she will receive from her payout annuity.

6. a. How large a monthly payment must James Magee (from Exercise 2) make if he saves for his payout annuity with an ordinary annuity, which he sets up twenty-five years before his retirement? (The two annuities pay the same interest rate.)

b. Find the total amount that James will pay into his ordinary annuity, and compare it with the total amount that he will receive from his payout annuity.

▶ 7. a. How large a monthly payment must Dean Gooch (from Exercise 3) make if he saves for his payout annuity with an ordinary annuity, which he sets up thirty years before his retirement? (The two annuities pay the same interest rate.)

b. How large a monthly payment must he make if he sets the ordinary annuity up twenty years before his retirement?

▶ 8. a. How large a monthly payment must Holly Krech (from Exercise 4) make if she saves for her payout annuity with an ordinary annuity, which she sets up thirty years before her retirement? (The two annuities pay the same interest rate.)

b. How large a monthly payment must she make if she sets the ordinary annuity up twenty years before her retirement?

9. Lily Chang is planning for her retirement, so she is setting up a payout annuity with her bank. For twenty years, she wishes to receive annual payouts that start at $14,000 and then receive an annual COLA of 4%.

a. How much money must she deposit if her money earns 8% interest per year?

b. How large will Lily's first annual payout be?

c. How large will Lily's second annual payout be?

d. How large will Lily's last annual payout be?

▶ 10. Wally Brown is planning for his retirement, so he is setting up a payout annuity with his bank. For twenty-five years, he wishes to receive annual payouts that start at $16,000 and then receive an annual COLA of 3.5%.

a. How much money must he deposit if his money earns 8.3% interest per year?

b. How large will Wally's first annual payout be?

c. How large will Wally's second annual payout be?

d. How large will Wally's last annual payout be?

11. Oshri Karmon is planning for his retirement, so he is setting up a payout annuity with his bank. He is now 30 years old, and he will retire when he is 60. He wants to receive annual payouts for twenty-five years, and he wants those payouts to receive an annual COLA of 3.5%.

a. He wants his first payout to have the same purchasing power as does $13,000 today. How big should that payout be if he assumes inflation of 3.5% per year?

b. How much money must he deposit when he is 60 if his money earns 7.2% interest per year?

c. How large a monthly payment must he make if he saves for his payout annuity with an ordinary annuity? (The two annuities pay the same interest rate.)

d. How large a monthly payment would he make if he waits until he is 40 before starting his ordinary annuity?

▶ 12. Shelly Franks is planning for her retirement, so she is setting up a payout annuity with her bank. She is now 35 years old, and she will retire when she is 65. She wants to receive annual payouts for twenty years, and she wants those payouts to receive an annual COLA of 4%.

a. She wants her first payout to have the same purchasing power as does $15,000 today. How big should that payout be if she assumes inflation of 4% per year?

b. How much money must she deposit when she is 65 if her money earns 8.3% interest per year?

c. How large a monthly payment must she make if she saves for her payout annuity with an ordinary annuity? (The two annuities pay the same interest rate.)

d. How large a monthly payment would she make if she waits until she is 40 before starting her ordinary annuity?

In Exercises 13–16, use the Annual Payout Annuity with COLA Formula to find the deposit necessary to receive monthly payouts with an annual cost-of-living adjustment. To use the formula, all figures must be annual figures, including the payout and the annual rate. You can adapt the formula for monthly payouts by using

- the future value of a one-year ordinary annuity in place of the annual payout, where *pymt* is the monthly payout, and
- the annual yield of the given compound interest rate in place of the annual rate *r*.

