ETS GRE®

Measuring the Power of Learning.®

GRE® Mathematics Test Practice Book

This practice book contains

- one actual, full-length *GRE*® Mathematics Test
- test-taking strategies

Become familiar with

- test structure and content
- test instructions and answering procedures

Compare your practice test results with the performance of those who took the test at a GRE administration.

www.ets.org/gre

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Test takers with disabilities or health-related needs who need test preparation materials in an alternate format should contact the ETS Office of Disability Services at **stassd@ets.org**. For additional information, visit **www.ets.org/gre/disabilities**.

Overview

The *GRE*® Mathematics Test consists of approximately 66 multiple-choice questions drawn from courses commonly offered at the undergraduate level. Testing time is 2 hours and 50 minutes; there are no separately-timed sections.

This publication provides a comprehensive overview of the GRE Mathematics Test to help you get ready for test day. It is designed to help you:

- Understand what is being tested
- Gain familiarity with the question types
- Review test-taking strategies
- Understand scoring
- Practice taking the test

To learn more about the GRE Subject Tests, visit **www.ets.org/gre.**

Test Content

Approximately 50 percent of the Mathematics Test questions involve calculus and its applications — subject matter that is assumed to be common to the backgrounds of almost all mathematics majors. About 25 percent of the questions in the test are in elementary algebra, linear algebra, abstract algebra, and number theory. The remaining questions deal with other areas of mathematics currently studied by undergraduates in many institutions.

The following content descriptions may assist students in preparing for the test. The percentages given are estimates; actual percentages will vary somewhat from one edition of the test to another.

I. Calculus (50%)

Material learned in the usual sequence of elementary calculus courses — differential and integral calculus of one and of several variables — including calculus-based applications and connections with coordinate geometry, trigonometry, differential equations, and other branches of mathematics

II. Algebra (25%)

Elementary algebra: basic algebraic techniques and manipulations acquired in high school and used throughout mathematics

Linear algebra: matrix algebra, systems of linear equations, vector spaces, linear transformations, characteristic polynomials, and eigenvalues and eigenvectors

Abstract algebra and number theory: elementary topics from group theory, theory of rings and modules, field theory, and number theory

III. Additional Topics (25%)

Introductory real analysis: sequences and series of numbers and functions, continuity, differentiability and integrability, and elementary topology of $\mathbb{R}$ and $\mathbb{R}^n$

Discrete mathematics: logic, set theory, combinatorics, graph theory, and algorithms

Other topics: general topology, geometry, complex variables, probability and statistics, and numerical analysis

The above descriptions of topics covered in the test should not be considered exhaustive; it is necessary to understand many other related concepts. Prospective test takers should be aware that questions requiring no more than a good precalculus background may be quite challenging; such questions can be among the most difficult questions on the test. In general, the questions are intended not only to test recall of information, but also to assess the understanding of fundamental concepts and the ability to apply those concepts in various situations.

Preparing for the Test

GRE Subject Test questions are designed to measure skills and knowledge gained over a long period of time. Although you might increase your scores to some extent through preparation a few weeks or months before you take the test, last minute cramming is unlikely to be of further help.

The following information may be helpful.

- A general review of your college courses is probably the best preparation for the test. However, the test covers a broad range of subject matter, and no one is expected to be familiar with the content of every question.
- Become familiar with the types of questions in the GRE Mathematics Test, paying special attention to the directions. If you thoroughly understand the directions before you take the test, you will have more time during the test to focus on the questions themselves.

Test-Taking Strategies

The questions in the practice test illustrate the types of multiple-choice questions in the test. When you take the actual test, you will mark your answers on a separate machine-scorable answer sheet.

The following are some general test-taking strategies you may want to consider.

- Read the test directions carefully, and work as rapidly as you can without being careless. For each question, choose the best answer from the available options.
- All questions are of equal value; do not waste time pondering individual questions you find extremely difficult or unfamiliar.
- You may want to work through the test quickly, first answering only the questions about which you feel confident, then going back and answering questions that require more thought, and concluding with the most difficult questions if there is time.
- If you decide to change an answer, make sure you completely erase it and fill in the oval corresponding to your desired answer.
- Your score will be determined by the number of questions you answer correctly. Questions you answer incorrectly or for which you mark no answer or more than one answer are counted as incorrect. Nothing is subtracted from a score if you answer a question incorrectly. Therefore, to maximize your score it is better for you to guess at an answer than not to respond at all.
- Record all answers on your answer sheet. Answers recorded in your test book will not be counted.
- Do not wait until the last few minutes of a testing session to record answers on your answer sheet.

What Your Scores Mean

The number of questions you answered correctly on the whole test (total correct score) is converted to the total reported scaled score. This conversion ensures that a scaled score reported for any edition of a GRE Mathematics Test is comparable to the same scaled score earned on any other edition of the test. Thus, equal scaled scores on a particular test indicate essentially equal levels of performance regardless of the test edition taken.

GRE Mathematics Test total scores are reported on a 200 to 990 score scale in ten-point increments.

Test scores should be compared only with other scores on the Mathematics Test. For example, a 680 on the Mathematics Test is not equivalent to a 680 on the Physics Test.

Taking the Practice Test

The practice test begins on page 6. The total time that you should allow for this practice test is 2 hours and 50 minutes. An answer sheet is provided for you to mark your answers to the test questions.

It is best to take this practice test under timed conditions. Find a quiet place to take the test and make sure you have a minimum of 2 hours and 50 minutes available.

To simulate how the administration will be conducted at the test center, print the answer sheet (pages 64 and 65). Then go to the back cover of the test book (page 61) and follow the instructions for completing the identification areas of the answer sheet. When you are ready to begin the test, note the time and begin marking your answers on the answer sheet. Stop working on the test when 2 hours and 50 minutes have elapsed.

Scoring the Practice Test

The worksheet on page 62 lists the correct answers to the questions on the practice test. The "Correct Response" columns are provided for you to mark those questions for which you chose the correct answer.

Mark each question that you answered correctly. Then, add up your correct answers and enter your total number of correct answers in the space labeled "Total Correct" at the bottom of the page. Next, use the "Total Score" conversion table on page 63 to find the corresponding scaled score. For example, suppose you chose the correct answers to 50 questions on the test. The "Total Correct" entry in the conversion table of 50 shows that your total scaled score is 790.

