- обичне диф. јне – ОДЈ , $x: \mathbb{R} \to \mathbb{R}$, $x(t)$
- парцијалне диф јна – ПДЈ , $u: \mathbb{R}^n \to \mathbb{R}$, $u(x_1, \dots, x_n)$

Примери: $\Delta u = 0$ – хармонијска фја

1) $u: \mathbb{R}^2 \to \mathbb{R}$, $u(x,y)$ $\quad \Delta u = 0 \iff \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ← 2. реда ($\frac{\partial^2}{\partial x^2}$ и $\frac{\partial^2}{\partial y^2}$)

2) $u, v: \mathbb{R}^2 \to \mathbb{R}$

$$\left.\begin{aligned} u_x &= v_y \\ v_x &= -u_y \end{aligned}\right\} \text{ Коши-Риманове јне}$$

↖ 1 реда ($\frac{\partial}{\partial x}$ и $\frac{\partial}{\partial y}$)

3) Јна проводења топлоте: $u(t, x_1, \dots, x_n)$, $u_t = k \Delta u$ ← 2 реда

$$\Delta u = \frac{\partial^2 u}{\partial x_1^2} + \dots + \frac{\partial^2 u}{\partial x_n^2}$$

нотација: $\frac{\partial u}{\partial x} = u_x = u'_x$

Општи облик ПДЈ 1. реда: $F\left(x_1, \dots, x_n, u, \frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_n}\right) = 0$

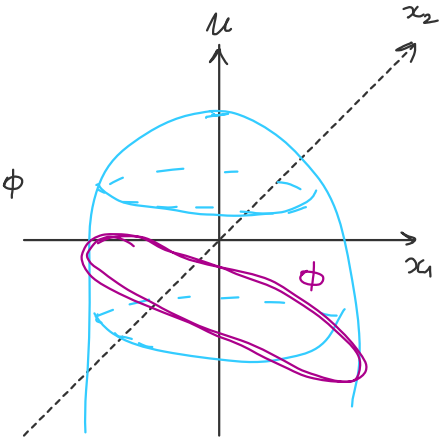
$F: \mathbb{R}^{2n+1} \to \mathbb{R}$, $u(x_1, \dots, x_n) = ?$

- хомогена линеарна (ХЛ): $a_1(x_1, \dots, x_n)\frac{\partial u}{\partial x_1} + \dots + a_n(x_1, \dots, x_n)\frac{\partial u}{\partial x_n} = 0$
- линеарна (Л): $a_1(x_1, \dots, x_n)\frac{\partial u}{\partial x_1} + \dots + a_n(x_1, \dots, x_n)\frac{\partial u}{\partial x_n} + a_{n+1}(x_1, \dots, x_n)u = c(x_1, \dots, x_n)$
- квазилинеарна (КЛ): $a_1(x_1, \dots, x_n, u)\frac{\partial u}{\partial x_1} + \dots + a_n(x_1, \dots, x_n, u)\frac{\partial u}{\partial x_n} = c(x_1, \dots, x_n, u)$

$$(ХЛ) \subseteq (Л) \subseteq (КЛ)$$

Кошијев проблем: наћи решење које садржи задату фигуру Φ

$$\Phi \subseteq \Gamma(u)$$



Метода карактеристика

Посматрајмо КЛ: $a(x,y,u)\frac{\partial u}{\partial x} + b(x,y,u)\frac{\partial u}{\partial y} = c(x,y,u)$ за $u(x,y)$

(ради једноставности $n=2$)

$S = \{(x,y,u(x,y))\} \subseteq \mathbb{R}^3$ → график решења $u(x,y)$ → површ у $\mathbb{R}^3$