13. Fabiola Macias is about to retire, so she is setting up a payout annuity with her bank. She wishes to receive a monthly payout for the next twenty-five years, where the payout starts at $1,000 per month and receives an annual COLA of 3%. Her money will earn 10% compounded monthly.

a. The annual payout is the future value of a one-year ordinary annuity. Find this future value.

b. The annual rate *r* is the annual yield of 10% interest compounded monthly. Find this annual yield. (*Do not* round it off.)

c. Use the Annual Payout Annuity with COLA Formula to find how much money she must deposit.

d. Fabiola could have saved for her payout annuity with an ordinary annuity. If she had started doing so thirty years ago, what would the required monthly payments have been? (The two annuities pay the same interest rate.)

▶ **14.** Gary Kersting is about to retire, so he is setting up a payout annuity with his bank. He wishes to receive a monthly payout for the next twenty years, where the payout starts at $1,300 per month and receives an annual COLA of 4%. His money will earn 8.7% compounded monthly.

a. The annual payout is the future value of a one-year ordinary annuity. Find this future value.

b. The annual rate *r* is the annual yield of 8.7% interest compounded monthly. Find this annual yield. (*Do not* round it off.)

c. Use the Annual Payout Annuity with COLA Formula to find how much money he must deposit.

d. Gary could have saved for his payout annuity with an ordinary annuity. If he had started doing so twenty-five years ago, what would the required monthly payments have been? (The two annuities pay the same interest rate.)

15. Conrad von Schtup is about to retire, so he is setting up a payout annuity with his bank. He wishes to receive a monthly payout for the next twenty-three years, where the payout starts at $1,400 per month and receives an annual COLA of 5%. His money will earn 8.9% compounded monthly.

a. The annual payout is the future value of a one-year ordinary annuity. Find this future value.

b. The annual rate *r* is the annual yield of 8.9% interest compounded monthly. Find this annual yield. (*Do not* round it off.)

c. Use the Annual Payout Annuity with COLA Formula to find how much money he must deposit.

d. Conrad could have saved for his payout annuity with an ordinary annuity. If he had started doing so twenty years ago, what would the required monthly payments have been? (The two annuities pay the same interest rate.)

▶ **16.** Mitch Martinez is about to retire, so he is setting up a payout annuity with his bank. He wishes to receive a monthly payout for the next thirty years, where the payout starts at $1,250 per month and receives an annual COLA of 4%. His money will earn 7.8% compounded monthly.

a. The annual payout is the future value of a one-year ordinary annuity. Find this future value.

b. The annual rate *r* is the annual yield of 7.8% interest compounded monthly. Find this annual yield.

c. Use the Annual Payout Annuity with COLA Formula to find how much money he must deposit.

d. Mitch could have saved for his payout annuity with an ordinary annuity. If he had started doing so twenty years ago, what would the required monthly payments have been? (The two annuities pay the same interest rate.)

17. Bob Pirtle won $1 million in a state lottery. He was surprised to learn that he will not receive a check for $1 million. Rather, for twenty years, he will receive an annual check from the state for $50,000. The state finances this series of checks by buying Bob a payout annuity. Find what the state pays for Bob's payout annuity if the interest rate is 8%.

▶ **18.** John-Paul Ramin won $2.3 million in a state lottery. He was surprised to learn that he will not receive a check for $2.3 million. Rather, for twenty years, he will receive an annual check from the state for $\frac{1}{20}$ of his winnings. The state finances this series of checks by buying John-Paul a payout annuity. Find what the state pays for John-Paul's payout annuity if the interest rate is 7.2%.

Answer the following questions using complete sentences and your own words.

• CONCEPT QUESTIONS

19. Compare and contrast a savings annuity with a payout annuity. How do they differ in purpose? How do they differ in structure? How do their definitions differ?

HINT: Compare Examples 1 and 2.

