ZADACI ZA VEŽBANJE

1.1

$$r_{De} = \sqrt{\frac{\varepsilon_0 k T_e}{n_e e^2}} \qquad \left.\begin{matrix} T_e = T_p = 10^7 K \\ n_e = n_p = 10^3 m^{-3} \end{matrix}\right\} \Rightarrow$$

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}} \left[\frac{rad}{s}\right] \qquad r_{De} \approx 6,9 \, km$$

$$\underline{\omega_{pe} = 2\pi \nu_{pe}} \Rightarrow \underline{\nu_{pe} [Hz]} = 284$$

$$\Lambda_e = r_{De}^3 n_e \qquad \Lambda_e \gg 1, \quad \Theta_{de} \gg 1$$

$$\Theta_{de} = \frac{kT_e}{\epsilon_F} \qquad v_{th,e} \approx 1,2 \cdot 10^7 \, m/s$$

$$\epsilon_F = \frac{h^2}{8 m_e} \left(\frac{3 n_e}{\pi}\right)^{2/3}$$

$$v_{th,e} = \sqrt{\frac{kT_e}{m_e}} \qquad \left.\begin{matrix} T_e = T_p = 10^8 K \\ n_e = n_p = 10^{20} m^{-3} \\ B = 3 \, T \end{matrix}\right| \Rightarrow$$

$$r_{c,e} = \frac{v_{th,e}}{\omega_{ce}} \qquad \frac{r_{De}}{r_{ce}} \approx 0,15 \qquad \frac{\omega_{pe}}{\omega_{ce}} \approx 1$$

$$\omega_{ce} = \frac{|e| B}{m_e}$$

$$p = nkT \qquad \beta < 1 \qquad p = p_e + p_p = 2 n_e k T_e$$

$$\beta = \frac{2 \mu_0 n k T}{B^2}$$

1.4

$$\frac{\omega_{pe}^2}{\omega_{ce}} = \frac{\dfrac{n_e e^{\cancel{2}}}{\varepsilon_0 \cancel{m_e}}}{\dfrac{\cancel{|e|} B}{\cancel{m_e}}} = \frac{n_e e}{\varepsilon_0 B}$$

$$\frac{\omega_{pp}^2}{\omega_{cp}} = \frac{\dfrac{n_p e^{\cancel{2}}}{\varepsilon_0 \cancel{m_p}}}{\dfrac{\cancel{|e|} B}{\cancel{m_p}}} = \frac{n_p e}{\varepsilon_0 B}$$

$$n_e = n_p \quad \Rightarrow \quad \underline{\frac{\omega_{pe}^2}{\omega_{ce}} = \frac{\omega_{pp}^2}{\omega_{cp}}}$$

1.14 $\vec{L}_\alpha (f_\alpha^M(v)) = ?$

$$f_\alpha^M = n_\alpha \left(\frac{m_\alpha}{2\pi k T_\alpha}\right)^{3/2} e^{-\frac{m_\alpha (\overbrace{\vec{v} - \vec{u}_\alpha}^{\vec{w}_\alpha})^2}{2kT_\alpha}}$$

$$\vec{L}_\alpha = \frac{1}{2} n_\alpha m_\alpha \langle w_\alpha^2 \vec{w}_\alpha \rangle_\alpha$$

$$\Rightarrow \vec{L}_\alpha = \frac{1}{2} n_\alpha m_\alpha \frac{1}{n_\alpha} n_\alpha \left(\frac{m_\alpha}{2\pi k T_\alpha}\right)^{3/2} \int w_\alpha^2 \vec{w}_\alpha \, e^{-\frac{m_\alpha w_\alpha^2}{2kT_\alpha}} d^3\vec{v} = \left| \begin{matrix} d^3\vec{v} = d^3(\vec{v} - \vec{u}_\alpha) \\ = d^3\vec{w}_\alpha \end{matrix} \right| \Rightarrow$$

$$\vec{L}_\alpha \propto \int_{V\vec{v}} w_\alpha^2 \vec{w}_\alpha \, e^{-\underbrace{\frac{m_\alpha}{2kT_\alpha}}_{a} w_\alpha^2} d^3\vec{w}_\alpha \Rightarrow I_1 \propto \int_{-\infty}^{\infty} w_{\alpha x}^3 e^{-a w_{\alpha x}^2} dw_{\alpha x}$$

$$I_2 \propto \int_{-\infty}^{\infty} w_{\alpha y}^3 e^{-a w_{\alpha y}^2} dw_{\alpha y}$$

$$I_3 \propto \int_{-\infty}^{\infty} w_{\alpha z}^3 e^{-a w_{\alpha z}^2} dw_{\alpha z}$$

NEPARNA F-JA

SIMETRIČNE GRANICE

$$I_1 = I_2 = I_3 = 0$$

$$\Rightarrow \underline{\vec{L}_\alpha = \vec{0}}$$

$$\hat{P}_\alpha (f_\alpha^M(v)) = ? \qquad \hat{P}_\alpha = n_\alpha m_\alpha \langle \vec{w}_\alpha \vec{w}_\alpha \rangle_\alpha = m_\alpha \int_{V\vec{v}} \vec{w}_\alpha \vec{w}_\alpha f_\alpha^M d^3\vec{v}$$

$$\vec{w_\alpha}\vec{w_\alpha} = \begin{pmatrix} w_x^2 & w_x w_y & w_x w_z \\ w_y w_x & w_y^2 & w_y w_z \\ w_z w_x & w_z w_y & w_z^2 \end{pmatrix}$$

$$\int_{-\infty}^{\infty} x e^{-ax^2} dx = 0 \Rightarrow \hat{\Pi}_\alpha = \hat{0}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{2a^{3/2}}, \quad p_\alpha = \frac{1}{3} \overbrace{n_\alpha m_\alpha}^{\rho_\alpha} \langle w_\alpha^2 \rangle_\alpha \Rightarrow$$

$$\hat{P}_\alpha = p_\alpha \hat{I}$$

1.17
1.18

$$\frac{\lambda_{mfp}}{\lambda_{De}} = ?$$

$$\lambda_{mfp,e} = v_{th,e} \tau_{coll,e} = \frac{\overbrace{v_{th,e}}^{\sqrt{\frac{kT_e}{m_e}}}}{w_{coll,e}} \cdot 2\pi \Rightarrow \frac{\lambda_{mfp,e}}{\lambda_{De}} = \frac{\frac{2\pi\sqrt{kT_e}}{\sqrt{m_e}\, w_{coll,e}}}{\sqrt{\frac{\varepsilon_0 kT_e}{n_e e^2}}} = 2\pi \frac{\sqrt{n_e e^2}}{\sqrt{\varepsilon_0 m_e}\, w_{coll,e}} \Rightarrow$$

$$w_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}} \quad \frac{\lambda_{mfp}}{\lambda_{De}} = 2\pi \frac{w_{pe}}{w_{coll}}$$

KAKO JE $w_{coll,e} \ll w_{pe} \Rightarrow \underline{\lambda_{mfp} \gg \lambda_{De}}$

$$\Lambda_e = \lambda_{De}^3 n_e$$

ŠTA JE $b_{90°}$?

$$b \in (\underbrace{b_{90°}}_{b_{min}\,(\text{KLASIČNO})}, \lambda_D) \quad \text{PARAMETAR SUDARA}$$

$$\lambda_{De}^2 = \frac{\varepsilon_0 kT_e}{n_e e^2}$$

$$\frac{\text{ELEKTROSTATIČKA POT. EN}}{\text{KINETIČKA ENERGIJA REL. KRETANJA}} = 2 \Rightarrow$$

$$\frac{e^2}{4\pi\varepsilon_0 b_{90°}} \cdot \frac{1}{\frac{mv^2}{2}} = 2 \Rightarrow b_{90°} = \frac{e^2}{4\pi\varepsilon_0 m v^2} \Big|\cdot\frac{n_e}{n_e} \Rightarrow$$

$$\frac{mv^2}{2} = \frac{3}{2} kT_e \Rightarrow b_{90°} = \frac{e^2}{4\pi\varepsilon_0 3kT_e}$$

$$b_{90°} \propto \frac{1}{n_e \lambda_{De}^2} \Rightarrow$$

$$\Lambda_e \propto (\lambda_{De}^2 \cdot n_e)\lambda_{De} \Rightarrow$$

$$\Lambda_e \propto \frac{\lambda_{De}}{b_{90°}}$$

3.1 J-NA KLIMONTOVIĆA

$$\frac{\partial K_\alpha}{\partial t} + \vec{v} \cdot \nabla K_\alpha + \underbrace{\frac{e_\alpha}{m_\alpha}(\vec{E}^m + \vec{v} \times \vec{B}^m)}_{\vec{a}} \cdot \nabla_v K_\alpha = 0$$

$$\left.\begin{aligned} \nabla_v \cdot (\vec{a} K_\alpha) &= \vec{a} \cdot \nabla_v K_\alpha + K_\alpha \nabla_v \cdot \vec{a} \overset{0}{} \\ \nabla \cdot (\vec{v} K_\alpha) &= \vec{v} \cdot \nabla K_\alpha + K_\alpha \cancel{\nabla \cdot \vec{v}} \overset{0}{} \end{aligned}\right\} \Rightarrow \frac{\partial K_\alpha}{\partial t} + \underbrace{\vec{v} \cdot \nabla K_\alpha}_{\nabla \cdot (\vec{v} K_\alpha)} + \underbrace{\vec{a} \cdot \nabla_v K_\alpha}_{\nabla_v \cdot (\vec{a} K_\alpha)} = 0$$

$$\Rightarrow \underline{\frac{\partial K_\alpha}{\partial t} + \nabla \cdot (\vec{v} K_\alpha) + \nabla_v \cdot (\vec{a} K_\alpha) = 0}$$

(4.1) $\frac{d}{dt}(p\rho^{-\gamma_g})=0 \Rightarrow \frac{\partial}{\partial t}(p\rho^{-\gamma_g}) + \vec{v}\cdot\nabla(p\rho^{-\gamma_g})=0$

$$\rho^{-\gamma_g}\frac{\partial p}{\partial t} - \gamma_g \rho^{-\gamma_g-1} p \frac{\partial\rho}{\partial t} + \vec{v}\cdot\rho^{-\gamma_g}\nabla p - \vec{v}\cdot p\gamma_g\rho^{-\gamma_g-1}\nabla\rho = 0$$

$$\rho\neq0 \Rightarrow \frac{\partial p}{\partial t} - \frac{\gamma_g p}{\rho}\frac{\partial\rho}{\partial t} - \frac{\gamma_g p}{\rho}\nabla\rho\cdot\vec{v} + \vec{v}\cdot\nabla p = 0$$

$$\left\| \frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v})=0 \Rightarrow \frac{\partial\rho}{\partial t} = -[\rho\nabla\cdot\vec{v} + \vec{v}\cdot\nabla\rho]\right\| \Rightarrow \frac{\partial p}{\partial t} + \frac{\gamma_g p}{\rho}[\rho\nabla\cdot\vec{v} + \cancel{\vec{v}\cdot\nabla\rho}] - \cancel{\frac{\gamma_g p}{\rho}\vec{v}\cdot\nabla\rho} + \vec{v}\cdot\nabla p = 0$$

$$\underline{\frac{\partial p}{\partial t} + \gamma_g p\nabla\cdot\vec{v} + \vec{v}\cdot\nabla p = 0}$$

NEKA JE $S = p\rho^{-\gamma_g}$

$$\frac{\partial}{\partial t}(\rho S) + \nabla\cdot(\rho S\vec{v}) = ?$$

$$\frac{\partial}{\partial t}(\rho p\rho^{-\gamma_g}) + \nabla\cdot(\rho p\rho^{-\gamma_g}\vec{v}) = \frac{\partial}{\partial t}(p\rho^{1-\gamma_g}) + \nabla\cdot(p\rho^{1-\gamma_g}\vec{v}) =$$

$$= p(1-\gamma_g)\rho^{-\gamma_g}\frac{\partial\rho}{\partial t} + \rho^{1-\gamma_g}\frac{\partial p}{\partial t} + p\rho^{1-\gamma_g}\nabla\cdot\vec{v} + \rho^{1-\gamma_g}\vec{v}\cdot\nabla p +$$

$$p(1-\gamma_g)\rho^{-\gamma_g}\vec{v}\cdot\nabla\rho = \left| \begin{array}{l} \frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{v})=0 \Rightarrow \\ \frac{\partial\rho}{\partial t} = -\rho\nabla\cdot\vec{v} - \vec{v}\cdot\nabla\rho \end{array} \right| =$$

$$= -p(1-\gamma_g)\rho^{-\gamma_g}\rho\nabla\cdot\vec{v} - \cancel{p(1-\gamma_g)\rho^{-\gamma_g}\vec{v}\cdot\nabla\rho} + \rho^{1-\gamma_g}\frac{\partial p}{\partial t} + \cancel{p\rho^{1-\gamma_g}\nabla\cdot\vec{v}} +$$

$$+\rho^{1-\gamma_g}\vec{v}\cdot\nabla p + \cancel{p(1-\gamma_g)\rho^{-\gamma_g}\vec{v}\cdot\nabla\rho} =$$

$$= p\gamma_g\rho^{1-\gamma_g}\nabla\cdot\vec{v} + \rho^{1-\gamma_g}\frac{\partial p}{\partial t} + \rho^{1-\gamma_g}\vec{v}\cdot\nabla p =$$

$$= \rho^{1-\gamma_g}\Big(\underbrace{\frac{\partial p}{\partial t} + \vec{v}\cdot\nabla p + p\gamma_g\nabla\cdot\vec{v}}_{0}\Big) = 0, \quad \rho\neq0$$