Evaluating Your Performance

Now that you have scored your test, you may wish to compare your performance with the performance of others who took this test.

The data in the worksheet on page 62 are based on the performance of a sample of the test takers who took the GRE Mathematics Test in the United States.

The numbers in the column labeled "P+" on the worksheet indicate the percentages of examinees in this sample who answered each question correctly. You may use these numbers as a guide for evaluating your performance on each test question.

Interpretive data based on the scores earned by a recent cohort of test takers are available on the GRE website at **www.ets.org/gre/subject/scores/understand**. The interpretive data shows, for selected scaled score, the percentage of test takers who received lower scores. To compare yourself with this population, look at the percentage next to the scaled score you earned on the practice test. Note that these interpretive data are updated annually and reported on GRE score reports.

It is important to realize that the conditions under which you tested yourself were not exactly the same as those you will encounter at a test center. It is impossible to predict how different test-taking conditions will affect test performance, and this is only one factor that may account for differences between your practice test scores and your actual test scores. By comparing your performance on this practice test with the performance of other individuals who took GRE Mathematics Test, however, you will be able to determine your strengths and weaknesses and can then plan a program of study to prepare yourself for taking the GRE Mathematics Test under standard conditions.

Note that question 53 was slightly modified for this practice test.

FORM GR1768

68

GRADUATE RECORD EXAMINATIONS®

MATHEMATICS TEST

MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is best and then completely fill in the corresponding space on the answer sheet.

Computation and scratch work may be done in this test book.

In this test:

(1) All logarithms with an unspecified base are natural logarithms, that is, with base e.
(2) The symbols $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ denote the sets of integers, rational numbers, real numbers, and complex numbers, respectively.

---

1. $\lim_{x \to 0} \dfrac{\cos(3x) - 1}{x^2} =$

(A) $\dfrac{9}{2}$ (B) $\dfrac{3}{2}$ (C) $-\dfrac{2}{3}$ (D) $-\dfrac{3}{2}$ (E) $-\dfrac{9}{2}$

---

2. What is the area of an equilateral triangle whose inscribed circle has radius 2 ?

(A) 12 (B) 16 (C) $12\sqrt{3}$ (D) $16\sqrt{3}$ (E) $4\left(3 + 2\sqrt{2}\right)$

---

3. $\int_{e^{-3}}^{e^{-2}} \dfrac{1}{x \log x}\, dx =$

(A) 1 (B) $\dfrac{2}{3}$ (C) $\dfrac{3}{2}$ (D) $\log\left(\dfrac{2}{3}\right)$ (E) $\log\left(\dfrac{3}{2}\right)$

GO ON TO THE NEXT PAGE.

SCRATCH WORK

4. Let V and W be 4-dimensional subspaces of a 7-dimensional vector space X. Which of the following CANNOT be the dimension of the subspace $V \cap W$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

5. Sofia and Tess will each randomly choose one of the 10 integers from 1 to 10. What is the probability that neither integer chosen will be the square of the other?

(A) 0.64 (B) 0.72 (C) 0.81 (D) 0.90 (E) 0.95

---

6. Which of the following shows the numbers $2^{1/2}$, $3^{1/3}$, and $6^{1/6}$ in increasing order?

(A) $2^{1/2} < 3^{1/3} < 6^{1/6}$

(B) $6^{1/6} < 3^{1/3} < 2^{1/2}$

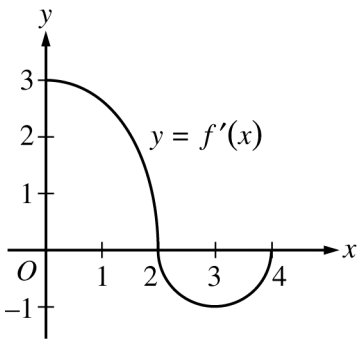
(C) $6^{1/6} < 2^{1/2} < 3^{1/3}$

(D) $3^{1/3} < 2^{1/2} < 6^{1/6}$

(E) $3^{1/3} < 6^{1/6} < 2^{1/2}$

GO ON TO THE NEXT PAGE.

SCRATCH WORK



7. The figure above shows the graph of the derivative f' of a function f, where f is continuous on the interval $[0, 4]$ and differentiable on the interval $(0, 4)$. Which of the following gives the correct ordering of the values $f(0)$, $f(2)$, and $f(4)$?

   (A) $f(0) < f(2) < f(4)$

   (B) $f(0) < f(4) = f(2)$

   (C) $f(0) < f(4) < f(2)$

   (D) $f(4) = f(2) < f(0)$

   (E) $f(4) < f(0) < f(2)$

---

8. Which of the following is NOT a group?

   (A) The integers under addition

   (B) The nonzero integers under multiplication

   (C) The nonzero real numbers under multiplication

   (D) The complex numbers under addition

   (E) The nonzero complex numbers under multiplication

SCRATCH WORK

9. Let g be a continuous real-valued function defined on $\mathbb{R}$ with the following properties.

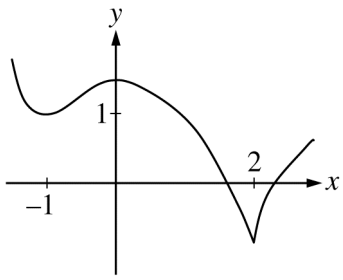
$$g'(0) = 0$$

$$g''(-1) > 0$$

$$g''(x) < 0 \text{ if } 0 < x < 2.$$

Which of the following could be part of the graph of g ?