20. Under what circumstances would a savings annuity and a payout annuity be combined?

WEB PROJECTS

21. This is an exercise in saving for your retirement. Write a paper describing all of the following points.

a. Go on the web and find out what the annual rate of inflation has been for each of the last ten years. Use the average of these figures as a prediction of the future annual rate of inflation.

b. Estimate the total monthly expenses you would have if you were retired now. Include housing, food, and utilities.

c. Use parts (a) and (b) to predict your total monthly expenses when you retire, assuming that you retire at age 65.

d. Plan on financing your monthly expenses with a payout annuity. How much money must you deposit when you are 65 if your money earns 7.5% interest per year?

e. How large a monthly payment must you make if you save for your payout annuity with an ordinary annuity, starting now? (The two annuities pay the same interest rate.)

f. How large a monthly payment must you make if you save for your payout annuity with an ordinary annuity, starting ten years from now?

g. How large a monthly payment must you make if you save for your payout annuity with an ordinary annuity, starting twenty years from now?

Some useful links for this web project are listed on the text web site:

www.cengage.com/math/johnson

5 CHAPTER REVIEW

TERMS

add-on interest loan
adjustable rate mortgage
amortization schedule
amortized loan
annual percentage rate (APR) of an add-on interest loan
annual percentage rate (APR) of a simple interest amortized loan
annual yield
annuity
annuity due
average daily balance
balance
compound interest
compounding period
cost-of-living adjustment (COLA)
doubling time
expire
finance
finance charges
future value
Individual Retirement Account (IRA)
interest
interest rate
line of credit
loan agreement
lump sum
maturity value
negative amortization
nominal rate
note
ordinary annuity
outstanding principal
payment period
payout annuity
periodic rate
point
prepaying a loan
prepayment penalty
present value
present value of an annuity
principal
simple interest
simple interest amortized loan
simple interest loan
sinking fund
term
tax-deferred annuity (TDA)
Truth in Lending Act
unpaid balance
yield

FORMULAS

The **simple interest** I on a principal P at a interest rate r for t years is

$$I = Prt$$

and the **future value** is

$$FV = P(1 + rt)$$

After n compounding periods, the future value FV of an initial principal P earning **compound interest** at a periodic interest rate i for n periods is

$$FV = P(1 + i)^n$$

The **annual yield** of a given compound rate is the simple interest rate r that has the same future value as the compound rate in 1 year. To find it, solve

$$FV(\text{compound interest}) = FV\,(\text{simple interest})$$
$$P(1 + i)^n = P(1 + rt)$$

for the simple interest rate r after making appropriate substitutions.

The future value FV of an **ordinary annuity** is

$$FV(\text{ordinary}) = pymt\frac{(1 + i)^n - 1}{i}$$

and the future value of an **annuity due** is

$$FV(\text{due}) = FV(\text{ordinary}) \cdot (1 + i) = pymt\frac{(1 + i)^n - 1}{i}(1 + i)$$

The **present value of an ordinary annuity** is the lump sum P such that

$$FV(\text{lump sum}) = FV(\text{annuity})$$
$$P(1 + i)^n = pymt\frac{(1 + i)^n - 1}{i}$$

Simple interest amortized loan:

$$FV(\text{annuity}) = FV(\text{loan amount})$$
$$pymt\frac{(1 + i)^n - 1}{i} = P(1 + i)^n$$

Unpaid balance:

$$\text{Unpaid balance} = \text{current value of loan amount} - \text{current value of annuity} \approx P(1 + i)^n - pymt\frac{(1 + i)^n - 1}{i}$$

where n is the number of periods *from the beginning of the loan to the present.*

Payout annuity: The total principal necessary to generate n payouts of size $pymt$ is P, where

$$P(1 + i)^n = pymt\frac{(1 + i)^n - 1}{i}$$

Annual payout annuity with COLA: A payout annuity with a term of t years, a first-year payout of $pymt$, and an annual COLA rate c, requires a principal of

$$P = (pymt)\frac{1 - \left(\frac{1 + c}{1 + r}\right)^t}{r - c}$$

STEPS

To find the **credit card finance charge** with the **average daily balance method:**

1. Find the balance for each day in the billing period and the number of days at that balance.
2. The average daily balance is the weighted average of these daily balances, weighted to reflect the number of days at that balance.
3. The finance charge is simple interest applied to the average daily balance.