4.1 DODATAK

$$e = \frac{1}{\gamma_g - 1}\frac{p}{\rho} \Rightarrow \underline{p = (\gamma_g - 1)\, e\rho}$$

$$S \equiv p\rho^{-\gamma_g} = (\gamma_g - 1)\, e\rho^{1-\gamma_g}$$

$$\frac{dS}{dt} = 0 \Rightarrow \frac{d}{dt}\left(p\rho^{-\gamma_g}\right) = 0 \Rightarrow$$

$$\frac{\partial p}{\partial t} + \vec{v}\cdot\nabla p + \gamma_g p\, \nabla\cdot\vec{v} = 0$$

$$\Rightarrow \frac{\partial}{\partial t}\left((\gamma_g-1)e\rho\right) + \vec{v}\cdot\nabla\left((\gamma_g-1)e\rho\right) + \gamma_g(\gamma_g-1)e\rho\,\nabla\cdot\vec{v} = 0$$

$$(\gamma_g-1)\rho\frac{\partial e}{\partial t} + (\gamma_g-1)e\frac{\partial\rho}{\partial t} + \vec{v}\cdot(\gamma_g-1)\rho\nabla e + \vec{v}\cdot(\gamma_g-1)e\nabla\rho + \gamma_g(\gamma_g-1)e\rho\nabla\cdot\vec{v} = 0$$

IF $\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$

$$\frac{\partial\rho}{\partial t} = -\rho\nabla\cdot\vec{v} - \vec{v}\cdot\nabla\rho \;//$$

$$\Rightarrow (\gamma_g-1)\rho\frac{\partial e}{\partial t} + (\gamma_g-1)e\left[-\rho\nabla\cdot\vec{v} - \vec{v}\cdot\nabla\rho\right] + \vec{v}\cdot(\gamma_g-1)\rho\nabla e + \vec{v}\cdot(\gamma_g-1)e\nabla\rho + \gamma_g(\gamma_g-1)e\rho\nabla\cdot\vec{v} = 0$$

$$\Rightarrow \left|\begin{array}{c}(\gamma_g-1)\rho\neq 0\\ \rho\neq 0\end{array}\right| \Rightarrow \frac{\partial e}{\partial t} - e\nabla\cdot\vec{v} - \cancel{\frac{e}{\rho}\vec{v}\cdot\nabla\rho} + \vec{v}\cdot\nabla e + \cancel{\vec{v}\cdot\nabla\rho\frac{e}{\rho}} + \gamma_g e\nabla\cdot\vec{v} = 0$$

$$\Rightarrow \underline{\frac{\partial e}{\partial t} + \vec{v}\cdot\nabla e + (\gamma_g-1)\, e\,\nabla\cdot\vec{v} = 0}$$

4.4 5

NAČIN I

$$\frac{dH}{dt} = \frac{d}{dt}\int_V \vec{A}\cdot\vec{B}\, dV' = \int_V \frac{d\vec{A}}{dt}\cdot\vec{B}\, dV' + \int_V \vec{A}\cdot\frac{d\vec{B}}{dt}\, dV' + \int_V \vec{A}\cdot\vec{B}\,\frac{d}{dt}(dV')$$

IΓ $\frac{d}{dt}(dV') = \nabla\cdot\vec{v}\, dV'$ $\Rightarrow$ $\frac{dH}{dt} = \int_V \left(\frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt} + \vec{A}\cdot\vec{B}\,\nabla\cdot\vec{v}\right) dV'$

(VIDI VEŽBE) ||

$$\frac{dH}{dt} = \int_V \left(\frac{\partial\vec{A}}{\partial t}\cdot\vec{B} + \vec{v}\cdot\nabla\vec{A}\cdot\vec{B} + \vec{A}\cdot\frac{\partial\vec{B}}{\partial t} + \vec{A}\cdot\vec{v}\cdot\nabla\vec{B} + \vec{A}\cdot\vec{B}\,(\nabla\cdot\vec{v})\right) dV'$$

IΓ $\nabla\cdot(T\vec{A}) = T\,\nabla\cdot\vec{A} + \vec{A}\cdot\nabla T$

$\nabla\cdot((\vec{A}\cdot\vec{B})\vec{v}) = (\vec{A}\cdot\vec{B})(\nabla\cdot\vec{v}) + \vec{v}\cdot\nabla(\vec{A}\cdot\vec{B})$ $\Rightarrow$

$(\vec{A}\cdot\vec{B})(\nabla\cdot\vec{v}) = \nabla\cdot((\vec{A}\cdot\vec{B})\vec{v}) - \vec{v}\cdot\nabla(\vec{A}\cdot\vec{B})$ ||

IΓ $\frac{\partial\vec{B}}{\partial t} = \nabla\times(\vec{v}\times\vec{B})$, $\vec{B} = \nabla\times\vec{A}$

$\frac{\partial}{\partial t}(\nabla\times\vec{A}) = \nabla\times\frac{\partial\vec{A}}{\partial t} = \nabla\times(\vec{v}\times\vec{B})$ $\Rightarrow$ $\frac{\partial\vec{A}}{\partial t} = \vec{v}\times\vec{B}$ ||

IΓ $\nabla(\vec{A}\cdot\vec{B}) = \cancel{\underset{\nabla\times\vec{B}}{\vec{A}}\times(\nabla\times\vec{B})} + \cancel{\vec{B}\times(\underbrace{\nabla\times\vec{A}}_{\vec{B}})} + \vec{A}\cdot\nabla\vec{B} + \vec{B}\cdot\nabla\vec{A}$ ||

$\nabla\times\nabla = \vec{0}$

$$\frac{dH}{dt} = \int_V \Big((\vec{v}\times\vec{B})\cdot\vec{B} + \cancel{\vec{v}\cdot\nabla\vec{A}\cdot\vec{B}} + \vec{A}\cdot\nabla\times(\vec{v}\times\vec{B}) + \cancel{\vec{A}\cdot\vec{v}\cdot\nabla\vec{B}} +$$
$$+ \nabla\cdot((\vec{A}\cdot\vec{B})\vec{v}) - \cancel{\vec{v}\cdot\vec{A}\cdot\nabla\vec{B}} - \cancel{\vec{v}\cdot\vec{B}\cdot\nabla\vec{A}} \Big) dV'$$

$$\frac{dH}{dt} = \int_V \left(\cancel{(\vec{v}\times\vec{B})\cdot\vec{B}} + \vec{A}\cdot\nabla\times(\vec{v}\times\vec{B}) + \nabla\cdot((\vec{A}\cdot\vec{B})\vec{v})\right) dV'$$

MEŠOVITI SA DVA ISTA

IΓ $\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$

$\nabla\cdot((\vec{v}\times\vec{B})\times\vec{A}) = \vec{A}\cdot(\nabla\times(\vec{v}\times\vec{B})) - \cancel{(\vec{v}\times\vec{B})\cdot(\underbrace{\nabla\times\vec{B}}_{\vec{B}})}$ MEŠOVITI SA ISTIM ||

$$\frac{dH}{dt} = \int_V \left(\nabla\cdot\left((\vec{v}\times\vec{B})\times\vec{A} + (\vec{A}\cdot\vec{B})\vec{v}\right)\right) dV' \Rightarrow$$

$$\frac{dH}{dt} = \oint_S \Big(\underbrace{(\vec{v}\times\vec{B})\times\vec{A}}_{\vec{B}(\vec{v}\cdot\vec{A}) - \vec{v}(\vec{B}\cdot\vec{A})} + \vec{A}\cdot\vec{B}\,\vec{v}\Big)\cdot\vec{n}\, dS$$

KAKO JE $\vec{B}\cdot\vec{n} = 0$ $\Rightarrow$ $\boxed{\frac{dH}{dt} = 0}$

NAČIN II

$$\frac{dH}{dt} = \frac{d}{dt}\int_V \overbrace{(\vec A\cdot\vec B)}^{F}\, dV' = \left|\begin{aligned}&\frac{d}{dt}\int_{V(t)} F\, dV' = \\ &= \int_{V(t)}\left(\frac{\partial F}{\partial t} + \nabla\cdot(F\vec v)\right)dV'\end{aligned}\right| = \int_V \left(\frac{\partial F}{\partial t} + \nabla\cdot(F\vec v)\right)dV'$$

$$\frac{dH}{dt} = \int_V\left(\frac{\partial}{\partial t}(\vec A\cdot\vec B) + \nabla\cdot(\vec A\cdot\vec B\,\vec v)\right)dV' = \int_V\left(\frac{\partial\vec A}{\partial t}\cdot\vec B + \vec A\cdot\frac{\partial\vec B}{\partial t} + \nabla\cdot(\vec A\cdot\vec B\,\vec v)\right)dV'$$

$$\frac{dH}{dt} = \int_V\left(\overbrace{\cancel{\underbrace{(\vec v\times\vec B)}_{\partial\vec A/\partial t}\cdot\vec B}}^{0} + \vec A\cdot\nabla\times(\vec v\times\vec B) + \nabla\cdot(\vec A\cdot\vec B\,\vec v)\right)dV'$$

IF $\nabla\cdot((\vec v\times\vec B)\times\vec A) = \vec A\cdot\nabla\times(\vec v\times\vec B)$ || $\Rightarrow$

$$\frac{dH}{dt} = \int_V \nabla\cdot\left((\vec v\times\vec B)\times\vec A + \vec A\cdot\vec B\,\vec v\right)dV' \xrightarrow[\text{I RANIJE}]{\text{KAO}}$$

$$\Rightarrow \frac{dH}{dt} = 0$$

DODATNI ZADACI

① POKAZATI DA $\vec{B} = \begin{pmatrix} 0 \\ B_0 \sin(\alpha x) \\ B_0 \cos(\alpha x) \end{pmatrix}$, $\alpha = const$, ZADOVOLJAVA USLOV POLJA NULTE SILE.

$$\nabla \times \vec{B} = \alpha \vec{B}, \quad \alpha = const$$

$$\nabla \times \vec{B} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & B_0 \sin(\alpha x) & B_0 \cos(\alpha x) \end{vmatrix} = \begin{pmatrix} 0 \\ B_0 \alpha \sin(\alpha x) \\ B_0 \alpha \cos(\alpha x) \end{pmatrix} = \alpha \vec{B}$$

② UBEDITI SE DA POLJE OBLIKA $\vec{B} = \begin{pmatrix} 0 \\ B_\varphi(\rho) \\ B_z(\rho) \end{pmatrix}$, $B_\varphi(\rho) = \frac{B_0 K \rho}{1 + K^2 \rho^2}$, $K = const$, $B_z(\rho) = \frac{B_0}{1 + K^2 \rho^2}$

ISPUNJAVA USLOV POLJA NULTE SILE SA $\alpha = \frac{2K}{1 + K^2 \rho^2}$. KOLIKO JE j_z?

$$\nabla \times \vec{B} = \begin{pmatrix} 0 \\ -\frac{\partial B_z}{\partial \rho} \\ \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho B_\varphi) \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ B_\varphi(\rho) \\ B_z(\rho) \end{pmatrix}$$

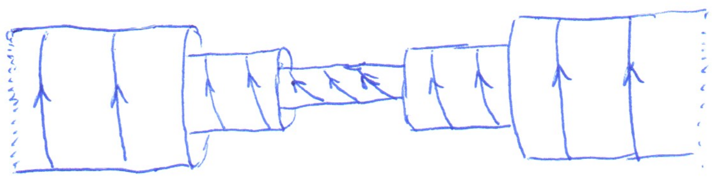
$$\vec{B} = \begin{pmatrix} 0 \\ B_0 K \rho / (1 + K^2 \rho^2) \\ B_0 / (1 + K^2 \rho^2) \end{pmatrix}$$

$$-\frac{\partial B_z}{\partial \rho} = -\frac{\partial}{\partial \rho}\left(\frac{B_0}{1 + K^2 \rho^2}\right) = \frac{2 B_0 K^2 \rho}{(1 + K^2 \rho^2)^2} = \alpha \frac{B_0 K \rho}{1 + K^2 \rho^2} \Rightarrow \alpha = \frac{2K}{1 + K^2 \rho^2} \checkmark$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{B_0 K \rho}{1 + K^2 \rho^2}\right) = \alpha \frac{B_0}{1 + K^2 \rho^2} \Rightarrow \alpha = \frac{2K}{1 + K^2 \rho^2} \checkmark$$

$$\mu_0 j_z = \alpha B_z(\rho) \Rightarrow \underline{j_z = \frac{2 K B_0}{\mu_0 (1 + K^2 \rho^2)^2}}$$

KONFIGURACIJA "FLUX ROPE" MAGNETNI KONOPAC



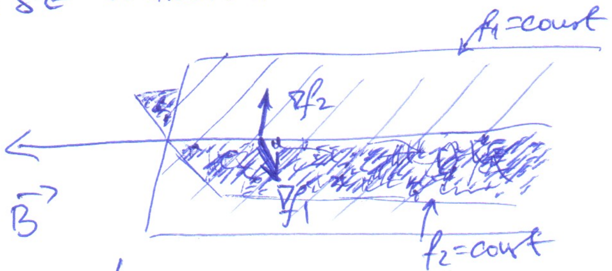
3 AKO SE NEKA LINIJA, KOJA SE KREĆE ZAJEDNO SA FLUIDOM, U JEDNOM TRENUTKU POKLAPA SA MAGNETNOM LINIJOM, ONDA ĆE SE UVEK POKLAPATI SA NJOM.