(A)



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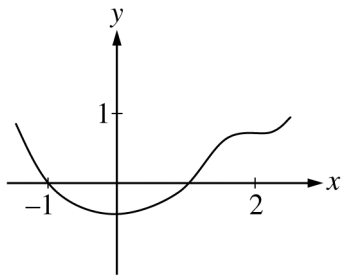
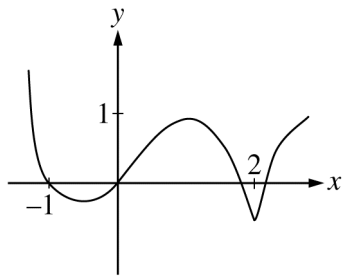


(C)



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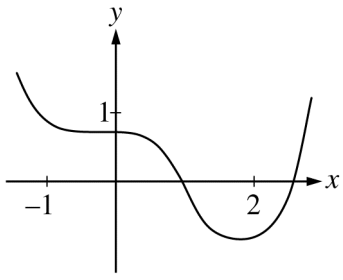
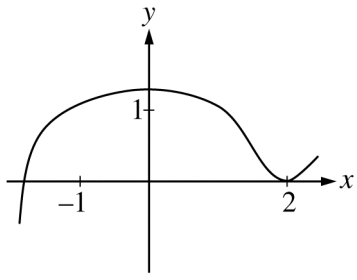


(E)



$$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-3)^2 + y^2}$$

10. In the xy-plane, the set of points whose coordinates satisfy the equation above is

(A) a line (B) a circle (C) an ellipse (D) a parabola (E) one branch of a hyperbola

---

GO ON TO THE NEXT PAGE.

SCRATCH WORK

11. The region bounded by the curves $y = x$ and $y = x^2$ in the first quadrant of the xy-plane is rotated about the y-axis. The volume of the resulting solid of revolution is

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) $\frac{3\pi}{2}$

---

12. For which integers n such that $3 \le n \le 11$ is there only one group of order n (up to isomorphism) ?

(A) For no such integer n

(B) For 3, 5, 7, and 11 only

(C) For 3, 5, 7, 9, and 11 only

(D) For 4, 6, 8, and 10 only

(E) For all such integers n

---

13. If f is a continuously differentiable real-valued function defined on the open interval $(-1, 4)$ such that $f(3) = 5$ and $f'(x) \ge -1$ for all x, what is the greatest possible value of $f(0)$?

(A) 3 (B) 4 (C) 5 (D) 8 (E) 11

GO ON TO THE NEXT PAGE.

SCRATCH WORK

14. Suppose g is a continuous real-valued function such that $3x^5 + 96 = \int_c^x g(t)\,dt$ for each $x \in \mathbb{R}$, where c is a constant. What is the value of c ?

(A) –96 (B) –2 (C) 4 (D) 15 (E) 32

---

15. Let S, T, and U be nonempty sets, and let $f : S \to T$ and $g : T \to U$ be functions such that the function $g \circ f : S \to U$ is one-to-one (injective). Which of the following must be true?

(A) f is one-to-one.

(B) f is onto.

(C) g is one-to-one.

(D) g is onto.

(E) $g \circ f$ is onto.

---

16. Suppose A, B, and C are statements such that C is true if exactly one of A and B is true. If C is false, which of the following statements must be true?

(A) If A is true, then B is false.

(B) If A is false, then B is false.

(C) If A is false, then B is true.

(D) Both A and B are true.

(E) Both A and B are false.

SCRATCH WORK

17. Which of the following equations has the greatest number of real solutions?

(A) $x^3 = 10 - x$

(B) $x^2 + 5x - 7 = x + 8$

(C) $7x + 5 = 1 - 3x$

(D) $e^x = x$

(E) $\sec x = e^{-x^2}$

---

18. Let f be the function defined by $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$ for all x such that $-1 < x < 1$. Then $f'(x) =$

(A) $\frac{1}{1-x}$ (B) $\frac{x}{1-x}$ (C) $\frac{1}{1+x}$ (D) $\frac{x}{1+x}$ (E) 0

---

19. If z is a complex variable and $\overline{z}$ denotes the complex conjugate of z, what is $\lim_{z \to 0} \frac{(\overline{z})^2}{z^2}$?

(A) 0 (B) 1 (C) i (D) ∞ (E) The limit does not exist.

GO ON TO THE NEXT PAGE.

SCRATCH WORK

20. Let g be the function defined by $g(x) = e^{2x+1}$ for all real x. Then $\lim_{x \to 0} \dfrac{g(g(x)) - g(e)}{x} =$

(A) $2e$ (B) $4e^2$ (C) e^{2e+1} (D) $2e^{2e+1}$ (E) $4e^{2e+2}$

---

21. What is the value of $\int_{-\pi/4}^{\pi/4} \left(\cos t + \sqrt{1 + t^2} \sin^3 t \cos^3 t \right) dt$?

(A) 0 (B) $\sqrt{2}$ (C) $\sqrt{2} - 1$ (D) $\dfrac{\sqrt{2}}{2}$ (E) $\dfrac{\sqrt{2} - 1}{2}$

---

22. What is the volume of the solid in xyz-space bounded by the surfaces $y = x^2$, $y = 2 - x^2$, $z = 0$, and $z = y + 3$?

(A) $\dfrac{8}{3}$ (B) $\dfrac{16}{3}$ (C) $\dfrac{32}{3}$ (D) $\dfrac{104}{105}$ (E) $\dfrac{208}{105}$

GO ON TO THE NEXT PAGE.

SCRATCH WORK

23. Let $(\mathbb{Z}_{10}, +, \cdot)$ be the ring of integers modulo 10, and let S be the subset of $\mathbb{Z}_{10}$ represented by $\{0, 2, 4, 6, 8\}$. Which of the following statements is FALSE?

(A) $(S, +, \cdot)$ is closed under addition modulo 10.

(B) $(S, +, \cdot)$ is closed under multiplication modulo 10.

(C) $(S, +, \cdot)$ has an identity under addition modulo 10.

(D) $(S, +, \cdot)$ has no identity under multiplication modulo 10.

(E) $(S, +, \cdot)$ is commutative under addition modulo 10.

---

24. Consider the system of linear equations

$$\begin{aligned} w + 3x + 2y + 2z &= 0 \\ w + 4x + y &= 0 \\ 3w + 5x + 10y + 14z &= 0 \\ 2w + 5x + 5y + 6z &= 0 \end{aligned}$$

with solutions of the form (w, x, y, z), where w, x, y, and z are real. Which of the following statements is FALSE?

(A) The system is consistent.

(B) The system has infinitely many solutions.

(C) The sum of any two solutions is a solution.

(D) $(-5, 1, 1, 0)$ is a solution.

(E) Every solution is a scalar multiple of $(-5, 1, 1, 0)$.