To create an **amortization schedule:**
For each payment, list the payment number, principal portion, interest portion, total payment and balance.

For each payment:

1. Find the interest on the balance—use the simple interest formula.

For each payment except the last:

2. The principal portion is the payment minus the interest portion.
3. The new balance is the previous balance minus the principal portion.

For the last payment:

4. The principal portion is the previous balance.
5. The total payment is the sum of the principal portion and interest portion.

Review Exercises

1. Find the interest earned by a deposit of $8,140 at $9\frac{3}{4}\%$ simple interest for eleven years.
2. Find the interest earned by a deposit of $10,620 at $8\frac{1}{2}\%$ simple interest for twenty-five years.
3. Find the future value of a deposit of $12,288 at $4\frac{1}{4}\%$ simple interest for fifteen years.
4. Find the future value of a deposit of $22,880 at $5\frac{3}{4}\%$ simple interest for thirty years.
5. Find the maturity value of a loan of $3,550 borrowed at $12\frac{1}{2}\%$ simple interest for one year and two months.
6. Find the maturity value of a loan of $12,250 borrowed at $5\frac{1}{2}\%$ simple interest for two years.
7. Find the present value of a future value of $84,120 at $7\frac{1}{4}\%$ simple interest for twenty-five years.
8. Find the present value of a future value of $10,250 at $5\frac{3}{4}\%$ simple interest for twenty years.
9. Find the future value of a deposit of $8,140 at $9\frac{3}{4}\%$ interest compounded monthly for eleven years.
10. Find the future value of a deposit of $7,250 at $5\frac{1}{4}\%$ interest compounded monthly for twenty years.
11. Find the interest earned by a deposit of $7,990 at $4\frac{3}{4}\%$ interest compounded monthly for eleven years.
12. Find the interest earned by a deposit of $22,250 at $9\frac{1}{4}\%$ interest compounded monthly for twenty years.
13. Find the present value of a future value of $33,120 at $6\frac{1}{4}\%$ interest compounded daily for twenty-five years.
14. Find the present value of a future value of $10,600 at $7\frac{7}{8}\%$ interest compounded daily for four years.
15. Find the annual yield corresponding to a nominal rate of 7% compounded daily.
16. Find the annual yield corresponding to a nominal rate of $8\frac{1}{2}\%$ compounded monthly.
17. Find the future value of a twenty-year ordinary annuity with monthly payments of $230 at 6.25% interest.
18. Find the future value of a thirty-year ordinary annuity with biweekly payments of $130 at 7.25% interest.
19. Find the future value of a twenty-year annuity due with monthly payments of $450 at 8.25% interest.
20. Find the future value of a thirty-year annuity due with biweekly payments of $240 at 6.75% interest.
21. Find (a) the monthly payment and (b) the total interest for a simple interest amortized loan of $25,000 for five years at $9\frac{1}{2}\%$ interest.
22. Find (a) the monthly payment and (b) the total interest for a simple interest amortized loan of $130,000 for twenty years at $8\frac{1}{4}\%$ interest.
23. The Square Wheel Bicycle store has found that they sell most of their bikes in the spring and early summer. On February 15, they borrowed $351,500 to buy bicycles. They are confident that they can sell most of these bikes by September 1. Their loan is at $5\frac{7}{8}\%$ simple interest. What size lump sum payment would they have to make on September 1 to pay off the loan?

24. Mike Taylor buys a four-year old Ford from a car dealer for $16,825. He puts 10% down and finances the rest through the dealer at 10.5% add-on interest. If he agrees to make thirty-six monthly payments, find the size of each payment.

25. The activity on Sue Washburn's MasterCard account for one billing period is shown below. Find the average daily balance and the finance charge if the billing period is August 26 through September 25, the previous balance was $3,472.38, and the annual interest rate is $19\frac{1}{2}$%.