IDEALNA MHD SE PODRAZUMEVA

UVEDIMO "FLUX COORDINATES" (KOORDINATE FLUKSA)

NEKA JE $\vec{B}\cdot\underbrace{\nabla f}_{\text{UVEK JE NORMALNO NA KRIVE } f=\text{const}}=0 \Rightarrow f=\text{const}$ DUŽ MAGNETNE LINIJE

ZA DVE FAMILIJE POVRŠI f_1 i f_2 KOJE ISPUNJAVAJU $\vec{B}\cdot\nabla f_1=\vec{B}\cdot\nabla f_2=0$, MOŽE SE MAGNETNA LINIJA UVESTI KAO PRESEK TIH POVRŠI



$$\left.\begin{array}{l}\vec{B}\perp\nabla f_1\\ \quad\perp\nabla f_2\end{array}\right\}\Rightarrow \vec{B}=\nabla f_1\times\nabla f_2$$

(f_1, f_2) SU "KOORDINATE FLUKSA"

$$\nabla\cdot\vec{B}=\nabla\cdot(\nabla f_1\times\nabla f_2)=0 \checkmark$$

ŠTA JE $\frac{d}{dt}(\vec{B}\cdot\nabla f)$?

$$\frac{d}{dt}(\vec{B}\cdot\nabla f)=\frac{\partial}{\partial t}(\vec{B}\cdot\nabla f)+(\vec{v}\cdot\nabla)(\vec{B}\cdot\nabla f)=\underbrace{\frac{\partial\vec{B}}{\partial t}}_{\nabla\times(\vec{v}\times\vec{B})}\cdot\nabla f+\vec{B}\cdot\nabla\frac{\partial f}{\partial t}+(\vec{v}\cdot\nabla)(\vec{B}\cdot\nabla f)$$

$$\boxed{\nabla\times(\vec{A}\times\vec{B})=\vec{A}(\nabla\cdot\vec{B})-\vec{B}(\nabla\cdot\vec{A})+(\vec{B}\cdot\nabla)\vec{A}-(\vec{A}\cdot\nabla)\vec{B}}$$

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times(\vec{v}\times\vec{B})=|\nabla\cdot\vec{B}=0|=-\vec{B}(\nabla\cdot\vec{v})+(\vec{B}\cdot\nabla)\vec{v}-(\vec{v}\cdot\nabla)\vec{B}$$

NEKA SE POVRŠ $f=\text{const}$ POMERA (KREĆE) ZAJEDNO SA FLUIDOM: $\frac{df}{dt}=\frac{\partial f}{\partial t}+\vec{v}\cdot\nabla f=0\Rightarrow\frac{\partial f}{\partial t}=-\vec{v}\cdot\nabla f\Rightarrow$

$$\frac{d}{dt}(\vec{B}\cdot\nabla f)=-\vec{B}(\nabla\cdot\vec{v})\cdot\nabla f+\underbrace{\vec{B}\cdot\nabla\vec{v}\cdot\nabla f}_{1}-\underbrace{(\vec{v}\cdot\nabla\vec{B})\cdot\nabla f}_{2}+\underbrace{\vec{B}\cdot\nabla(-\vec{v}\cdot\nabla f)}_{3}+(\vec{v}\cdot\nabla)(\vec{B}\cdot\nabla f)$$

$$1+2+3=\vec{B}\cdot\nabla\vec{v}\cdot\nabla f-\vec{v}\cdot\nabla\vec{B}\cdot\nabla f-\vec{B}\cdot\nabla(\vec{v}\cdot\nabla f)=$$

$$=\cancel{\vec{B}\cdot\nabla\vec{v}\cdot\nabla f}-\vec{v}\cdot\nabla\vec{B}\cdot\nabla f-\cancel{\vec{B}\cdot\nabla\vec{v}\cdot\nabla f}-\vec{B}\cdot\vec{v}\cdot\nabla^2 f$$

SA DRUGE STRANE, $-(\vec{v}\cdot\nabla)(\vec{B}\cdot\nabla f)=-\vec{v}\cdot\nabla\vec{B}\cdot\nabla f-\vec{v}\cdot\vec{B}\cdot\nabla^2 f\Rightarrow$

$$1+2+3=-(\vec{v}\cdot\nabla)(\vec{B}\cdot\nabla f)\Rightarrow$$

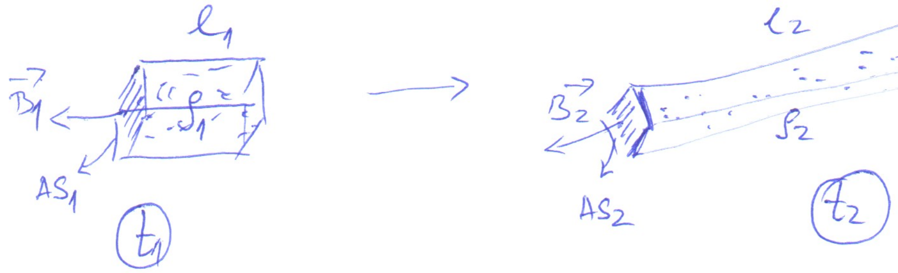
$$\frac{d}{dt}(\vec{B}\cdot\nabla f)=-\vec{B}(\nabla\cdot\vec{v})\cdot\nabla f\cancel{-(\vec{v}\cdot\nabla)(\vec{B}\cdot\nabla f)}+\cancel{(\vec{v}\cdot\nabla)(\vec{B}\cdot\nabla f)}$$

$$\boxed{\frac{d}{dt}(\vec{B}\cdot\nabla f)=-(\nabla\cdot\vec{v})(\vec{B}\cdot\nabla f)}$$

AKO JE $\vec{B}\cdot\nabla f=0$ U $t=0$ $\Rightarrow \vec{B}\cdot\nabla f=0 \ \forall t$

AKO $\underset{\text{const}}{\overset{\shortparallel}{f_1}}$ i $\underset{\text{const}}{\overset{\shortparallel}{f_2}}$ OZNAČAVAJU MAGNETNU LINIJU U $t=0\Rightarrow$ OZNAČAVAJU JE $\forall t$

AKO U $t=0$ OZNAČIMO MAGNETNU LINIJU TAKO ŠTO JOJ "PRILEPIMO" ČESTICE FLUIDA, ONDA ĆE ISTE ČESTICE BITI "ZALEPLJENE" ZA TU LINIJU $\forall t$. TOPOLOGIJA POLJA SE NE MENJA.



$$\Phi_m = \text{const} \Rightarrow$$

$$B_1 \Delta S_1 = B_2 \Delta S_2$$

$$M = \text{const} \Rightarrow$$

$$\rho_1 \Delta S_1 l_1 = \rho_2 \Delta S_2 l_2$$

$$\Leftrightarrow \frac{\Delta S_1}{\Delta S_2} = \frac{\rho_2 l_2}{\rho_1 l_1}$$

$$\Rightarrow \underline{B_2} = \frac{B_1 \Delta S_1}{\Delta S_2} = \underline{B_1\left(\frac{\rho_2}{\rho_1}\right)\left(\frac{l_2}{l_1}\right)}$$

(a) KOLAPS U NEUTRONSKU ZVEZDU

$$\rho \propto \frac{1}{l^3}$$

$$B_2 = B_1\left(\frac{\frac{1}{l_2^3}}{\frac{1}{l_1^3}}\right)\left(\frac{l_2}{l_1}\right) = \left(\frac{l_1}{l_2}\right)^2$$

(b) DIFERENCIJALNA ROTACIJA NA SUNCU – RASTEZANJE MAGNETNIH LINIJA DUŽ EKVATORA SE RAZVLAČE SKORO HORIZONTALNO ($\rho_2 \approx \rho_1$) $\Rightarrow$

$$B_2 = B_1\left(\frac{l_2}{l_1}\right)$$

(4) POLJE NULTE SILE I MAGNETOSTATIČKA KONFIGURACIJA

ZANEMARE SE ∇p SILE

$$\vec{j} \times \vec{B} = \vec{0} \quad \text{UZ} \quad \nabla \times \vec{B} = \mu_0 \vec{j} = \alpha \vec{B}$$

$$\nabla \cdot (\nabla \times \vec{B}) = 0 = \nabla \cdot (\alpha \vec{B}) = \alpha \cancel{\nabla \cdot \vec{B}} + \vec{B} \cdot \nabla \alpha \Rightarrow \underline{\vec{B} \cdot \nabla \alpha = 0}$$

(PO DEFINICIJI $\nabla \cdot (\nabla \times \vec{A}) = 0$)

MAGNETNE LINIJE MORAJU LEŽATI NA POVRŠIMA $\alpha = \text{const}$.

POVRŠ $\alpha = \text{const}$ NE MOŽE BITI PROSTA ZATVORENA POVRŠ (KAO ŠTO JE SFERA) !?

NEKA JE C ZATVORENA KRIVA SVUDA PARALELNA $\vec{B} \Rightarrow \oint_C \vec{B} \cdot d\vec{l} \neq 0$

$$\Rightarrow \oint_C \vec{B} \cdot d\vec{l} = \int_{S'} (\nabla \times \vec{B}) \cdot d\vec{S} = \int_{S'} \alpha \vec{B} \cdot d\vec{S} \neq 0$$

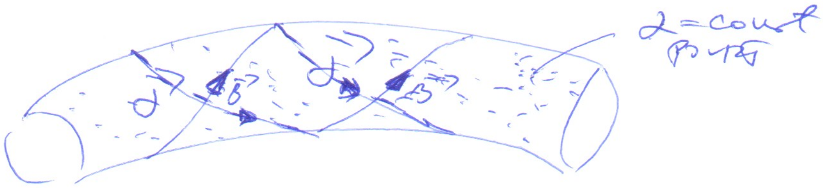
STOKSOVA TEOREMA

$S' \to$ PROSTA POVRŠ (BEZ RUPA) OGRANIČENA KONTUROM C (TADA VAŽI STOKSOVA TEOREMA)

NEKA SE S' POKLAPA SA $\alpha = \text{const}$ POVRŠI.

$$\alpha = \text{const} \text{ NA } S' \Rightarrow \oint_C \vec{B} \cdot d\vec{l} = \alpha \int_{S'} \vec{B} \cdot d\vec{S} \neq 0$$

IPAK, MAGNETNE LINIJE MORAJU LEŽATI NA $\alpha = \text{CONST}$ POVRŠIMA $\Rightarrow$ $\vec{B} \cdot d\vec{S} = 0 \Rightarrow$ ↯ KONTRADIKCIJA $\Rightarrow$ POVRŠ $\alpha = \text{const}$ NE MOŽE BITI PROSTA ZATVORENA POVRŠ !?



HOPFOVA TEOREMA (IZ TOPOLOGIJE): NAJPROSTIJA TOPOLOGIJA ZA $\alpha=const$ POVRŠ (OD RANIJE) JE TORUS.

TOROIDALNA GEOMETRIJA

TO VAŽI I ZA $\vec{j}\times\vec{B}=\vec{0}$ I ZA $\vec{j}\times\vec{B}=\nabla p$

$\vec{j}\times\vec{B}=\nabla p \,/\cdot\vec{j}, \cdot\vec{B} \Rightarrow \vec{j}\cdot\nabla p=0$

$\vec{B}\cdot\nabla p=0$ → OVDE UMESTO POVRŠI $\alpha=const$ IMAMO POVRŠI $p=const$

MAGNETNE LINIJE ZA MAGNETOSTATIKU U RAVNOTEŽI MORAJU LEŽATI NA POVRŠI TORUSA

5) NEKA JE MAGNETNO POLJE TZV. KORONALNIH LUKOVA OPISANO PREKO: $\vec{B} = B_0 e^{-\ell z} \begin{pmatrix} (\ell/k)\cos kx \\ (\alpha/k)\cos kx \\ -\sin kx \end{pmatrix}$, $\alpha^2 = k^2 - \ell^2$

POKAZATI DA JE REČ O POLJU NULTE SILE. U XOY RAVNI MAGNETNE LINIJE SU NAGNUTE NA X-OSU POD UGLOM $\operatorname{arctg}(\alpha/\ell)$. UBEDITI SE U TO TVRĐENJE.

$$\nabla \times \vec{B} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ B_x(z,x) & B_y(z,x) & B_z(z,x) \end{vmatrix} = \begin{pmatrix} -\partial B_y/\partial z \\ \partial B_x/\partial z - \partial B_z/\partial x \\ \partial B_y/\partial x \end{pmatrix} =$$

$$= \begin{pmatrix} B_0 \ell\, e^{-\ell z}(\alpha/k)\cos kx \\ -B_0 \ell e^{-\ell z}(\ell/k)\cos kx + B_0 e^{-\ell z}\, k \cos kx \\ -B_0 e^{-\ell z}(\alpha/k)\, k \sin kx \end{pmatrix} = \alpha \begin{pmatrix} B_x \\ \frac{1}{\alpha} B_0 e^{-\ell z}\cos kx\left(\frac{k^2}{k} - \frac{\ell^2}{k}\right) \\ B_z \end{pmatrix} =$$

$$= \alpha \vec{B}$$

$$\frac{dx}{B_x} = \frac{dy}{B_y} \Rightarrow \frac{dx}{\ell} = \frac{dy}{\alpha} \Rightarrow \alpha\, dx = \ell\, dy \Rightarrow dy = \left(\frac{\alpha}{\ell}\right) dx \Rightarrow$$

$$y = \underbrace{\left(\frac{\alpha}{\ell}\right)} x + \text{const}$$

$$\operatorname{tg}\theta = \frac{\alpha}{\ell} \Rightarrow$$

$$\theta = \operatorname{arctg}\left(\frac{\alpha}{\ell}\right)$$

(6) NEKA JE U OBLASTI RAZMATRANJA PROTUBERANCE $T = \text{const}$, KAO I "HORIZONTALNA" KOMPONENTA MAGNETNOG POLJA (B_x, B_y). TAKOĐE, NEKA SU $p = p(x)$, $\rho = \rho(x)$ I $B_z = B_z(x)$. NAPISATI IZRAZ ZA RAVNOTEŽNU MAGNETOHIDROSTATIČKU KONFIGURACIJU AKO JE PRISUTNO GRAVITACIONO POLJE $\vec{g} = -g\,\vec{e_z}$, $g = \text{const}$. VAŽI IDEALNA MHD.