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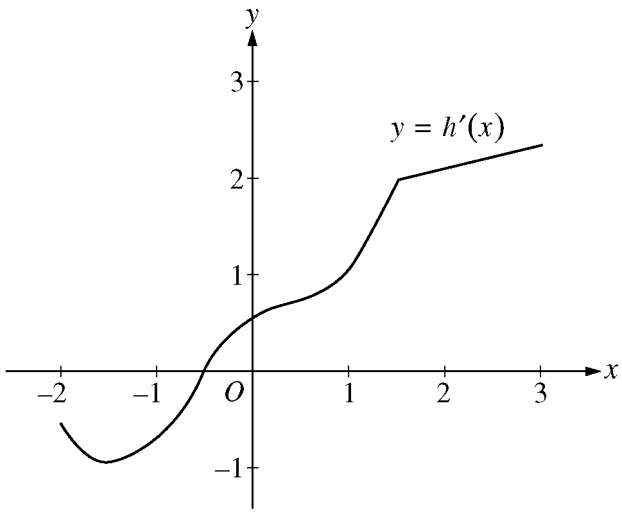
SCRATCH WORK



25. The graph of the derivative h' is shown above, where h is a real-valued function. Which of the following open intervals contains a value c for which the point $(c, h(c))$ is an inflection point of h ?

(A) $(-2, -1)$ (B) $(-1, 0)$ (C) $(0, 1)$ (D) $(1, 2)$ (E) $(2, 3)$

$$3x \equiv 5 \pmod{11}$$
$$2y \equiv 7 \pmod{11}$$

26. If x and y are integers that satisfy the congruences above, then $x + y$ is congruent modulo 11 to which of the following?

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

---

27. $(1 + i)^{10} =$

(A) 1 (B) i (C) 32 (D) $32i$ (E) $32(i + 1)$

GO ON TO THE NEXT PAGE.

SCRATCH WORK

28. Let f be a one-to-one (injective), positive-valued function defined on $\mathbb{R}$. Assume that f is differentiable at $x = 1$ and that in the xy-plane the line $y - 4 = 3(x - 1)$ is tangent to the graph of f at $x = 1$. Let g be the function defined by $g(x) = \sqrt{x}$ for $x \geq 0$. Which of the following is FALSE?

(A) $f'(1) = 3$

(B) $\left(f^{-1}\right)'(4) = \frac{1}{3}$

(C) $(fg)'(1) = 5$

(D) $(g \circ f)'(1) = \frac{1}{2}$

(E) $(g \circ f)(1) = 2$

---

29. A tree is a connected graph with no cycles. How many nonisomorphic trees with 5 vertices exist?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

30. For what positive value of c does the equation $\log x = cx^4$ have exactly one real solution for x ?

(A) $\frac{1}{4e}$ (B) $\frac{1}{4e^4}$ (C) $\frac{e^4}{4}$ (D) $\frac{4}{e^{1/4}}$ (E) $4e^{1/4}$

GO ON TO THE NEXT PAGE.

SCRATCH WORK

31. Of the numbers 2, 3, and 5, which are eigenvalues of the matrix $\begin{pmatrix} 3 & 5 & 3 \\ 1 & 7 & 3 \\ 1 & 2 & 8 \end{pmatrix}$?

(A) None (B) 2 and 3 only (C) 2 and 5 only (D) 3 and 5 only (E) 2, 3, and 5

---

32. $\dfrac{d}{dx}\displaystyle\int_{x^3}^{x^4} e^{t^2}\,dt =$

(A) $e^{x^6}\left(e^{x^8-x^6}-1\right)$ (B) $4x^3e^{x^8}$ (C) $\dfrac{1}{\sqrt{1-e^{x^2}}}$ (D) $\dfrac{e^{x^2}}{x^2}-1$ (E) $x^2e^{x^6}\left(4xe^{x^8-x^6}-3\right)$

---

33. What is the 19th derivative of $\dfrac{x-1}{e^x}$?

(A) $(18-x)e^{-x}$

(B) $(19-x)e^{-x}$

(C) $(20-x)e^{-x}$

(D) $(x-19)e^{-x}$

(E) $(x-20)e^{-x}$

GO ON TO THE NEXT PAGE.

SCRATCH WORK

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

34. Which of the following statements about the real matrix shown above is FALSE?

(A) A is invertible.

(B) If $\mathbf{x} \in \mathbb{R}^5$ and $A\mathbf{x} = \mathbf{x}$, then $\mathbf{x} = \mathbf{0}$.

(C) The last row of A^2 is $\begin{pmatrix} 0 & 0 & 0 & 0 & 25 \end{pmatrix}$.

(D) A can be transformed into the 5×5 identity matrix by a sequence of elementary row operations.

(E) $\det(A) = 120$

---

35. In xyz-space, what are the coordinates of the point on the plane $2x + y + 3z = 3$ that is closest to the origin?

(A) $(0, 0, 1)$ (B) $\left(\frac{3}{7}, \frac{3}{14}, \frac{9}{14}\right)$ (C) $\left(\frac{7}{15}, \frac{8}{15}, \frac{1}{15}\right)$ (D) $\left(\frac{5}{6}, \frac{1}{3}, \frac{1}{3}\right)$ (E) $\left(1, 1, \frac{1}{3}\right)$

GO ON TO THE NEXT PAGE.

SCRATCH WORK

36. Suppose S is a nonempty subset of $\mathbb{R}$. Which of the following is necessarily true?

(A) For each $s, t \in S$, there exists a continuous function f mapping $[0, 1]$ into S with $f(0) = s$ and $f(1) = t$.

(B) For each $u \notin S$, there exists an open subset U of $\mathbb{R}$ such that $u \in U$ and $U \cap S = \varnothing$.

(C) $\{v \in S$: there exists an open subset V of $\mathbb{R}$ with $v \in V \subseteq S\}$ is an open subset of $\mathbb{R}$.

(D) $\{w \notin S$: there exists an open subset W of $\mathbb{R}$ with $w \in W$ and $W \cap S = \varnothing\}$ is a closed subset of $\mathbb{R}$.

(E) S is the intersection of all closed subsets of $\mathbb{R}$ that contain S.

---

37. Let V be a finite-dimensional real vector space and let P be a linear transformation of V such that $P^2 = P$. Which of the following must be true?