August 30	payment	$100.00
September 2	gasoline	$34.12
September 10	restaurant	$62.00

26. George and Martha Simpson bought a house from Sue Sanchez for $205,500. In lieu of a 20% down payment, Ms. Sanchez accepted 5% down at the time of the sale and a promissory note from the Simpsons for the remaining 15%, due in eight years. The Simpsons also agreed to make monthly interest payments to Ms. Sanchez at 12% simple interest until the note expires. The Simpsons borrowed the remaining 80% of the purchase price from their bank. The bank paid that amount, less a commission of 6% of the purchase price, to Ms. Sanchez.

a. Find the Simpsons' monthly interest-only payment to Ms. Sanchez.

b. Find Ms. Sanchez's total income from all aspects of the down payment.

c. Find Ms. Sanchez's total income from all aspects of the sale of the house, including the down payment.

27. Tien Ren Chiang wants to have an IRA that will be worth $250,000 when he retires at age 65.

a. How much must he deposit at age 25 at $8\frac{1}{8}$% compounded quarterly?

b. If he arranges for the monthly interest to be sent to him starting at age 65, how much would he receive each month? (Assume that he will continue to receive $8\frac{1}{8}$% interest, compounded monthly.)

28. Extremely Trustworthy Savings offers five-year CDs at 7.63% compounded annually, and Bank of the South offers five-year CDs at 7.59% compounded daily. Compute the annual yield for each institution and determine which offering is more advantageous for the consumer.

29. You are 32, and you have just set up an ordinary annuity to save for retirement. You make monthly payments of $200 that earn $6\frac{1}{8}$% interest. Find the future value when you reach age 65.

30. Find and interpret the present value of the annuity in Exercise 29.

31. Find the future value of an annuity due with monthly payments of $200 that earns $6\frac{1}{8}$% interest, after eleven years.

32. Matt and Leslie Silver want to set up a TDA that will generate sufficient interest on maturity to meet their living expenses, which they project to be $1,300 per month.

a. Find the amount needed at maturity to generate $1,300 per month interest if they can get $8\frac{1}{4}$% interest compounded monthly.

b. Find the monthly payment they would have to make into an ordinary annuity to obtain the future value found in part (a) if their money earns $9\frac{3}{4}$% and the term is thirty years.

33. Mr. and Mrs. Liberatore set up a TDA to save for their retirement. They agreed to have $100 deducted from each of Mrs. Liberatore's monthly paychecks, which will earn $6\frac{1}{8}$% interest.

a. Find the future value of their ordinary annuity, if it comes to term after they retire in thirty years.

b. After retiring, the Liberatores convert their annuity to a savings account, which earns 5.75% interest compounded monthly. At the end of each month, they withdraw $1,000 for living expenses. Complete the chart in Figure 5.29 for their postretirement account.

Month Number	Account Balance at Beginning of the Month	Interest for the Month	With-drawal	Account Balance at End of the Month
1				
2				
3				
4				
5				

FIGURE 5.29 Chart for Exercise 33.

34. Delores Lopez buys some land in Nevada. She agrees to pay the seller a lump sum of $235,000 in five years. Until then, she will make monthly simple interest payments to the seller at 10% interest.

a. Find the amount of each interest payment.

b. Delores sets up a sinking fund to save the $235,000. Find the size of her monthly payments if her payments are due at the end of every month and her money earns $9\frac{3}{8}$% interest.

c. Prepare a table showing the amount in the sinking fund after each of the first two deposits.

35. Maude Frickett bought a house for $225,600. She put 20% down and obtains a simple interest amortized loan for the rest at $7\frac{3}{8}$% for thirty years.

a. Find her monthly payment.

b. Find the total interest.

c. Prepare an amortization schedule for the first two months of the loan.