$$\vec{j} \times \vec{B} + \rho \vec{g} - \nabla p = 0 \Rightarrow \frac{1}{\mu_0}(\nabla \times \vec{B}) \times \vec{B} - \rho g \vec{e_z} - \frac{dp(x)}{dx}\vec{e_x} = \vec{0}$$

$$\nabla \times \vec{B} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ B_x & B_y & B_z \end{vmatrix} = \begin{pmatrix} 0 \\ -\partial B_z/\partial x \\ 0 \end{pmatrix} = -\frac{dB_z}{dx}\vec{e_y}$$

$$(\nabla \times \vec{B}) \times \vec{B} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0 & -\frac{dB_z}{dx} & 0 \\ B_x & B_y & B_z \end{vmatrix} = \begin{pmatrix} -B_z \frac{dB_z}{dx} \\ 0 \\ B_x \frac{dB_z}{dx} \end{pmatrix} \Rightarrow$$

$$\frac{1}{\mu_0}\begin{pmatrix} -B_z \frac{dB_z}{dx} \\ 0 \\ B_x \frac{dB_z}{dx} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \rho g \end{pmatrix} + \begin{pmatrix} dp/dx \\ 0 \\ 0 \end{pmatrix} \Rightarrow \boxed{\begin{aligned} -\frac{1}{\mu_0} B_z \frac{dB_z}{dx} &= \frac{dp(x)}{dx} \\ \frac{1}{\mu_0} B_x \frac{dB_z}{dx} &= \rho(x) g \end{aligned}} \Rightarrow$$

$$\Rightarrow \frac{d}{dx}\left(p(x) + \frac{B_z^2(x)}{2\mu_0}\right) = 0$$

$$\Rightarrow \frac{d}{dx}\left(p(x) + \frac{B^2}{2\mu_0}\right) = 0 \quad \text{KAKO SU } B_x, B_y = \text{const}$$

DODATNO:

$$\frac{1}{\mu_0} B_x \frac{dB_z}{dx} = \rho g \Rightarrow \frac{dB_z}{dx} = \frac{\mu_0}{B_x} g \rho(x) \Big/ \frac{d}{dx}, \quad p = \rho R \underset{\text{const}}{\overset{=\text{const}}{T}} \Rightarrow$$

$$\frac{dB_z}{dx} = \frac{\mu_0}{B_x}\frac{g}{RT} p(x) \Rightarrow \frac{d^2 B_z}{dx^2} = \frac{\mu_0}{B_x}\frac{g}{RT}\frac{dp(x)}{dx}$$

$$\Rightarrow \frac{d^2 B_z}{dx^2} = \frac{\cancel{\mu_0}}{B_x}\frac{g}{RT}(-)\frac{1}{\cancel{\mu_0}} B_z \frac{dB_z}{dx}, \quad \text{AKO JE "SKALA VISINE" } \Lambda = \frac{RT}{g} \Rightarrow$$

$$\boxed{\frac{d^2 B_z}{dx^2} + \overset{=\text{const}}{\frac{1}{\Lambda B_x}} B_z \frac{dB_z}{dx} = 0}$$

B_z MORA DA ZADOVOLJAVA OVAJ IZRAZ

$$\text{KAKO JE } \frac{dB_z}{dx} = \frac{\mu_0 g \rho(x)}{B_x} \Rightarrow \frac{dp(x)}{dx} = -\frac{1}{\mu_0} B_z \frac{\mu_0 g}{B_x}\frac{p(x)}{RT} \Rightarrow$$

$$\boxed{\frac{dp(x)}{dx} + \frac{B_z(x)}{\underset{=\text{const}}{B_x \Lambda}} p(x) = 0}$$

p MORA DA ZADOVOLJAVA OVAJ IZRAZ

NEKA JE $B_z(x) = \underset{\text{const}}{B_0} \, th\left(\frac{x}{\underset{\text{= const}}{w}}\right)$ 13

DA LI OVO POLJE ISPUNJAVA PRETHODNI USLOV?

$$\left\| \Gamma \; th\, x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \right\|$$

$$\frac{dB_z}{dx} = \frac{w}{w}\frac{d}{dx}\left(B_0\, th\left(\frac{x}{w}\right)\right) = \frac{1}{w}\frac{d}{d(x/w)}\left(B_0\, th\left(\frac{x}{w}\right)\right) = \frac{1}{w}\frac{d}{d\xi}\left(B_0\, th\,\xi\right) =$$

$$= \frac{B_0}{w}\frac{d}{d\xi}\left(\frac{e^{x/w} - e^{-x/w}}{e^{x/w} + e^{-x/w}}\right) = \frac{B_0}{w}\frac{(e^{x/w}+e^{-x/w})^2 - (e^{x/w}-e^{-x/w})^2}{(e^{x/w}+e^{-x/w})^2} = \frac{B_0}{w}\frac{(e^{2\xi}+e^{-2\xi}+2e^{\xi}e^{-\xi}) - (e^{2\xi}+e^{-2\xi}-2)}{(e^{\xi}+e^{-\xi})^2} =$$

$$= \frac{4}{(e^{\xi}+e^{-\xi})^2}\frac{B_0}{w} = \frac{B_0}{w}\frac{1}{ch^2\xi} = \frac{B_0}{w}\frac{1}{ch^2(x/w)} \qquad \left\| \Gamma \; ch\,\xi = \frac{e^{\xi}+e^{-\xi}}{2} \right\|$$

$$\frac{d^2B_z}{dx^2} = \frac{d}{dx}\left(\frac{dB_z}{dx}\right) = \frac{1}{w}\frac{d}{d\xi}\left(\frac{B_0}{w}\frac{1}{ch^2\xi}\right) = \frac{B_0}{w^2}\frac{d}{d\xi}\left(\frac{4}{(e^{\xi}+e^{-\xi})^2}\right) =$$

$$= \frac{B_0}{w^2}\frac{(-)4\cdot 2\,(\cancel{e^{\xi}+e^{-\xi}})}{(e^{\xi}+e^{-\xi})^{\cancel{4}\,3}}(e^{\xi}-e^{-\xi}) \Rightarrow$$

$$\frac{d^2B_z}{dx^2} + \frac{1}{\Lambda B_x}B_z\frac{dB_z}{dx} = 0 \Rightarrow$$

$$-\frac{\overset{2}{\cancel{8}}B_0}{w^{\cancel{2}}}\frac{\cancel{(e^{\xi}-e^{-\xi})}}{\cancel{(e^{\xi}+e^{-\xi})^3}} + \frac{1}{\Lambda B_x}B_0\frac{\cancel{e^{\xi}-e^{-\xi}}}{\cancel{e^{\xi}+e^{-\xi}}}\frac{\cancel{B_0}}{\cancel{w}}\frac{\cancel{4}}{\cancel{(e^{\xi}+e^{-\xi})^2}} = 0$$

$$-\frac{2}{w} + \frac{B_0}{\Lambda B_x} = 0 \Rightarrow \frac{2}{w} = \frac{B_0}{\Lambda B_x} \Rightarrow w = \frac{2\Lambda B_x}{B_0}$$

DA, ZA $w = \frac{2\Lambda B_x}{B_0}$.

SKICIRATI OVO POLJE!

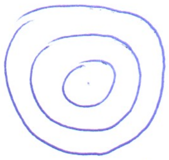
ODREDITI SADA $\rho(x)$

(7) NEKA JE $\vec{B} = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} = -y\vec{e_x} + x\vec{e_y}$, ODREDITI MAGNETNE LINIJE.

$$\frac{dx}{B_x} = \frac{dy}{B_y} \Rightarrow \frac{dx}{-y} = \frac{dy}{x} \Rightarrow x\,dx = -y\,dy \Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = const \Rightarrow$$

$$\underline{x^2 + y^2 = C}$$

ZA NEKE C SE CRTAJU



RAZLIČITA VEKTORSKA POLJA MOGU IMATI ISTE VEKTORSKE LINIJE. RECIMO,

$$\vec{B} = \begin{pmatrix} \frac{-y}{x^2+y^2} \\ \frac{x}{x^2+y^2} \\ 0 \end{pmatrix} \Rightarrow \frac{dx}{\frac{-y}{x^2+y^2}} = \frac{dy}{\frac{x}{x^2+y^2}} \Rightarrow$$

$$x\,dx = -y\,dy \Rightarrow x^2 + y^2 = C$$

DA LI OVA POLJA UOPŠTE ZADOVOLJAVAJU USLOV $\nabla \cdot \vec{B} = 0$?

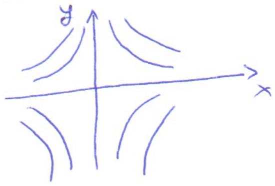
$$\frac{\partial}{\partial x}(-y) + \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial z}(0) = 0$$

$$\frac{\partial}{\partial x}\left(-\frac{y}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right) + \frac{\partial}{\partial z}(0) = -\frac{(0 - y2x)}{(x^2+y^2)^2} + \frac{(0 - x2y)}{(x^2+y^2)^2} = 0$$

(8) $\vec{B} = \begin{pmatrix} x \\ -y \\ 0 \end{pmatrix}$, $\nabla \cdot \vec{B} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(-y) + \frac{\partial}{\partial z}(0) = 0$

$$\frac{dx}{x} = \frac{dy}{-y} \Rightarrow \ln x = -\ln y + const \Rightarrow \underline{xy = C}$$

ZA NEKE C



(9) $\vec{B} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$, $\nabla \cdot \vec{B} \neq 0$!

$$\frac{dx}{x} = \frac{dy}{y} \Rightarrow \ln y = \ln x + const$$

$$y = C \cdot x$$

(10) $\vec{B} = \begin{pmatrix} B_0 \\ -2B_0 x \\ 0 \end{pmatrix}$, $\nabla \cdot \vec{B} = 0$

$$\frac{dx}{\cancel{B_0}} = \frac{dy}{-2\cancel{B_0}x} \Rightarrow -2x\,dx = dy \Rightarrow \underline{y = -x^2 + C}$$

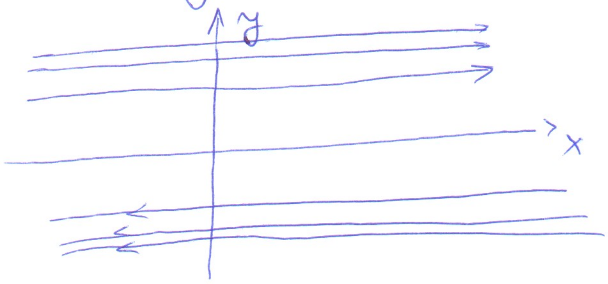
(11) $\vec{B} = \begin{pmatrix} 2B_0 y \\ -B_0 \\ 0 \end{pmatrix}$, $\nabla \cdot \vec{B} = 0$

$$\frac{dx}{2\cancel{B_0}y} = \frac{dy}{-\cancel{B_0}} \Rightarrow -dx = 2y\,dy \Rightarrow x = -y^2 + C$$

(12) $\vec{B} = \begin{pmatrix} y \\ 0 \\ 0 \end{pmatrix} = y\vec{e_x}$, $\nabla \cdot \vec{B} = 0$

$$\frac{dx}{y} = \frac{dy}{0} \Rightarrow y = const$$

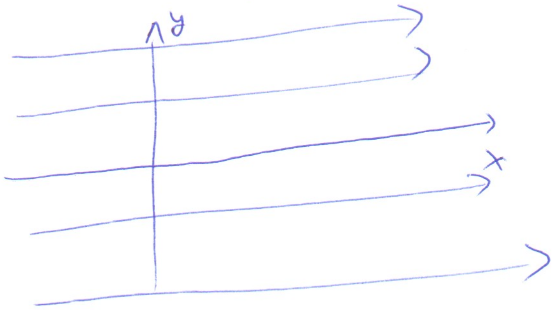
$$\|\vec{B}\| = y$$



(13) $\vec{B} = B_0 \vec{e_x}$, $\nabla \cdot \vec{B} = 0$

$$\frac{dx}{B_0} = \frac{dy}{0} \Rightarrow y = const$$

$$\|\vec{B}\| = B_0$$



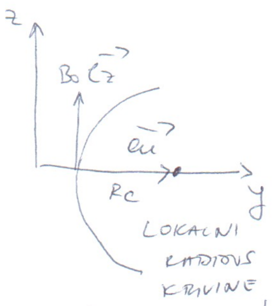
(14) $\vec{B} = B_0 e^y \vec{e_x}$, $\nabla \cdot \vec{B} = 0$

$$\Rightarrow y = const$$

$$\|\vec{B}\| = B_0 e^y$$

SKICIRATI ZA VEŽBU

(2.15) $\vec{B} = B_0\vec{e_z} + \underbrace{B_0\left(\frac{z}{R_c}\right)\vec{e_y}}_{\vec{B_1}}, \quad \left|\frac{z}{R_c}\right| \ll 1$



$\vec{B} = \vec{B_0} + \vec{B_1}, \quad \|\vec{B_1}\| \ll \|\vec{B_0}\|$

$m\dot{\vec{v}} = \mathcal{L}\vec{v}\times\vec{B} \;/\; \langle\,\rangle_{\tau_c} \Rightarrow, \quad \tau_c \equiv c$ CIKLOTRONSKA ROTACIJA

$m\dot{\vec{v}}_{gc} = \langle\mathcal{L}\vec{v}\times\vec{B}\rangle_{\tau_c}$

$m\dot{\vec{v}}_{gc} = \langle\mathcal{L}\vec{v}\times\vec{B_0}\rangle_{\tau_c} + \langle\mathcal{L}\vec{v}\times\vec{B_1}\rangle_{\tau_c} \Rightarrow$

$\langle\dots\rangle_{\tau_c} = \frac{1}{\tau_c}\int_0^{\tau_c}(\dots)\,dt, \quad \langle\vec{v}\rangle_{\tau_c} = \vec{v_{gc}}$