I. P is invertible.

II. P is diagonalizable.

III. P is either the identity transformation or the zero transformation.

(A) None (B) I only (C) II only (D) III only (E) II and III

GO ON TO THE NEXT PAGE.

SCRATCH WORK

38. The maximum number of acute angles in a convex 10-gon in the Euclidean plane is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

39. Consider the following algorithm, which takes an input integer `n > 2` and prints one or more integers.

```
input(n)
set i = 1
while i < n
  begin
    replace i by i + 1
    set k = n
    while k ≥ i
      begin
        if i = k then print(i)
        replace k by k - 1
      end
  end
```

If the input integer is `88`, what integers will be printed?

(A) Only the integer `2`

(B) Only the integer `88`

(C) Only the divisors of `88` that are greater than `1`

(D) The integers from `2` to `88` in increasing order

(E) The integers from `88` to `2` in decreasing order

SCRATCH WORK

40. Let S be the set of all functions $f : \mathbb{R} \to \mathbb{R}$. Consider the two binary operations $+$ and $\circ$ on S defined as pointwise addition and composition of functions, as follows.

$$(f + g)(x) = f(x) + g(x)$$
$$(f \circ g)(x) = f(g(x))$$

Which of the following statements are true?

I. $\circ$ is commutative.

II. $+$ and $\circ$ satisfy the left distributive law $f \circ (g + h) = (f \circ g) + (f \circ h)$.

III. $+$ and $\circ$ satisfy the right distributive law $(g + h) \circ f = (g \circ f) + (h \circ f)$.

(A) None (B) II only (C) III only (D) II and III only (E) I, II, and III

---

41. Let ℓ be the line that is the intersection of the planes $x + y + z = 3$ and $x - y + z = 5$ in $\mathbb{R}^3$. An equation of the plane that contains $(0, 0, 0)$ and is perpendicular to ℓ is

(A) $x - z = 0$

(B) $x + y + z = 0$

(C) $x - y - z = 0$

(D) $x + z = 0$

(E) $x + y - z = 0$

GO ON TO THE NEXT PAGE.

SCRATCH WORK

42. Let $\mathbb{Z}^+$ be the set of positive integers and let d be the metric on $\mathbb{Z}^+$ defined by

$$d(m, n) = \begin{cases} 0 & \text{if } m = n \\ 1 & \text{if } m \neq n \end{cases}$$

for all $m, n \in \mathbb{Z}^+$. Which of the following statements are true about the metric space $\left(\mathbb{Z}^+, d\right)$?

I. If $n \in \mathbb{Z}^+$, then $\{n\}$ is an open subset of $\mathbb{Z}^+$.

II. Every subset of $\mathbb{Z}^+$ is closed.

III. Every real-valued function defined on $\mathbb{Z}^+$ is continuous.

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

---

43. A curve in the xy-plane is given parametrically by

$$x = t^2 + 2t$$
$$y = 3t^4 + 4t^3$$

for all $t > 0$. The value of $\dfrac{d^2y}{dx^2}$ at the point $(8, 80)$ is

(A) 4 (B) 24 (C) 32 (D) 96 (E) 192

GO ON TO THE NEXT PAGE.

SCRATCH WORK

$$y' + xy = x$$
$$y(0) = -1$$

44. If y is a real-valued function defined on the real line and satisfying the initial value problem above, then $\lim_{x \to -\infty} y(x) =$

(A) 0 (B) 1 (C) -1 (D) ∞ (E) $-\infty$

---

45. How many positive numbers x satisfy the equation $\cos(97x) = x$?

(A) 1 (B) 15 (C) 31 (D) 49 (E) 96

---

46. A ladder 9 meters in length is leaning against a vertical wall on level ground. As the bottom end of the ladder is moved away from the wall at a constant rate of 2 meters per second, the top end slides downward along the wall. How fast, in meters per second, will the top end of the ladder be sliding downward at the moment the top end is 3 meters above the ground?

(A) $12\sqrt{2}$ (B) $6\sqrt{2}$ (C) $4\sqrt{2}$ (D) $\frac{1}{2\sqrt{2}}$ (E) $\frac{2}{3}$

SCRATCH WORK

47. The function $f : \mathbb{R} \to \mathbb{R}$ is defined as follows.

$$f(x) = \begin{cases} 3x^2 & \text{if } x \in \mathbb{Q} \\ -5x^2 & \text{if } x \notin \mathbb{Q} \end{cases}$$

Which of the following is true?

(A) f is discontinuous at all $x \in \mathbb{R}$.

(B) f is continuous only at $x = 0$ and differentiable only at $x = 0$.

(C) f is continuous only at $x = 0$ and nondifferentiable at all $x \in \mathbb{R}$.

(D) f is continuous at all $x \in \mathbb{Q}$ and nondifferentiable at all $x \in \mathbb{R}$.

(E) f is continuous at all $x \notin \mathbb{Q}$ and nondifferentiable at all $x \in \mathbb{R}$.

---

48. Let g be the function defined by $g(x, y, z) = 3x^2y + z$ for all real x, y, and z. Which of the following is the best approximation of the directional derivative of g at the point $(0, 0, \pi)$ in the direction of the vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$? (Note: $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the standard basis vectors in $\mathbb{R}^3$.)

(A) 0.2 (B) 0.8 (C) 1.4 (D) 2.0 (E) 2.6

---

49. What is the largest order of an element in the group of permutations of 5 objects?

(A) 5 (B) 6 (C) 12 (D) 15 (E) 120

GO ON TO THE NEXT PAGE.

SCRATCH WORK

50. Let R be a ring and let U and V be (two-sided) ideals of R. Which of the following must also be ideals of R ?

I. $U + V = \{u + v : u \in U \text{ and } v \in V\}$

II. $U \cdot V = \{uv : u \in U \text{ and } v \in V\}$

III. $U \cap V$

(A) II only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

---

51. Which of the following is an orthonormal basis for the column space of the real matrix $\begin{pmatrix} 1 & -1 & 2 & -3 \\ -1 & 1 & -3 & 2 \\ 2 & -2 & 5 & -5 \end{pmatrix}$?