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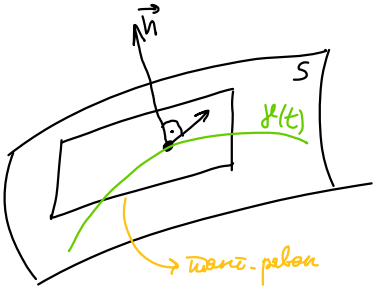
$\vec{n} = (u_x(x,y), u_y(x,y), -1)$ – нормала на површ S у $(x,y,u(x,y))$



u решење $\Rightarrow$ $a(x,y,u)\frac{\partial u}{\partial x} + b(x,y,u)\frac{\partial u}{\partial y} = c(x,y,u)$

$\Rightarrow$ $a(x,y,u)\frac{\partial u}{\partial x} + b(x,y,u)\frac{\partial u}{\partial y} - c(x,y,u) = 0$

$\Rightarrow$ $\langle (a,b,c), (u_x, u_y, -1) \rangle = 0$ ($(u_x,u_y,-1) = \vec{n}$)

$\Rightarrow$ $\langle (a,b,c), \vec{n} \rangle = 0$

$\Rightarrow$ $(a,b,c) \in$ тангентној равни на S

$\Rightarrow$ $\left.\begin{aligned} x'(t) &= a(x,y,u) \\ y'(t) &= b(x,y,u) \\ u'(t) &= c(x,y,u) \end{aligned}\right\}$ $\circledast$ – систем карактеристика

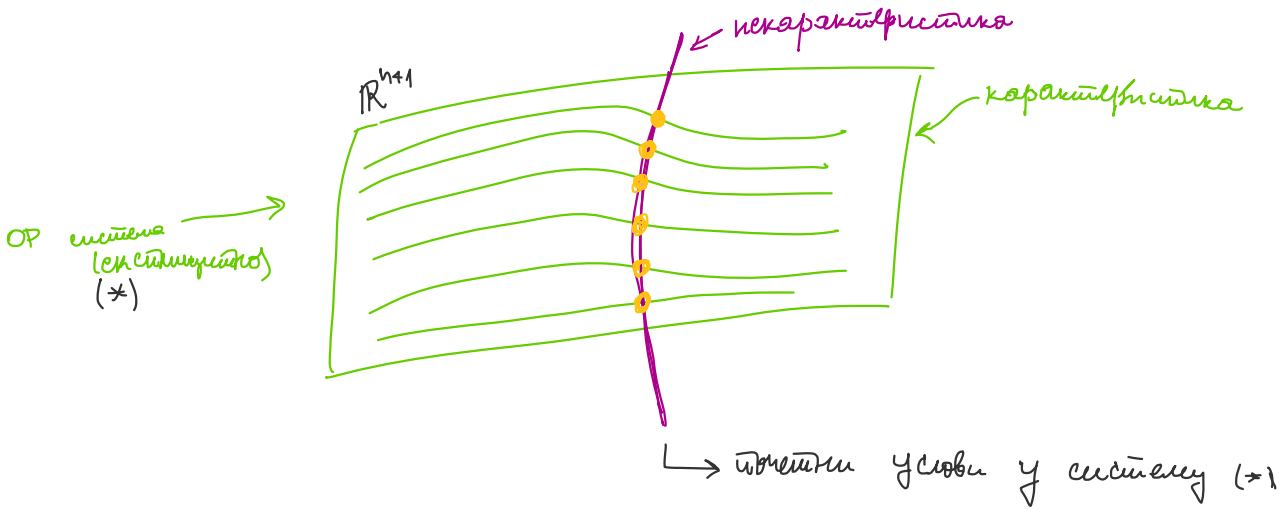
$\gamma(t) = (x(t), y(t), u(t))$ – решење система $\circledast$ $\Rightarrow \gamma(t) \in S, \forall t$

(KJ) $\Rightarrow$ $x_j'(t) = a_j(x_1, \dots, x_n, u)$

$u'(t) = c(x_1, \dots, x_n, u)$

Систем карактеристика $(*)$

(NJ) $\Rightarrow$ $x_j'(t) = a_j(x_1, \dots, x_n)$



↳ почетни услови у систему $(*)$

(1) Решити Кошијев проблем $u_x' + u_y' + u = 1$, $u(x, x+x^2) = \sin x$, $x > 0$