SAMO PRVI RED SE ZADRŽAVA

$\frac{dz}{B_0} = \frac{dy}{B_0\left(\frac{z}{R_c}\right)} \Rightarrow$

$z\,dz = R_c\,dy \Rightarrow$

$y = \frac{z^2}{2R_c} + C$

$m\frac{d\vec{v_{gc}}}{dt} = \mathcal{L}\vec{v_{gc}}\times\vec{B_0} + \langle\mathcal{L}\vec{v_0}\times\vec{B_1}\rangle_{\tau_c}$

$\vec{v_0} = \vec{v_\parallel} + \vec{v_c} \Rightarrow m\frac{d\vec{v_{gc}}}{dt} = \mathcal{L}\vec{v_{gc}}\times\vec{B_0} + \langle\mathcal{L}\vec{v_\parallel}\times\vec{B_1}\rangle_{\tau_c} + \langle\mathcal{L}\vec{v_c}\times\vec{B_1}\rangle_{\tau_c}$

NEKA SU POČETNI USLOVI KAO KOD PRIMERA U POGLAVLJU 2.3.1 $\Rightarrow$

$\vec{v_c} = v_c\left(\cos(\omega_c t)\vec{e_x} - \sin(\omega_c t)\vec{e_y}\right)$

$z^{"0"} = v_\parallel^0 t$ (HOMOGENO)

$r_c(0) = r_c, \quad \vec{v_c}(0) = v_c\vec{e_x}$

$$m\frac{d\vec{v_{gc}}}{dt} = \mathcal{L}\vec{v_{gc}}\times\vec{B_0} + \underbrace{\left\langle\mathcal{L}v_\parallel^0\vec{e_z}\times B_0\frac{z^{"0"}}{R_c}\vec{e_y}\right\rangle_{\tau_c}}_{I} + \underbrace{\left\langle\mathcal{L}\vec{v_c}\times B_0\frac{z^{"0"}}{R_c}\vec{e_y}\right\rangle_{\tau_c}}_{II}$$

$$I = \left\langle\frac{\mathcal{L}v_\parallel^{0\,2}B_0}{R_c}t(-)\vec{e_x}\right\rangle_{\tau_c} = -\frac{\mathcal{L}v_\parallel^{0\,2}B_0}{R_c}\langle t\rangle_{\tau_c}\vec{e_x}$$

$$II = \left\langle\mathcal{L}B_0\frac{v_\parallel^0}{R_c}v_c t\cos(\omega_c t)\vec{e_z}\right\rangle_{\tau_c} = \frac{\mathcal{L}B_0 v_\parallel^0 v_c}{R_c}\underbrace{\langle t\cos(\omega_c t)\rangle_{\tau_c}}_{(*)}\vec{e_z}$$

$$(*)\ \langle t\cos(\omega_c t)\rangle_{\tau_c} = \frac{1}{\tau_c}\int_0^{\tau_c} t\cos(\omega_c t)\,dt = \left|\begin{matrix}\text{PARCIJALNA}\\ \text{INTEGRACIJA}\\ \omega_c = 2\pi/\tau_c\end{matrix}\right| = 0$$

$\Rightarrow$

$$\boxed{\frac{d\vec{v_{gc}}}{dt} = \frac{\mathcal{L}}{m}\underbrace{\vec{v_{gc}}\times\vec{B_0}}_{\vec{V_D}\times\vec{B_0}} - \frac{\omega_c v_\parallel^{0\,2}}{R_c}\langle t\rangle_{\tau_c}\vec{e_x}} \;/\times\vec{B_0}$$

OD RANIJE ZNAMO DA JE $\frac{d\vec{v_{gc}}}{dt} = \frac{dv_\parallel}{dt}\vec{b} + v_\parallel^2\frac{\vec{e_u}}{R_c} + \cancel{\frac{d\vec{V_D}}{dt}}^{\,0}$ STACIONARNO SE PODRAZUMEVA

$$\frac{d\vec{v_{gc}}}{dt}\times\vec{B_0} = \frac{\mathcal{L}}{m}\underbrace{(\vec{V_D}\times\vec{B_0})\times\vec{B_0}}_{\vec{B_0}(\cancel{\vec{V_D}\cdot\vec{B_0}}) - \vec{V_D}B_0^2} - \frac{\omega_c v_\parallel^{0\,2}}{R_c}\langle t\rangle_{\tau_c}\vec{e_x}\times\vec{B_0} \Rightarrow$$

$$v_\parallel^2\frac{\vec{e_u}}{R_c}\times\vec{B_0} = -\frac{\mathcal{L}}{m}B_0^2\vec{V_D} + \frac{\omega_c v_\parallel^{0\,2}}{R_c}\langle t\rangle_{\tau_c}B_0\vec{e_y} \Rightarrow$$

$$\vec{V_D} = \frac{\cancel{\mathcal{L}B_0}}{\cancel{m}}\frac{v_\parallel^{0\,2}}{R_c}\langle t\rangle_{\tau_c}\cancel{B_0}\frac{\cancel{m}}{\cancel{\mathcal{L}B_0^2}}\vec{e_y} - \frac{m}{\mathcal{L}B_0^2}v_\parallel^2\frac{\vec{e_u}}{R_c}\times\cancel{B_0}\vec{e_z}$$

$$\vec{V_D} = \frac{v_\parallel^{0\,2}}{R_c}\langle t\rangle_{\tau_c}\vec{e_y} - \frac{m}{\mathcal{L}B_0}\frac{v_\parallel^2}{R_c}\vec{e_u}\times\vec{e_z}$$

$$(\vec{b}\cdot\nabla)\vec{b} = \frac{\vec{e_y}}{R_c}, \quad \vec{e_n} = \vec{e_y}$$

!! $\frac{z}{|R_c|} \ll 1 \Rightarrow \alpha^2 \lll 1$!!

$$\boxed{\vec{V_D} = \frac{v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c}\vec{e_y} - \underline{\frac{1}{\omega_c}\frac{v_{\parallel}^2}{R_c}\vec{e_x}}}$$

PRAVI DRIFT

$$\langle t\rangle_{\tau_c} = \frac{1}{\tau_c}\int_0^{\tau_c} t\,dt = \frac{\tau_c}{2}$$

$v_{\parallel} = v_{\parallel}^o$

DODATNO: $\frac{d\vec{v_{gc}}}{dt} = \frac{q}{m}\underbrace{\vec{V_D}}_{\vec{v_{gc}}}\times\vec{B_o} - \frac{\omega_c v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c}\vec{e_x}$

$$\vec{v_{gc}}\times\vec{B_o} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ v_{gcx} & v_{gcy} & v_{gcz} \\ 0 & 0 & B_o \end{vmatrix} = \begin{pmatrix} v_{gcy}B_o \\ -v_{gcx}B_o \\ 0 \end{pmatrix} \Rightarrow$$

$$\frac{dv_{gcx}}{dt} = \omega_c v_{gcy} - \frac{\omega_c v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c}$$

$$\frac{dv_{gcy}}{dt} = -\omega_c v_{gcx}$$

$$\frac{dv_{gcz}}{dt} = 0$$

$$\frac{d}{dt}\left(\frac{dv_{gcx}}{dt}\right) = \omega_c\frac{d}{dt}v_{gcy} - \frac{\omega_c v_{\parallel}^{o2}}{R_c}\frac{d}{dt}\left(\frac{1}{\tau_c}\int_0^{\tau_c} t\,dt\right)$$

$$\ddot{v}_{gcx} = \omega_c\dot{v}_{gcy} - \frac{\omega_c v_{\parallel}^{o2}}{R_c} \Rightarrow$$

$$\ddot{v}_{gcx} + \omega_c^2 v_{gcx} = -\frac{\omega_c v_{\parallel}^{o2}}{R_c}$$

$$v_{gcx} = C_1\sin(\omega_c t) + C_2\cos(\omega_c t) - \frac{v_{\parallel}^{o2}}{\omega_c R_c}$$

VIDIMO DA I VODEĆI CENTAR IMA KVAZI KRUŽNO KRETANJE

ZA $t=0$, $v_{gcx}=0$, $v_{gcy}=0 \Rightarrow 0 = C_2 - \frac{v_{\parallel}^{o2}}{\omega_c R_c} \Rightarrow C_2 = \frac{v_{\parallel}^{o2}}{\omega_c R_c}$

$$\dot{v}_{gcx} = C_1\omega_c\cos(\omega_c t) - C_2\omega_c\sin(\omega_c t)\Big|_0 \Rightarrow \quad \Rightarrow$$

$$C_1\cancel{\omega_c} = -\frac{\cancel{\omega_c} v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c} \Rightarrow C_1 = -\frac{v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c}$$

$$v_{gcx} = -\frac{v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c}(\sin(\omega_c t)) + \frac{v_{\parallel}^{o2}}{\omega_c R_c}(\cos(\omega_c t)) - \underline{\underline{\frac{v_{\parallel}^{o2}}{\omega_c R_c}}}$$

$$\ddot{v}_{gcy} = -\omega_c\dot{v}_{gcx} = -\omega_c\left(\omega_c v_{gcy} - \frac{\omega_c v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c}\right) \Rightarrow$$

$$\ddot{v}_{gcy} + \omega_c^2 v_{gcy} = \frac{\omega_c^2 v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c} \Rightarrow v_{gcy} = C_1\sin(\omega_c t) + C_2\cos(\omega_c t) + \frac{v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c}$$

$$0 = C_2 + \frac{v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c} \Rightarrow C_2 = -\frac{v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c}$$

$$\dot{v}_{gcy} = C_1\omega_c\cos(\omega_c t) - C_2\omega_c\sin(\omega_c t)\Big|_0 \Rightarrow 0 = C_1 \Rightarrow$$

$$v_{gcy} = \frac{v_{\parallel}^{o2}}{R_c}\langle t\rangle_{\tau_c}(\underline{\underline{1}} - \cos(\omega_c t))$$

KADA SE DODATNO USREDNJI DOBIJE SE ISTO

(#) DODATNI ZADATAK Bittencourt 3.7

$\vec{B} = B_0 \alpha z \vec{e_x} + B_0(1+\alpha x)\vec{e_z}$, $B_0, \alpha =$ const

$|\alpha x| \ll 1, \ |\alpha z| \ll 1$

$$\vec{B} = \underbrace{B_0\vec{e_z}}_{\vec{B_0}} + \underbrace{B_0\alpha z \vec{e_x} + B_0 \alpha x \vec{e_z}}_{\vec{B_1}} \Rightarrow$$

$$m\frac{d\vec{v_{gc}}}{dt} = q\vec{V_D}\times\vec{B_0} + \langle q(\underbrace{\vec{v_\parallel}+\vec{v_c}}_{\vec{v_0}})\times\vec{B_1}\rangle_{\tau_c} \Rightarrow$$

$$m\frac{d\vec{v_{gc}}}{dt} = q\vec{V_D}\times\vec{B_0} + \langle q\vec{v_c}\times B_0\alpha x^{"0"}\vec{e_z}\rangle_{\tau_c} + \langle q\vec{v_\parallel}^{"0"} \times B_0 \alpha z^{"0"}\vec{e_x}\rangle_{\tau_c} + \langle q\vec{v_c}\times B_0\alpha z^{"0"}\vec{e_x}\rangle_{\tau_c}$$

KAO I RANIJE, NEKA JE

$x^{"0"} = r_c \sin(\omega_c t)$ $\Rightarrow$

$z^{"0"} = v_\parallel^0 t$

$$\vec{v_c} = v_c(\cos(\omega_c t)\vec{e_x} - \sin(\omega_c t)\vec{e_y}) \Rightarrow$$

$$m\frac{d\vec{v_{gc}}}{dt} = q\vec{V_D}\times\vec{B_0} + q v_c r_c B_0 \alpha \langle \cancelto{0}{\cos(\omega_c t)\sin(\omega_c t)}\rangle_{\tau_c}(-)\vec{e_y} - \underline{q v_c r_c B_0\alpha\langle\sin^2(\omega_c t)\rangle\vec{e_x}} + \underline{q v_\parallel^{0\,2} B_0\alpha\langle t\rangle_{\tau_c}\vec{e_y}} + q v_c v_\parallel^0 B_0\alpha\langle t \cancel{\sin(\omega_c t)}\rangle_{\tau_c}\vec{e_z}$$

$$m\frac{d\vec{v_{gc}}}{dt} = q\vec{V_D}\times\vec{B_0} - \underbrace{\frac{q r_c v_c B_0\alpha}{2}}_{\text{grad drift}}\vec{e_x} + q v_\parallel^{0\,2}B_0\alpha\langle t\rangle_{\tau_c}\vec{e_y} \quad /\times\vec{B_0}$$

↓ centrifugalni se tu krije

↓ DA BI SE KRETALA PO ZAKRIVLJENIM MAGNETNIM LINIJAMA

$\Rightarrow \vec{V_D} =$ - - - - - - - - - -

(##) PRIMER IZ UDŽBENIKA ZA PRAVE LINIJE (TU JE $1/R_c = 0$)

$$\vec{B} = \left(B_0 + \left(\frac{\partial B}{\partial x}\right)_0 x\right)\vec{e_z} = \underbrace{B_0\vec{e_z}}_{\vec{B_0}} + \underbrace{\left(\frac{\partial B}{\partial x}\right)_0 x\,\vec{e_z}}_{\vec{B_1}}$$

$$m\frac{d\vec{v_{gc}}}{dt} = q\vec{V_D}\times\vec{B_0} + \langle q\vec{v_0}\times\left(\frac{\partial B}{\partial x}\right)_0 x^{"0"}\vec{e_z}\rangle_{\tau_c}$$

$$m\frac{d\vec{v_{gc}}}{dt} = q\vec{V_D}\times\vec{B_0} + \langle q\underbrace{\vec{v_c}}_{\begin{pmatrix} v_c\cos(\omega_c t) \\ -v_c\sin(\omega_c t)\end{pmatrix}}\times\left(\frac{\partial B}{\partial x}\right)_0 r_c\sin(\omega_c t)\,\vec{e_z}\rangle_{\tau_c} \quad /\times\vec{B_0} \Rightarrow$$

$$\cancelto{0}{m\frac{d\vec{v_{gc}}}{dt}} = q\vec{V_D}\times\vec{B_0} - q v_c\left(\frac{\partial B}{\partial x}\right)_0 r_c\langle\cancelto{0}{\cos(\omega_c t)\sin(\omega_c t)}\rangle_{\tau_c}\vec{e_y} - q v_c \left(\frac{\partial B}{\partial x}\right)_0 r_c\langle\underbrace{\sin^2(\omega_c t)}_{1/2}\rangle_{\tau_c}\vec{e_x} \quad /\times\vec{B_0} \Rightarrow$$