(A) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

(B) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

(C) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{pmatrix} \right\}$

(D) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} \right\}$

(E) $\left\{ \begin{pmatrix} \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \right\}$

SCRATCH WORK

52. A university's mathematics department has 10 professors and will offer 20 different courses next semester. Each professor will be assigned to teach exactly 2 of the courses, and each course will have exactly one professor assigned to teach it. If any professor can be assigned to teach any course, how many different complete assignments of the 10 professors to the 20 courses are possible?

(A) $\frac{20!}{2^{10}}$ (B) $\frac{10!}{2^9}$ (C) $10^{20} - 2^{10}$ (D) $10^{20} - 100$ (E) $\frac{20!10!}{2^{10}}$

---

53. Let f and g be continuous functions of a real variable such that $g(x) = \int_0^x f(y)(y - x)\,dy$ for all x. If g is three times continuously differentiable, what is the greatest integer n for which f must be n times continuously differentiable?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

54. If a real number x is chosen at random in the interval $[0, 3]$ and a real number y is chosen at random in the interval $[0, 4]$, what is the probability that $x < y$?

(A) $\frac{1}{2}$ (B) $\frac{7}{12}$ (C) $\frac{5}{8}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

GO ON TO THE NEXT PAGE.

SCRATCH WORK

55. If a and b are positive numbers, what is the value of $\int_0^\infty \frac{e^{ax} - e^{bx}}{\left(1 + e^{ax}\right)\left(1 + e^{bx}\right)}\,dx$?

(A) 0 (B) 1 (C) $a - b$ (D) $(a - b)\log 2$ (E) $\frac{a - b}{ab}\log 2$

---

56. Which of the following statements are true?

I. There exists a constant C such that $\log x \le C\sqrt{x}$ for all $x \ge 1$.

II. There exists a constant C such that $\sum_{k=1}^{n} k^2 \le Cn^2$ for all integers $n \ge 1$.

III. There exists a constant C such that $\left|\sin x - x\right| \le C\left|x^3\right|$ for all real x.

(A) None (B) I only (C) III only (D) I and III only (E) I, II, and III

SCRATCH WORK

57. For each positive integer n, let x_n be a real number in the open interval $\left(0, \frac{1}{n}\right)$. Which of the following statements must be true?

I. $\lim_{n\to\infty} x_n = 0$

II. If f is a continuous real-valued function defined on $(0, 1)$, then $\{f(x_n)\}_{n=1}^{\infty}$ is a Cauchy sequence.

III. If g is a uniformly continuous real-valued function defined on $(0, 1)$, then $\lim_{n\to\infty} g(x_n)$ exists.

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

---

58. A circular helix in *xyz*-space has the following parametric equations, where $\theta \in \mathbb{R}$.

$$x(\theta) = 5\cos\theta$$
$$y(\theta) = 5\sin\theta$$
$$z(\theta) = \theta$$

Let $L(\theta)$ be the arc length of the helix from the point $P(\theta) = (x(\theta), y(\theta), z(\theta))$ to the point $(5, 0, 0)$, and let $D(\theta)$ be the distance between $P(\theta)$ and the origin. If $L(\theta_0) = 26$, then $D(\theta_0) =$

(A) 6 (B) $\sqrt{51}$ (C) $\sqrt{52}$ (D) $14\sqrt{3}$ (E) $15\sqrt{3}$

SCRATCH WORK

59. Let A be a real 3×3 matrix. Which of the following conditions does NOT imply that A is invertible?

(A) $-A$ is invertible.

(B) There exists a positive integer k such that $\det\left(A^k\right) \neq 0$.

(C) There exists a positive integer k such that $(I - A)^k = 0$, where I is the 3×3 identity matrix.

(D) The set of all vectors of the form $A\mathbf{v}$, where $\mathbf{v} \in \mathbb{R}^3$, is $\mathbb{R}^3$.

(E) There exist 3 linearly independent vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{R}^3$ such that $A\mathbf{v}_i \neq \mathbf{0}$ for each i.

---

60. A real-valued function f defined on $\mathbb{R}$ has the following property.

For every positive number ϵ, there exists a positive number δ such that
$|f(x) - f(1)| \geq \epsilon$ whenever $|x - 1| \geq \delta$.

This property is <u>equivalent</u> to which of the following statements about f?

(A) f is continuous at $x = 1$.

(B) f is discontinuous at $x = 1$.

(C) f is unbounded.

(D) $\lim\limits_{|x| \to \infty} |f(x)| = \infty$

(E) $\int_0^\infty |f(x)| dx = \infty$

SCRATCH WORK

61. A tank initially contains a salt solution of 3 grams of salt dissolved in 100 liters of water. A salt solution containing 0.02 grams of salt per liter of water is sprayed into the tank at a rate of 4 liters per minute. The sprayed solution is continually mixed with the salt solution in the tank, and the mixture flows out of the tank at a rate of 4 liters per minute. If the mixing is instantaneous, how many grams of salt are in the tank after 100 minutes have elapsed?

(A) 2 (B) $2 - e^{-2}$ (C) $2 + e^{-2}$ (D) $2 - e^{-4}$ (E) $2 + e^{-4}$

---

62. Let S be the subset of $\mathbb{R}^2$ consisting of all points (x, y) in the unit square $[0, 1] \times [0, 1]$ for which x or y, or both, are irrational. With respect to the standard topology on $\mathbb{R}^2$, S is

(A) closed

(B) open

(C) connected

(D) totally disconnected

(E) compact

GO ON TO THE NEXT PAGE.

SCRATCH WORK

63. For any nonempty sets A and B of real numbers, let $A \cdot B$ be the set defined by

$$A \cdot B = \{xy : x \in A \text{ and } y \in B\}.$$

If A and B are nonempty bounded sets of real numbers and if $\sup(A) > \sup(B)$, then $\sup(A \cdot B) =$

(A) $\sup(A)\sup(B)$

(B) $\sup(A)\inf(B)$

(C) $\max\{\sup(A)\sup(B),\ \inf(A)\inf(B)\}$

(D) $\max\{\sup(A)\sup(B),\ \sup(A)\inf(B)\}$

(E) $\max\{\sup(A)\sup(B),\ \inf(A)\sup(B),\ \inf(A)\inf(B)\}$

---

64. What is the value of the flux of the vector field $\mathbf{F}$, defined on $\mathbb{R}^3$ by $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, through the surface $z = \sqrt{1 - x^2 - y^2}$ oriented with upward-pointing normal vector field? (Note: $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the standard basis vectors in $\mathbb{R}^3$.)