(1) Решити Кошијев проблем $u_x' + u u_y' + 1 \cdot u = 1$, $u(x, x+x^2) = \sin x$, $x > 0$

↳ Кошијев услов

(КЛ) $u_x' + u u_y' = 1 - u$

$1 \cdot u_x' + 1 \cdot u_y' = 1 - u$

a_1 a_2 c

$$\left.\begin{aligned} x'(t) &= 1 \\ y'(t) &= 1 \\ u'(t) &= 1 - u \end{aligned}\right\} \quad \begin{aligned} x(t) &= t + c_1 \\ y(t) &= t + c_2 \\ u(t) &= 1 + c_3 e^{-t} \end{aligned}$$

$u(x, x+x^2) = \sin x, \; x > 0$

C-некарактеристика $\rightsquigarrow \gamma(s) = (s, s+s^2, \sin s), \; s > 0$

$\subseteq \Gamma(u)$

$x(t, s) = t + c_1(s)$ $\quad x_0(s) = s$ $\quad x_0(s) = x(0, s)$

$y(t, s) = t + c_2(s)$ $\quad y_0(s) = s + s^2$

$u(t, s) = 1 + c_3(s) \cdot e^{-t}$ $\quad u_0(s) = \sin s$

$s = c_1(s)$

$s + s^2 = y_0(s) = c_2(s)$

$\sin s = u_0(s) = 1 + c_3(s) \; e^0 \Rightarrow c_3(s) = \sin s - 1$

$\big(x(t, s), y(t, s), u(t, s)\big) = \big(t + s, \; t + s + s^2, \; 1 + (\sin s - 1)\, e^{-t}\big)$

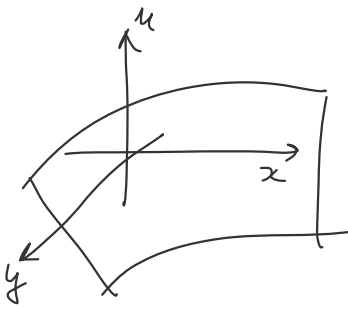
↑ ова тврди је решење

$(s > 0)$

$y-x=s^2 \Rightarrow s=\sqrt{y-x}$

$t=x-s=x-\sqrt{y-x}$

$u=1+(u_0-1)\cdot e^{-t}=1+(\sin\sqrt{y-x}-1)\cdot e^{-x+\sqrt{y-x}}$



проверка: $u(s,1,s+1)=\sin s$

$u'_x+u'_y+u=1$

② $(y+u)u'_x+yu'_y=x-y$ $\qquad u|_{y=1}=x+1$

$u(x,1)=x+1$

y

$(s,1,s+1)\subseteq\Gamma(u)$

$x'=y+u$, $x_0(s)=s$

$y'=y$, $y_0(s)=1$

$u'=x-y$, $u_0(s)=s+1$

$y=c_1e^t$

$x=c_2e^t+c_3e^{-t}$

$u=c_2e^t-c_3e^{-t}-e^t$

$t=0$:

$c_2+c_3=s$

$c_1=1$

$c_2-c_3-1=s+1$

$c_1=1$

$c_2=s+1$

$c_3=-1$

решение $\left((s+1)e^t-e^{-t},\ e^t,\ se^t+e^{-t}\right)$

$\qquad\qquad\quad =x \qquad\quad =y \quad =u$

$$t = \ln y \Rightarrow e^{-t} = e^{-\ln y} = (y)^{-1} = \frac{1}{y}$$

$$x = (1+\lambda)y - \frac{1}{y} \Rightarrow \lambda = -1 + \frac{x}{y} + \frac{1}{y^2}$$

$$u(x,y) = \left(-1 + \frac{x}{y} + \frac{1}{y^2}\right) \cdot y + \frac{1}{y} = x - y + \frac{2}{y} .$$