$\frac{d\vec{v_{gc}}}{dt}$

$$\boxed{\vec{V_D} = \frac{q}{2m} r_c^2 \left(\frac{\partial B}{\partial x}\right)_0 \vec{e_y}}$$

$r_c = \frac{v_c}{\omega_c}$, $\omega_c = \frac{qB_0}{m}$

2.7 Freidberg 8.5.2
+ ZADATAK 8.3

NEKA JE NAJPRE $\vec{E} = E_x(x)\vec{e_x}$. TADA VAŽI $\nabla \times \vec{E} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x(x) & 0 & 0 \end{vmatrix} = \vec{0}$

$\vec{B} = \vec{B_0} = B_0 \vec{e_z}$

$m\dot{\vec{v}} = q\vec{v}\times\vec{B} + q\vec{E} \; / \langle\rangle_{\tau_c}$ NEKA JE $\vec{E} = (E_0 + (\frac{\partial E}{\partial x})_0 x)\vec{e_x} = \vec{E_0} + \vec{E_1}$

$\|\vec{E_1}\| \ll \|\vec{E_0}\|$

$m\dot{\vec{v}}_{gc} = q\vec{V_D}\times\vec{B_0} + \langle q\vec{E}\rangle_{\tau_c}$

$m\dot{\vec{v}}_{gc} = q\vec{V_D}\times\vec{B_0} + qE_0\vec{e_x} + \langle q(\frac{\partial E}{\partial x})_0 \, x\rangle_{\tau_c}\vec{e_x} \; / \times\vec{B_0}$

$x'' 0'' = \tau_c \sin(\omega_c t)$

SA JEDNE STRANE JE:

$m\dot{\vec{v}}_{gc}\times\vec{B_0} = -qB_0^2\vec{V_D} - qE_0B_0\vec{e_y} + \cancel{\langle q(\frac{\partial E}{\partial x})_0 \tau_c \sin(\omega_c t)\rangle_{\tau_c}\vec{e_x}\times B_0\vec{e_z}} \Rightarrow$

$\underbrace{m\dot{\vec{v}}_{gc}}_{\substack{\text{PRAVE SU MAG. LINIJE} \\ \text{+STAC} \\ = 0}}$

$$\boxed{\vec{V_D} = -\frac{E_0}{B_0}\vec{e_y}}$$

SA DRUGE STRANE JE:

$$\dot{\vec{v}}_{gc} = \omega_c\begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ v_{gcx} & v_{gcy} & v_{gcz} \\ 0 & 0 & 1 \end{vmatrix} + \begin{pmatrix} qE_0/m \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} (q(\frac{\partial E}{\partial x})_0\tau_c/m)\langle\sin(\omega_c t)\rangle_{\tau_c} \\ 0 \\ 0 \end{pmatrix} \Rightarrow$$

$$\left.\begin{aligned} \dot{v}_{gcx} &= \omega_c v_{gcy} + \frac{qE_0}{m} + \frac{q(\frac{\partial E}{\partial x})_0\tau_c}{m}\langle\sin(\omega_c t)\rangle_{\tau_c} \\ \dot{v}_{gcy} &= -\omega_c v_{gcx} \\ \dot{v}_{gcz} &= 0 \end{aligned}\right| \Rightarrow$$

$$\ddot{v}_{gcx} = \omega_c\underbrace{\dot{v}_{gcy}}_{-\omega_c v_{gcx}} + \frac{q(\frac{\partial E}{\partial x})_0\tau_c}{m}\frac{1}{\tau_c}\frac{d}{dt}\int_0^{\tau_c}\underbrace{\sin(\omega_c t)}_{x''0''}dt \; / \frac{B_0}{B_0} \Rightarrow$$

$$\ddot{v}_{gcx} + \omega_c^2 v_{gcx} = \omega_c\left(\frac{\partial E}{\partial x}\right)_0\frac{1}{B_0}\underbrace{\frac{1}{\tau_c}\int_0^{\tau_c}\overbrace{\frac{dx}{dt}}^{v_x}dt}_{\langle v_x\rangle_{\tau_c} = v_{gcx}} \Rightarrow$$

$$\ddot{v}_{gcx} + \omega_c^2 v_{gcx} - \omega_c\left(\frac{\partial E}{\partial x}\right)_0\frac{1}{B_0}v_{gcx} = 0 \Rightarrow$$

$$\ddot{v}_{gcx} + \omega_c^2 v_{gcx}\left(1 - \left(\frac{\partial E}{\partial x}\right)_0\frac{1}{B_0\omega_c}\right) = 0$$

NEMA NOVOG DRIFTA
JEDINO KOREKCIJA
ω_c

SVE OVO MOŽE, ZA VEŽBU, DA SE UOPŠTI NA $\vec{E} = \vec{E_0} + (\vec{r_c}\cdot\nabla)\vec{E}$

ŠTA AKO SE ZADRŽI KVADRATNI ČLAN ?

NEKA JE $\vec{E} = (E_0 + (\frac{\partial E}{\partial x})_0 x + \frac{1}{2}(\frac{\partial^2 E}{\partial x^2})_0 x^2)\vec{e_x}$, $x \equiv x''0'' \Rightarrow$

$m\dot{\vec{v}}_{gc} = q\vec{V_D}\times\vec{B_0} + qE_0\vec{e_x} + \langle q(\frac{\partial E}{\partial x})_0 \cancel{(\tau_c\sin(\omega_c t))}\rangle_{\tau_c}\vec{e_x} + \frac{q}{2}\langle(\frac{\partial^2 E}{\partial x^2})_0 \underbrace{x^2}_{x''0''}\rangle_{\tau_c}\vec{e_x} \; / \times\vec{B_0} \Rightarrow$

$\langle\sin^2(\omega_c t)\rangle_{\tau_c} = 1/2$

$\frac{qB_0^{\cancel{2}}}{\cancel{m}}\vec{V_D} = -\frac{qE_0\cancel{B_0}}{\cancel{m}}\vec{e_y} - \frac{q}{2\cancel{m}}\left(\frac{\partial^2 E}{\partial x^2}\right)_0 \tau_c^2 \cancel{B_0}\frac{1}{2}\vec{e_y} \Rightarrow$

$$\underline{\vec{V_D} = -\frac{E_0}{B_0}\vec{e_y} - \frac{1}{4}\left(\frac{\partial^2 E}{\partial x^2}\right)_0\frac{\tau_c^2}{B_0}\vec{e_y}}$$

NEKA JE DOJ $\vec{E} = \vec{E_0} + \underbrace{(\vec{r_c}\cdot\nabla)\vec{E}}_{\vec{E_1}} + \underbrace{\frac{1}{2}\vec{r_c}\vec{r_c}\cdot\Delta\vec{E}}_{\vec{E_2}}$ $\quad \|\vec{E_1}\| \ll \|\vec{E_0}\|$ $\quad \|\vec{E_2}\| \ll \|\vec{E_1}\|$ $\Rightarrow$

(2)

$$m\dot{\vec{v}}_{gc} = q\vec{V_D}\times\vec{B_0} + q\vec{E_0} + \langle q(\vec{r_c}\cdot\nabla)\vec{E}\rangle_{\tau_c} + \frac{1}{2}q\langle\vec{r_c}\vec{r_c}\cdot\Delta\vec{E}\rangle_{\tau_c} \Big/ \times\vec{B_0}$$

$r_c\begin{pmatrix}\sin(\omega_c t)\\ \cos(\omega_c t)\\ 0\end{pmatrix}$

PRAVE MAG. LINIJE $\Rightarrow$

$\langle\sin(\omega_c t)\rangle_{\tau_c} = \langle\cos(\omega_c t)\rangle_{\tau_c} = 0 \Rightarrow$

$$\frac{qB_0^2}{m}\vec{V_D} = \frac{q}{m}\vec{E_0}\times\vec{B_0} + \frac{q}{2m}B_0\langle\vec{r_c}\vec{r_c}\cdot\Delta\vec{E}\rangle_{\tau_c}\times\vec{e_z}$$

$$\vec{r_c}\vec{r_c} = r_c^2\begin{pmatrix}\sin^2(\omega_c t) & \sin(\omega_c t)\cos(\omega_c t) & 0\\ \sin(\omega_c t)\cos(\omega_c t) & \cos^2(\omega_c t) & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$\langle\vec{r_c}\vec{r_c}\rangle_{\tau_c} = \frac{r_c^2}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix} \Rightarrow$$

$$\vec{V_D} = \frac{\vec{E_0}\times\vec{B_0}}{B_0^2}\left(1 + \frac{r_c^2}{4}\Delta\right)$$

$$\vec{V_D} = \left(1 + \frac{r_c^2}{4}\Delta\right)\frac{\vec{E_0}\times\vec{B_0}}{B_0^2}$$

2.6 Freidberg 8.8.2 (VIDI ZA DETALJE)

NEKA JE $\vec{B} = B(t)\,\vec{e_z}$

$\vec{E} = E_x(t)\,\vec{e_x} + E_y(t)\,\vec{e_y} = \vec{E_\perp}$

SLABO NESTACIONARNO

$B(t) \Rightarrow E(x,y,t)$ ALI $E(x,y)$ SADA NE RAZMATRAMO

PA GLEDAMO SAMO DRIFT OD „t"

AKO SE I ZADRŽI SAMO PRVI ČLAN URAČUNJ $\vec{E}$ TO OPET NE DADE DRIFT

$$\vec{V_D} = \vec{V_D}^{(I)} + \vec{V_D}^{(II)}$$

$$\vec{V_D} = \frac{\vec{F_\perp} \times \vec{B}}{\rho B^2}, \quad \vec{F_\perp} = \underbrace{-M\nabla_\perp B - \frac{2W_\parallel}{RC}\vec{e_n}}_{\langle \vec{F_\perp}\rangle} - m\left(\frac{d\vec{V_D}}{dt}\right)_\perp$$

$$\vec{V_D}^{(I)} + \vec{V_D}^{(II)} = \frac{\langle \vec{F_\perp}\rangle \times \vec{B}}{\rho B^2} - \frac{m\left(\frac{d\vec{V_D}}{dt}\right)_\perp \times \vec{B}}{\rho B^2}$$

$$\vec{V_D}^{(II)} = -\frac{m}{\rho B^2}\left(\frac{d\vec{V_D}^{(I)}}{dt}\right)_\perp \times \vec{B}$$

$$\vec{V_D} = \frac{\vec{E} \times \vec{B}}{B^2} = \frac{1}{B^2}\begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ E_x & E_y & 0 \\ 0 & 0 & B \end{vmatrix} = \frac{1}{B^2}\begin{pmatrix} B E_y \\ -E_x B \\ 0 \end{pmatrix} \Rightarrow$$

$$\vec{V_D}^{(II)} = -\frac{m}{\rho B^2}\frac{d}{dt}\begin{pmatrix} E_y/B \\ -E_x/B \\ 0 \end{pmatrix} \times \vec{B} = -\frac{m}{\rho B^2}\begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \frac{d}{dt}\left(\frac{E_y}{B}\right) & -\frac{d}{dt}\left(\frac{E_x}{B}\right) & 0 \\ 0 & 0 & B \end{vmatrix} =$$

$$= -\frac{m}{\rho B^2}\begin{pmatrix} -B\frac{d}{dt}\left(\frac{E_x}{B}\right) \\ -B\frac{d}{dt}\left(\frac{E_y}{B}\right) \\ 0 \end{pmatrix} \Rightarrow$$

$$\vec{V_D}^{(II)} = \frac{m}{\rho B}\begin{pmatrix} \frac{d}{dt}(E_x/B) \\ d/dt\,(E_y/B) \\ 0 \end{pmatrix} \Rightarrow \underline{\underline{\vec{V_D}^{(II)} = \frac{m}{\rho B}\frac{d}{dt}\left(\frac{\vec{E_\perp}}{B}\right)}}$$

Chen 2-8. (DEO ZADATKA)

POSMATRAMO IZOTROPNU RASPODELU PROTONA ENERGIJA OD 1 eV NA $r = 5R_\oplus$, $R_\oplus \approx 6400\,km$ U RAVNI MAGNETNOG EKVATORA. MAGNETNO POLJE ZEMLJE SE SMATRA IDEALNIM DIPOLOM I U RAZMATRANOJ OBLASTI JE $\vec{B} = B_0 \left(\frac{R_\oplus}{r}\right)^3 \vec{e_\theta}$, $B_0 = 3\cdot10^{-5}\,T$. ODREDITI $\vec{V_D}^{\nabla B}$.