(A) 0 (B) $\frac{2\pi}{3}$ (C) π (D) $\frac{4\pi}{3}$ (E) 2π

GO ON TO THE NEXT PAGE.

SCRATCH WORK

65. Let g be a differentiable function of two real variables, and let f be the function of a complex variable z defined by

$$f(z) = e^x \sin y + ig(x, y),$$

where x and y are the real and imaginary parts of z, respectively. If f is an analytic function on the complex plane, then $g(3, 2) - g(1, 2) =$

(A) e^2

(B) $e^2(\sin 3 - \sin 1)$

(C) $e^2(\cos 3 - \cos 1)$

(D) $e - e^3 \sin 2$

(E) $\left(e - e^3\right)\cos 2$

---

66. Let $\mathbb{Z}_{17}$ be the ring of integers modulo 17, and let $\mathbb{Z}_{17}{}^{\times}$ be the group of units of $\mathbb{Z}_{17}$ under multiplication. Which of the following are generators of $\mathbb{Z}_{17}{}^{\times}$?

I. 5

II. 8

III. 16

(A) None (B) I only (C) II only (D) III only (E) I, II, and III

S T O P

If you finish before time is called, you may check your work on this test.

SCRATCH WORK

I

NOTE: To ensure prompt processing of test results, it is important that you fill in the blanks exactly as directed.

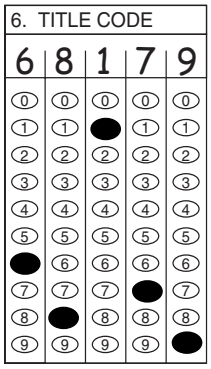
SUBJECT TEST

A. Print and sign your full name in this box:

PRINT: _____
(LAST) (FIRST) (MIDDLE)

SIGN: _____

Copy this code in box 6 on your answer sheet. Then fill in the corresponding ovals exactly as shown.



Copy the Test Name and Form Code in box 7 on your answer sheet.

TEST NAME Mathematics

FORM CODE GR1768

GRADUATE RECORD EXAMINATIONS SUBJECT TEST

B. The Subject Tests are intended to measure your achievement in a specialized field of study. Most of the questions are concerned with subject matter that is probably familiar to you, but some of the questions may refer to areas that you have not studied.

Your score will be determined by the number of questions you answer correctly. Questions you answer incorrectly or for which you mark no answer or more than one answer are counted as incorrect. Nothing is subtracted from a score if you answer a question incorrectly. Therefore, to maximize your score, it is better for you to guess at an answer than not to respond at all.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult for you. Go on to the other questions and come back to the difficult ones later if you can.

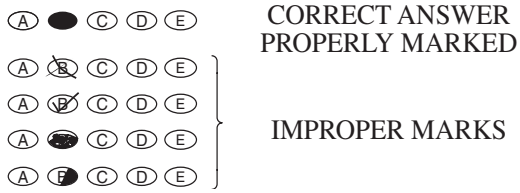
YOU MUST INDICATE ALL YOUR ANSWERS ON THE SEPARATE ANSWER SHEET. No credit will be given for anything written in this examination book, but you may write in the book as much as you wish to work out your answers. After you have decided on your response to a question, fill in the corresponding oval on the answer sheet. BE SURE THAT EACH MARK IS DARK AND COMPLETELY FILLS THE OVAL. Mark only one answer to each question. No credit will be given for multiple answers. Erase all stray marks. If you change an answer, be sure that all previous marks are erased completely. Incomplete erasures may be read as intended answers. Do not be concerned that the answer sheet provides spaces for more answers than there are questions in the test.

Example:

What city is the capital of France?

(A) Rome
(B) Paris
(C) London
(D) Cairo
(E) Oslo

Sample Answer



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Educational Testing Service
Princeton, New Jersey 08541

Worksheet for the GRE Mathematics Test, Form GR1768
Answer Key and Percentages* of Test Takers Answering Each Question Correctly

QUESTION		P+	CORRECT RESPONSE
Number	Answer		
1	E	88	
2	C	62	
3	D	80	
4	A	80	
5	E	77	
6	C	58	
7	C	73	
8	B	80	
9	A	82	
10	A	87	
11	B	72	
12	B	78	
13	D	86	
14	B	80	
15	A	58	
16	B	75	
17	B	49	
18	A	75	
19	E	53	
20	E	78	
21	B	44	
22	C	45	
23	D	80	
24	E	59	
25	A	68	
26	D	77	
27	D	85	
28	D	65	
29	C	45	
30	A	41	
31	C	58	
32	E	72	
33	C	72	
34	B	68	
35	B	65	

QUESTION		P+	CORRECT RESPONSE
Number	Answer		
36	C	49	
37	C	31	
38	C	37	
39	D	63	
40	C	64	
41	A	56	
42	E	43	
43	A	27	
44	B	44	
45	C	35	
46	C	67	
47	B	52	
48	B	43	
49	B	30	
50	D	24	
51	E	43	
52	A	53	
53	A	22	
54	C	53	
55	E	33	
56	D	38	
57	C	36	
58	B	38	
59	E	27	
60	D	25	
61	E	36	
62	C	32	
63	E	41	
64	E	24	
65	E	53	
66	B	27	

Total Correct:_____

Total Scaled: _____

* The numbers in the P+ column indicate the percentages of test takers in the United States who answer each question correctly.

Score Conversions for the GRE Mathematics Test, Form GR1768

Total Score			
Total Correct	**Scaled Score**	**Total Correct**	**Scaled Score**
63-66	910	33	590
61-62	900	32	580
		31	570
59-60	890	30	560
58	880	29	550
57	870		
56	860	28	540
55	850	27	520
		26	510
54	840	25	500
53	830		
52	820	24	490
51	810	22-23	480
		21	470
50	790	20	460
49	780	19	450
48	770		
47	760	18	440
		17	430
46	740	16	420
45	730	15	410
44	720	14	400
43	710		
		12-13	390
42	690	11	380
41	680	9-10	370
40	670	8	360
39	660	7	350
38	650		
		6	340
37	630	4-5	330
36	620	3	320
35	610	2	310
34	600	1	300
		0	280

GRADUATE RECORD EXAMINATIONS® - GRE - SUBJECT TEST

SIDE 1

DO NOT USE INK

Use only a pencil with soft, black lead (No. 2 or HB) to complete this answer sheet. Be sure to fill in completely the space that corresponds to your answer choice. Completely erase any errors or stray marks.