36. Navlet's Nursery needs to borrow $228,000 to increase its inventory for the upcoming spring season. The owner is confident that he will sell most if not all of the new plants during the summer, so he wishes to borrow the money for only six months. His bank has offered him a simple interest amortized loan at $8\frac{1}{4}\%$ interest.

a. Find the size of the monthly bank payment.

b. Prepare an amortization schedule for all six months of the loan.

37. Harry Carry had his kitchen remodeled. He did not have sufficient cash to pay for it. However, he had previously set up a line of credit with his bank. On May 16, he wrote a check to his contractor on his line of credit for $41,519. The line's interest rate is $7\frac{3}{4}\%$.

a. Find the size of the required monthly interest payment.

b. Harry decided that it would be in his best interests to get this loan paid off in seven months. Find the size of the monthly principal-plus-interest payment that would accomplish this.

(*HINT:* In effect, Carry is converting the loan to an amortized loan.)

c. Prepare an amortization schedule for all seven months of the loan.

d. Find the amount of line of credit interest that Carry could deduct from his taxes next year.

38. Ben Suico buys a car for $13,487.31. He puts 10% down and obtains a simple interest amortized loan for the rest at $10\frac{7}{8}\%$ interest for five years.

a. Find his monthly payment.

b. Find the total interest.

c. Prepare an amortization schedule for the first two months of the loan.

d. Mr. Suico decides to sell his car two years and six months after he bought it. Find the unpaid balance on his loan.

39. Scott Frei wants to buy a used car that costs $6,200. The used car dealer has offered him a four-year add-on interest loan that requires a $200 down payment at 9.9% annual interest with an APR of 10%.

a. Find the monthly payment.

b. Verify the APR.

40. Miles Archer bought a house for $112,660. He put 20% down and obtains a simple interest amortized loan for the rest at $9\frac{7}{8}\%$ for thirty years. If Miles paid two points and $5,738.22 in fees, $1,419.23 of which are included in the finance charge, find the APR.

41. Susan and Steven Tamchin are thinking of buying a home for $198,000. A potential lender advertises an 80%, thirty-year simple interest amortized loan at $8\frac{1}{2}\%$ interest, with and APR of 9.02%.

a. Find the size of the Tamchin's monthly payment.

b. Use the APR to approximate the fees included in the finance charge.

42. Fred Rodgers is planning for his retirement, so he is setting up a payout annuity with his bank. He wishes to receive a payout of $1,700 per month for twenty-five years.

a. How much money must he deposit if his money earns 6.1% interest compounded monthly?

b. How large a monthly payment would Fred have made if he had saved for his payout annuity with an ordinary annuity, set up thirty years before his retirement? (The two annuities pay the same interest rate.)

c. Find the total amount that Fred will pay into his ordinary annuity and the total amount that he will receive from his payout annuity.

43. Sue West is planning for her retirement, so she is setting up a payout annuity with her bank. She is now 30 years old, and she will retire when she is 60. She wants to receive annual payouts for twenty-five years, and she wants those payouts to have an annual COLA of 4.2%.

a. She wants her first payout to have the same purchasing power as does $17,000 today. How big should that payout be if she assumes inflation of 4.2% per year?

b. How much money must she deposit when she is sixty if her money earns 8.3% interest per year?

c. How large a monthly payment must she make if she saves for her payout annuity with an ordinary annuity? (The two annuities pay the same interest rate.)

Answer the following questions using complete sentences and your own words.

• CONCEPT QUESTIONS

44. What is the difference between simple interest and compound interest?

45. Describe a situation in which simple interest, rather than compound interest, would be expected.

46. Describe a situation in which compound interest, rather than simple interest, would be expected.

47. What is the difference between an account that earns compound interest and an annuity that earns compound interest?

48. What is the difference between a simple interest amortized loan and an add-on interest loan?

• HISTORY QUESTIONS

49. What does the Truth in Lending Act do for borrowers?

50. Who offered the first credit card?

51. How did the first credit card differ from the first post–World War II credit card?