①

$KT = 1\,eV$

$$\vec{V_D}^{\nabla B} = \frac{m\langle v_\perp^2\rangle}{2|q|} \frac{\vec{B}\times\nabla_\perp B}{B^3} = \frac{KT}{e} \frac{\vec{B}\times\nabla_\perp B}{B^3}$$

$$\left.\begin{array}{l} E = \frac{3}{2}KT \\ E_i = \frac{KT}{2} \end{array}\right\} \Rightarrow \frac{m\langle v_\perp^2\rangle}{2} = KT \qquad \langle v_\perp^2\rangle = \frac{2KT}{m}$$

$$B = B_0\left(\frac{R_\oplus}{r}\right)^3 = B_0 \frac{1}{5^3}\,[T]$$

$$\nabla_\perp B = \frac{\partial}{\partial r}\left(\frac{B_0 R_\oplus^3}{r^3}\right)\vec{e_r} = -3\frac{B}{r}\vec{e_r}$$

$$\vec{B}\times\nabla_\perp B = B\vec{e_\theta}\times(-)\frac{3B}{r}\vec{e_r} = \frac{3}{r}B^2\vec{e_\varphi} \Rightarrow$$

$$\underline{\vec{V_D}^{\nabla B} = \frac{3\cdot5^3(KT)}{5\,e\,B_0 R_\oplus}\vec{e_\varphi}}$$

$$\underline{\underline{V_D^{\nabla B}}} = \frac{3\cdot5^3\cdot 1\cdot\cancel{1{,}603\cdot10^{-19}}}{5\cdot\cancel{1{,}603\cdot10^{-19}}\cdot6400000\cdot3\cdot10^{-5}} = \underline{\underline{0{,}39\,\frac{m}{s}}}$$

ZA VEŽBU ISTOTO ZA

e^- ENERGIJE OD 30 keV

$\Rightarrow V_D^{\nabla B} = 1{,}17\cdot10^4\,m/s$

ZA KOLIKO e^- OBIĐU OKO ZEMLJE?

$$t = \frac{2\pi r}{V_D^{\nabla B}} = \Big| r = 5R_\oplus \Big| = 4{,}8\,h$$

NEKI ZADACI ZA VEŽBANJE

(1) U POČETNOM TRENUTKU ANALIZE $t=0$, ELEKTRON SE NALAZI U $z=0$ SA v_0. POČETNI NAGIBNI UGAO JE θ. ELEKTRON SE KREĆE U MAGNETNOM POLJU OBLIKA $B(z) = B_0(1+(\gamma z)^2)$, $\gamma = const$ (MERA GRADIJENTA MAGNETNOG POLJA)
$\vec{B} = B(z)\vec{e_z}$

ODREDITI z_t, GDE „t" OZNAČAVA MESTO MAGNETNOG OGLEDALA.

▽ ZAKON ODRŽANJA ENERGIJE (KINETIČKE) $\Rightarrow v_0^2 = v_t^2$

ZAKON ODRŽANJA MAGNETNOG MOMENTA

z KOMPONENTA (PARALELNA) BRZINE JE U $s=s_t$ JEDNAKA NULI $\Rightarrow v_t = v_\perp(s=s_t)$ $(v_\parallel(s=s_t)=0)$

$$\mu = \frac{w_\perp}{B} = \frac{m_e v_\perp^2}{2B} \Rightarrow \frac{m_e v_0^2 \sin^2\theta}{2\underbrace{B_0}_{B(z=0)}} = \frac{m_e v_t^2}{2B_0(1+(\gamma z_t)^2)} \Rightarrow$$

$$\Rightarrow v_0^2 \sin^2\theta(1+(\gamma z_t)^2) = \underbrace{v_t^2}_{v_0^2} \Rightarrow \sin^2\theta(1+(\gamma z_t)^2) = 1 \Rightarrow$$

$$z_t = \frac{1}{\gamma}\sqrt{\frac{1}{\sin^2\theta}-1} \Rightarrow \underline{z_t = \frac{1}{\gamma}\frac{1}{tg\theta}}$$

POLOŽAJ TAČKE REFLEKSIJE ZAVISI SAMO OD NAGIBNOG UGLA θ I GRADIJENTA MAGNETNOG POLJA △

(2) ISTO KAO (1) ALI SA $B(z) = B_0(1+(\gamma z)^4)$

▽ $z_t = \frac{1}{\gamma}\frac{1}{\sqrt{tg\theta}}$ △

(3) ODREDITI SREDNJU BRZINU ZA FUNKCIJU RASPODELE OBLIKA
$f(v) = \frac{n}{2}\sin(v)$, SA $v \in [0,\pi]$ I $f(v)=0$ INAČE.
(n – KONCENTRACIJA)

▽ $\langle v \rangle = 1$ △

(4) DATA JE LORENCOVA (KOŠIJEVA) FUNKCIJA RASPODELE OBLIKA $f(v) = \frac{C}{v^2+\frac{kT}{m}}$. ODREDITI C I SREDNJU BRZINU.

▽ $C = n\sqrt{\frac{kT}{m\pi^2}}$, $\langle v \rangle$ NIJE DEFINISANA △

(5) NEKA JE $\vec{B} = B_0\, th\left(\frac{x}{d}\right)\vec{e_y}$. PRI RAVNOTEŽNOJ KONFIGURACIJI, VAŽI MODEL MHD HSR, ODREDITI $\vec{j}$ I ρ, AKO VAŽI DA JE $p \propto \rho^\gamma$.

▽ $\vec{j} = \frac{1}{\mu_0}\nabla\times\vec{B} = \text{-----}$

$\vec{j}\times\vec{B} = \frac{dp}{dx}\vec{e_x} \Rightarrow p = \text{-----}$

$p \propto \rho^\gamma \Rightarrow \rho = \text{-----}$

△

⑥ NAĆI VEZU IZMEĐU HELICITETA I MAGNETNE ENERGIJE (2)
U POLJU NULTE SILE SA KONSTANTNIM PARAMETROM α.

R: HELICITET GLATKOG VEKTORSKOG POLJA $\vec{V}$ U 3D PROSTORU JE MERA UMOTAVANJA VEKTORSKIH LINIJA JEDNIH OKO DRUGIH:

$$\mathcal{H}^{\vec{V}} = \int_V \vec{V}\cdot(\nabla\times\vec{V})\, dV' \text{, PO ZAPREMINI CELOG PROSTORA !!}$$

$$\mathcal{H}^{\vec{B}} = \int_V \vec{A}\cdot\underbrace{(\nabla\times\vec{A})}_{\vec{B}}\, dV'$$

$\vec{A}$ – VEKTORSKI POTENCIJAL

SETIMO SE USLOVA

$$\nabla\times\vec{B} = \alpha\vec{B}, \quad \alpha = const$$

$$\nabla\times\vec{B} = \nabla\times(\nabla\times\vec{A}) = \alpha\vec{B} = \alpha\,\nabla\times\vec{A} = \nabla\times(\alpha\vec{A})$$

$$\Rightarrow \nabla\times\vec{A} = \alpha\vec{A} \Rightarrow \vec{A} = \frac{1}{\alpha}\nabla\times\vec{A}$$

$$\vec{A}\cdot\underbrace{(\nabla\times\vec{A})}_{\vec{B}} = \frac{1}{\alpha}\underbrace{(\nabla\times\vec{A})}_{\vec{B}}\cdot\vec{B} = \frac{1}{\alpha}B^2 \Rightarrow \mathcal{H}^{\vec{B}} = \int_V \frac{1}{\alpha}B^2\, dV'$$

⑦ CILINDAR RADIJUSA a POSTAVLJEN TAKO DA MU JE OSA DUŽ Z-OSE IMA RAVNOTEŽNU GUSTINU STRUJE, KOJA TEČE, OBLIKA $\vec{j} = j_0\left(1-\frac{\rho^2}{a^2}\right)\vec{e_z}$. ODREDITI $\vec{B}$, p.

R: $\mu_0\vec{j} = \nabla\times\vec{B} \Rightarrow B_\varphi = \frac{\mu_0 j_0}{2}\rho\left(1-\frac{\rho^2}{2a^2}\right)$

KAKO JE $\nabla p = \vec{j}\times\vec{B} \Rightarrow \frac{\partial p}{\partial\rho} = -j_0\left(1-\frac{\rho^2}{a^2}\right)\frac{\mu_0 j_0}{2}\rho\left(1-\frac{\rho^2}{2a^2}\right)$

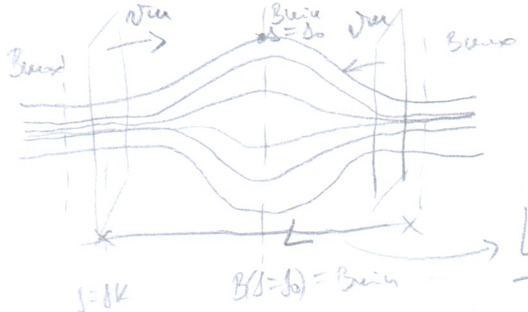
$$\Rightarrow p = p_0 - \frac{\mu_0 j_0^2}{4}\rho^2\left(1-\frac{3\rho^2}{4a^2}+\frac{\rho^4}{6a^4}\right)$$

8 PROTON KOSMIČKOG ZRAČENJA ZAROBLJEN JE U MAGNETNOJ KLOPCI. POČETNA ENERGIJA MU JE $1\,keV$ I VAŽI $v_{\perp,i} = v_{\parallel,i}$ U $t=0$. U TOM POČETNOM TRENUTKU, PROTON SE NALAZI U $s = s_0$ GDE JE $B(s=s_0) = B_{min}$. SVAKO MAGNETNO OGLEDALO SE KREĆE KA $s=s_0$ BRZINOM $v_m = 10\,\frac{km}{s}$. VAŽI DA JE $\frac{B_{max}}{B_{min}} \equiv R_m = 5$. 3

$B_{min} \leftarrow B(s=s_0)$

(a) UPOTREBOM IZRAZA ZA DEFINISANJE OTVORA KONUSA GUBITKA, KAO I ODRŽANJA MAGNETNOG MOMENTA μ, ODREDITI ENERGIJU DO KOJE ĆE PROTON BITI UBRZAN PRE NEGO ŠTO NAPUSTI DATU KONFIGURACIJU.

(b) AKO SE MAGNETNA OGLEDALA TRETIRAJU KAO DVE RAVNI KOJE SE PRIBLIŽAVAJU, POKAZATI DA SE PORAST BRZINE $v_\parallel$ U SVAKOJ REFLEKSIJI MOŽE PREDSTAVITI PREKO $2v_m$. KOLIKO REFLEKSIJA SE DESI DOK PROTON NE NAPUSTI OVU KLOPKU? KOLIKO IZNOSI VREME KOJE PROTEKNE DA SE ODIGRA PUNI BROJ REFLEKSIJA (VIDI SLIKU)?



$\rightarrow L = 10^{10}\,km$

PRETPOSTAVKA DA JE B UNIFORMNO IZMEĐU OGLEDALA I DA SE SKOKOVITO MENJA U BLIZINI OGLEDALA

8 (a) $\sin^2\alpha_{loss} = \frac{B_{min}}{B_{max}} = \frac{1}{R_m} = \frac{1}{5}$

$tg\,\alpha = \frac{v_\perp}{v_\parallel}$, $v_\perp = v\sin\alpha$, $v_\parallel = v\cos\alpha$

$\sin\alpha = \frac{v_\perp}{v}$

$\sin^2\alpha_0$

FINALNA VREDNOST

$$\sin^2\alpha_{loss} = \frac{v_{\perp,f}^2}{v_{\perp,f}^2 + v_{\parallel,f}^2} = \frac{1}{1+\left(\frac{v_{\parallel,f}}{v_{\perp,f}}\right)^2}$$

U $t=0$ PROTON IMA NEKO POČETNO α_0, ALI TO NAM NIJE BITNO, VEĆ BRZINA (ENERGIJA) KADA α POSTANE GRANIČNO α_{loss}. DAKLE, $\alpha=\alpha(t)$, A NAS ZANIMA KADA α POSTANE α_{loss}

$= \Big|\ \mu = const \Rightarrow \frac{W_\perp}{B} = const \Rightarrow$

$\frac{m v_\perp^2 (s=s_0 \text{ u } t=t_i)}{2B(s=s_0)} = \frac{m v_\perp^2 (s=s_0 \text{ u } t_f)}{2B(s=s_0)}$

$\Rightarrow v_{\perp,i}^2 = v_{\perp,f}^2$

$v_{\perp,i} = v_{\perp,f}$

$$\Rightarrow \frac{1}{5} = \frac{1}{1+\left(\frac{v_{\parallel,f}}{v_{\perp,i}}\right)^2} \Rightarrow 5 = 1 + \left(\frac{v_{\parallel,f}}{v_{\perp,i}}\right)^2 \Rightarrow$$

$v_{\parallel,f} = 2v_{\perp,i} \Rightarrow$

$W_f = \frac{1}{2} m_p \left(v_{\perp,f}^2 + v_{\parallel,f}^2\right) = \frac{5}{2} m v_{\perp,i}^2$

POŠTO JE $v_{\perp,i} = v_{\parallel,i}$, A POZNATA JE $W_i = 1\,keV \Rightarrow 1\,keV = \frac{1}{2} m_p v_{\perp,i}^2 \cdot 2 \Rightarrow v_{\perp,i}^2$

$\Rightarrow W_f = \frac{5}{2} W_i = 2{,}5\,keV$

(6) $v_{\parallel i} = \sqrt{\frac{2 W_i}{2 m_p}} \approx 310 \frac{km}{s}$ $1\,keV$

$W_f = 2{,}5\,keV \Rightarrow v_{\parallel f} = 2 v_{\perp,i} \approx 620 \frac{km}{s}$

$\Delta v_{\parallel} = v_{\parallel f} - v_{\parallel i} = 310 \frac{km}{s}$

$N_{ODBIJANJA} = \frac{\Delta v_{\parallel}}{\Delta v_{JEDNOG\ ODBIJANJA}}$, $\Delta v_{JEDNOG\ ODBIJANJA} \equiv \Delta v_1$

(4)

ŠTA JE Δv_1 ?