1. NAME Enter your last name, first name initial (given name), and middle initial if you have one. Omit spaces, apostrophes, Jr., II., etc.

Last Name only (Family Name or Surname) - First 15 Letters | First Name Initial | Middle Initial

2.

YOUR NAME: (Print) Last Name (Family or Surname) First Name (Given) M.I.

MAILING ADDRESS: (Print) P.O. Box or Street Address

City State or Province

Country ZIP or Postal Code

CENTER: City State or Province

Country Center Number Room Number

SIGNATURE:

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ● .
YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

1	A B C D E	24	A B C D E	47	A B C D E	70	A B C D E	93	A B C D E
2	A B C D E	25	A B C D E	48	A B C D E	71	A B C D E	94	A B C D E
3	A B C D E	26	A B C D E	49	A B C D E	72	A B C D E	95	A B C D E
4	A B C D E	27	A B C D E	50	A B C D E	73	A B C D E	96	A B C D E
5	A B C D E	28	A B C D E	51	A B C D E	74	A B C D E	97	A B C D E
6	A B C D E	29	A B C D E	52	A B C D E	75	A B C D E	98	A B C D E
7	A B C D E	30	A B C D E	53	A B C D E	76	A B C D E	99	A B C D E
8	A B C D E	31	A B C D E	54	A B C D E	77	A B C D E	100	A B C D E
9	A B C D E	32	A B C D E	55	A B C D E	78	A B C D E	101	A B C D E
10	A B C D E	33	A B C D E	56	A B C D E	79	A B C D E	102	A B C D E
11	A B C D E	34	A B C D E	57	A B C D E	80	A B C D E	103	A B C D E
12	A B C D E	35	A B C D E	58	A B C D E	81	A B C D E	104	A B C D E
13	A B C D E	36	A B C D E	59	A B C D E	82	A B C D E	105	A B C D E
14	A B C D E	37	A B C D E	60	A B C D E	83	A B C D E	106	A B C D E
15	A B C D E	38	A B C D E	61	A B C D E	84	A B C D E	107	A B C D E
16	A B C D E	39	A B C D E	62	A B C D E	85	A B C D E	108	A B C D E
17	A B C D E	40	A B C D E	63	A B C D E	86	A B C D E	109	A B C D E
18	A B C D E	41	A B C D E	64	A B C D E	87	A B C D E	110	A B C D E
19	A B C D E	42	A B C D E	65	A B C D E	88	A B C D E	111	A B C D E
20	A B C D E	43	A B C D E	66	A B C D E	89	A B C D E	112	A B C D E
21	A B C D E	44	A B C D E	67	A B C D E	90	A B C D E	113	A B C D E
22	A B C D E	45	A B C D E	68	A B C D E	91	A B C D E	114	A B C D E
23	A B C D E	46	A B C D E	69	A B C D E	92	A B C D E	115	A B C D E

Item responses continued on reverse side.

3. DATE OF BIRTH

Month	Day	Year
Jan.		
Feb.	0 0	0 0
Mar.	1 1	1 1
April	2 2	2 2
May	3 3	3 3
June	4 4	4 4
July	5 5	5 5
Aug.	6 6	6 6
Sept.	7 7	7 7
Oct.	8 8	8 8
Nov.	9 9	9 9
Dec.		

4. LAST FOUR DIGITS OF SOCIAL SECURITY NUMBER (U.S.A. only)

5. REGISTRATION NUMBER (from your admission ticket)

6. TITLE CODE (on back cover of your test book)

7. TEST NAME (on back cover of your test book)

FORM CODE (on back cover of your test book)

8. TEST BOOK SERIAL NUMBER (number in upper right corner of front cover of your test book)

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SIDE 2

SUBJECT TEST

COMPLETE THE CERTIFICATION STATEMENT, THEN TURN ANSWER SHEET OVER TO SIDE 1.

CERTIFICATION STATEMENT

Please write the following statement below, DO NOT PRINT.
"I certify that I am the person whose name appears on this answer sheet. I also agree not to disclose the contents of the test I am taking today to anyone."
Sign and date where indicated.

SIGNATURE: _____ **DATE:** ___/___/___ (Month / Day / Year)

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●.
YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

116 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	148 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	180 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	212 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
117 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	149 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	181 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	213 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
118 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	150 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	182 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	214 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
119 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	151 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	183 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	215 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
120 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	152 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	184 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	216 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
121 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	153 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	185 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	217 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
122 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	154 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	186 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	218 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
123 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	155 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	187 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	219 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
124 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	156 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	188 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	220 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
125 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	157 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	189 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	221 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
126 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	158 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	190 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	222 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
127 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	159 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	191 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	223 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
128 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	160 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	192 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	224 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
129 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	161 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	193 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	225 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
130 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	162 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	194 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	226 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
131 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	163 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	195 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	227 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
132 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	164 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	196 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	228 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
133 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	165 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	197 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	229 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
134 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	166 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	198 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	230 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
135 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	167 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	199 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	231 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
136 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	168 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	200 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	232 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
137 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	169 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	201 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	233 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
138 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	170 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	202 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	234 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
139 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	171 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	203 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	235 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
140 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	172 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	204 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	236 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
141 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	173 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	205 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	237 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
142 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	174 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	206 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	238 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
143 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	175 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	207 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	239 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
144 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	176 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	208 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	240 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
145 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	177 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	209 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	241 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
146 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	178 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	210 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	242 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
147 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	179 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	211 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	

TR	TW	TFS	TCS	1R	1W	1FS	1CS	2R	2W	2FS	2CS
FOR ETS USE ONLY				3R	3W	3FS	3CS	4R	4W	4FS	4CS
				5R	5W	5FS	5CS	6R	6W	6FS	6CS

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