$\Gamma_{OOLO} = $ NEDEFINISANO)

ELASTIČNI SUDAR $\Rightarrow$

$m_1 \vec{v}_1 + m_2 \vec{v}_{m_1} = m_1 \vec{v}_2 + m_2 \vec{v}_{m_2}$

$m_1 v_1 - m_2 v_{m_1} = -m_1 v_2 - m_2 v_{m_2}$

$m_1 (v_1 + v_2) = \overset{\infty}{m_2} (\overset{0}{v_{m_1}} - v_{m_2})$

$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_{m_1}^2 = \frac{1}{2} m_1 v_2^2 + \frac{1}{2} m_2 v_{m_2}^2$

$m_1 (v_1^2 - v_2^2) = \overset{\infty}{m_2} (v_{m_2}^2 - v_{m_1}^2)$

$m_1 (v_1 - v_2)(v_1 + v_2) = \overset{\infty}{m_2} (\overset{0}{v_{m_2} - v_{m_1}})(v_{m_2} + v_{m_1})$

$\Rightarrow v_1 - v_2 = -(v_{m_2} + v_{m_1}) \Rightarrow v_2 - v_1 \equiv \underset{\Delta v_1}{\Delta v} = \underbrace{v_{m_1} + v_{m_2}}_{2 v_m} \Rightarrow \Delta v_1 = 2 v_m$

$\Delta v_1 = 20 \frac{km}{s} \Rightarrow N_{ODBIJANJA} \approx 15$

PRETPOSTAVLJAMO SVE VREME $v_m \ll v_{\parallel PROTONA} \Rightarrow$ KRETANJE OGLEDALA, PREĐENI PUT OGLEDALA JE ZANEMARLJIV PREMA PREĐENOM PUTU ČESTICE $\Rightarrow$

PUT DUŽINE $N_{ODBIJANJA} \cdot L$ SE PREĐE ZA $\langle t \rangle = \frac{N_{ODBIJANJA} \cdot L}{\langle v \rangle}$

$\langle v \rangle = \frac{v_1 + v_2}{2} = 465 \frac{km}{s} \Rightarrow \langle t \rangle \approx 10\ god$

IPAK, L SE MENJA $\rightarrow \Delta L = 2 v_m t \approx 6{,}4 \cdot 10^{12}\,m$ ($10^9\,km$) ALI TO JE MALA KOREKCIJA

(t: $32 \cdot 10^{8}\,s$)

(9) NEKA JE $\vec{B} = B(t)\vec{e_z}$ SLABO NESTACIONARNO → DAKLE, PROMENE SU SLABE NA SKALI ŽIRO-PERIODA (5)

NASTAJE $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ KOJE JE NEHOMOGENO

SA JEDNE STRANE MORA BITI $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$, A SA DRUGE $\nabla \cdot \vec{E} = 0$

U ORBITALNOM METODU $\Rightarrow \vec{E} = E_\varphi(\rho)\vec{e_\varphi}$ ZADOVOLJAVA OBA USLOVA

$$\Gamma\ \nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \vec{e_\rho} & \rho\vec{e_\varphi} & \vec{e_z} \\ \partial/\partial\rho & \partial/\partial\varphi & \partial/\partial z \\ A_\rho & \rho A_\varphi & A_z \end{vmatrix} \Rightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho E_\varphi) = -\frac{\partial B(t)}{\partial t} \Rightarrow /\int \rho\, d\rho$$

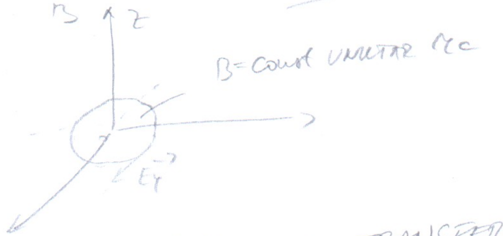
$*$ B(t) SE SPORO MENJA

$$\Rightarrow \int_0^\rho \frac{\partial}{\partial\rho'}(\rho' E_\varphi(\rho'))d\rho' = -\frac{\partial B}{\partial t}\int_0^\rho \rho' d\rho' \Rightarrow$$

$$\rho E_\varphi(\rho) = -\frac{\partial B}{\partial t}\frac{\rho^2}{2} \Rightarrow E_\varphi(\rho) = -\frac{1}{2}\frac{\partial B}{\partial t}\rho$$

$$\Rightarrow \vec{E_\varphi} = -\frac{1}{2}\frac{\partial B}{\partial t}\rho\,\vec{e_\varphi}$$

$$\Gamma\ \vec{\rho} = \rho\vec{e_\rho}, \quad \vec{e_\rho} \times \vec{e_z} = -\vec{e_\varphi} \Rightarrow \vec{E_\varphi} = \frac{1}{2}\vec{\rho} \times \frac{\partial \vec{B}}{\partial t}$$



KOLIKA JE PROMENA TRANSFERZALNE KINETIČKE ENERGIJE TOKOM JEDNOG ŽIRO-PERIODA

SPORA PROMENA POLJA ⇒ ZATVORENA PUTANJA ⇒ PRIBLIŽNO

$$\delta\left(\frac{1}{2}mv_\perp^2\right) = \mathcal{L}\oint \vec{E_\varphi}\cdot d\vec{\rho}, \quad \vec{v_\perp} = \frac{d\vec{\rho}}{dt}$$

$$\Rightarrow \delta\left(\frac{1}{2}mv_\perp^2\right) = \mathcal{L}\int_S (\nabla \times \vec{E_\varphi})\cdot d\vec{S} = -\mathcal{L}\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} = |\mathcal{L}|\frac{\partial B}{\partial t} r_c^2 \pi$$

STOKSOVA TEOREMA

POVRŠINA OGRANIČENA CIKLOTRONSKOM PUTANJOM

$$\vec{B}\cdot d\vec{S} = \begin{cases} <0, & \text{JONI} \\ >0, & \text{ELEKTRONI} \end{cases}$$

ZA JEDNO $\tau_c = \frac{2\pi}{\omega_c} \Rightarrow \delta B = \left(\frac{\partial B}{\partial t}\right)\frac{2\pi}{\omega_c}$, A KAKO JE $r_c^2 = \frac{v_\perp^2}{\omega_c^2}$, $\omega_c = \frac{|\mathcal{L}|B}{m}$

$$\Rightarrow \delta\left(\frac{1}{2}mv_\perp^2\right) = |\mathcal{L}|\frac{v_\perp^2}{\omega_c^{\cancel{2}}}\cancel{\pi}\frac{\delta B}{2\cancel{\pi}}\cancel{\omega_c} = \cancel{|\mathcal{L}|}v_\perp^2\frac{m\,\delta B}{\cancel{|\mathcal{L}|}B} = \frac{mv_\perp^2}{2}\frac{\delta B}{B} = |\mu|\delta B$$

$$\Rightarrow \delta(\mu B) = \mu\,\delta B \Rightarrow \mu = \text{const} \quad \text{ZA } \frac{\partial B}{\partial t}\frac{2\pi}{\omega_c} \ll B$$

$$\mu = \frac{mv_\perp^2\,|\mathcal{L}|}{2B\,|\mathcal{L}|} = \frac{v_\perp^2}{\omega_c}\frac{|\mathcal{L}|\pi\,\omega_c}{2\pi\,\omega_c} = r_c^2\frac{|\mathcal{L}|\pi}{2\pi}\frac{|\mathcal{L}|B}{m} =$$

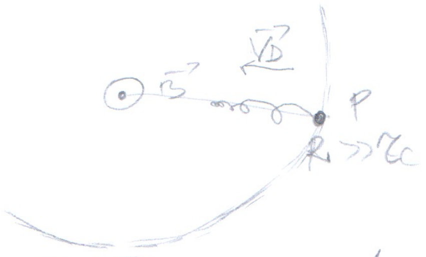
$$= \pi r_c^2 B\frac{\mathcal{L}^2}{2\pi m} \propto r_c^2 B = \text{const}$$

$r_c\uparrow \Rightarrow B\downarrow$

$B\uparrow \Rightarrow r_c\downarrow$

Φ_m KROZ CIKLOTRONSKU ORBITU $\rightarrow \Phi_m = BS = B\pi r_c^2 = \text{const}$

KADA $B\uparrow \Rightarrow r_c\downarrow$ TAKO DA ČESTICA UVEK KRUŽI OKO ISTOG BROJA MAGNETNIH LINIJA



CILINDRIČNA SIMETRIJA U RADIJUSU $\rho = R \gg r_c$

U TAČKI P $\Rightarrow$ $\vec{V}_D = \frac{\vec{E}_\varphi \times \vec{B}}{B^2} = \frac{1}{2}\left(\vec{R} \times \frac{\partial \vec{B}}{\partial t}\right) \times \frac{\vec{B}}{B^2} =$

$\underline{\vec{R} \perp \vec{B}}$

$= \frac{1}{2B^2}\left(\frac{\partial B}{\partial t}\right) \cancel{B}\, \vec{R} \Rightarrow \underline{\vec{V}_D = -\frac{1}{2}\left(\frac{\partial B}{\partial t}\right)\frac{\vec{R}}{B}}$

(10) ZADATAK 3.2

$\Theta = -k \sum_\alpha \int f_\alpha \ln f_\alpha \, d^3\vec{r} \, d^3\vec{v}$

$\frac{d\Theta}{dt} = ?$

(7)

$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \nabla f_\alpha + \vec{a} \cdot \nabla_v f_\alpha = 0$ — JEDNAČINA VLASOVA

$\Big/ \cdot \ln f_\alpha d^3\vec{r} d^3\vec{v}$

$$\underbrace{\int \frac{\partial f_\alpha}{\partial t} \ln f_\alpha d^3\vec{r} d^3\vec{v}}_{I_1} + \underbrace{\int \vec{v} \cdot \nabla f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v}}_{I_2} + \underbrace{\int \vec{a} \cdot \nabla_v f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v}}_{I_3} = 0$$

$$I_1 \equiv \int \frac{\partial f_\alpha}{\partial t} \ln f_\alpha d^3\vec{r} d^3\vec{v} = \int \frac{\partial}{\partial t}(f_\alpha \ln f_\alpha - f_\alpha) d^3\vec{r} d^3\vec{v} = \frac{\partial}{\partial t}\left(\int f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v} - \int f_\alpha d^3\vec{r} d^3\vec{v}\right)$$

$$= \frac{\partial}{\partial t}\left(\int f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v}\right) - \frac{\partial}{\partial t} N_\alpha$$

$$\frac{\partial}{\partial t}(f_\alpha \ln f_\alpha) = \frac{\partial f_\alpha}{\partial t} \ln f_\alpha + f_\alpha \frac{\partial \ln f_\alpha}{\partial t} = \frac{\partial f_\alpha}{\partial t} \ln f_\alpha + \frac{\partial f_\alpha}{\partial t}$$

SKRAĆENO NAPISANO SAMO $\int$

N_α - UKUPAN BROJ ČESTICA TIPA α

KAKO $\iint d^3\vec{r} d^3\vec{v}$ KONAČNO DAJE FUNKCIJU KOJA ZAVISI SAMO OD $t \Rightarrow \frac{\partial}{\partial t} \to \frac{d}{dt} \Rightarrow$

$$I_1 = \frac{d}{dt}\left(\int f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v}\right) - \frac{d}{dt} N_\alpha$$

AJNŠTAJNOVA KONVENCIJA O SUMIRANJU

$$I_2 \equiv \int \vec{v} \cdot \nabla f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v} = \int v_i \frac{\partial f_\alpha}{\partial x_i} \ln f_\alpha d^3\vec{r} d^3\vec{v} = \int v_i d^2\vec{r} d^3\vec{v} \int \frac{\partial f_\alpha}{\partial x_i} \ln f_\alpha dx_i =$$

$$= \int v_i d^2\vec{r} d^3\vec{v} \int \frac{\partial}{\partial x_i}(f_\alpha \ln f_\alpha - f_\alpha) dx_i = \left| \begin{array}{l} x_i = \pm\infty \\ \Rightarrow f_\alpha = 0 \end{array} \right| = 0$$

$$I_3 \equiv \int \vec{a} \cdot \nabla_v f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v} = \frac{q_\alpha}{m_\alpha} \int \vec{E} \cdot \nabla_v f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v} + \int \frac{q_\alpha}{m_\alpha} (\vec{v} \times \vec{B}) \cdot \nabla_v f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v} =$$

$\vec{a} = \frac{q_\alpha}{m_\alpha}(\vec{E} + \vec{v} \times \vec{B})$

$$+ \frac{q_\alpha}{m_\alpha} \int \epsilon_{ijk} v_j B_k \frac{\partial f_\alpha}{\partial v_i} \ln f_\alpha d^3\vec{r} d^3\vec{v} =$$

$$= \frac{q_\alpha}{m_\alpha} \int E_i \frac{\partial f_\alpha}{\partial v_i} \ln f_\alpha d^3\vec{r} d^2\vec{v} dv_i + \frac{q_\alpha}{m_\alpha} \int \epsilon_{ijk} B_k d^3\vec{r} d^2\vec{v} \int v_j \frac{\partial f_\alpha}{\partial v_i} dv_i = 0$$

$$= \frac{q_\alpha}{m_\alpha} \int E_i d^3\vec{r} d^2\vec{v} \int \frac{\partial}{\partial v_i}(f_\alpha \ln f_\alpha - f_\alpha) dv_i \quad v_i = \pm\infty \Rightarrow f_\alpha \to 0$$

$(\vec{a} \times \vec{b})_i = \epsilon_{ijk} a_j b_k$

$$\Rightarrow \frac{d}{dt}\left(\int f_\alpha \ln f_\alpha d^3\vec{r} d^3\vec{v}\right) - \frac{dN_\alpha}{dt} = 0 \quad \Big/ -k \sum_\alpha \Rightarrow$$

$$\frac{d\Theta}{dt} = -k \frac{d}{dt}\left(\sum_\alpha N_\alpha\right)$$

UKUPAN BROJ ČESTICA U SISTEMU

AKO SE UKUPAN BROJ ČESTICA U SISTEMU NE MENJA TOKOM VREMENA (NPR. POTPUNO JONIZOVANE PLAZME) $\Rightarrow$

$$\frac{d\Theta}{dt} = 0$$