NERELATIVISTIČKA JEDNAČINA KRETANJA – VERLEOV METOD INTEGRACIJE ①

NEKA JE Δt DOVOLJNO MALO (U ODNOSU NA $\tau_c = \frac{2\pi}{\omega_c} = \frac{2\pi}{qB} m$, NPR. $\tau_c/1000$), ONDA SE MOŽE NAĆI TEJLOROV RAZVOJ U RED POLOŽAJA ČESTICE U $t \pm \Delta t$:

$$\vec{r}(t \pm \Delta t) \approx \vec{r}(t) \pm \vec{v}(t)\Delta t + \frac{1}{2}\vec{a}(t)(\Delta t)^2 \pm \frac{1}{6}\dddot{\vec{r}}(t)(\Delta t)^3 + \cdots$$

NA KLASIČNI VERLEOV METOD INTEGRACIJE J-NE KRETANJA SE IZVODI NA SLEDEĆI NAČIN:

NEKA JE $m\underbrace{\ddot{\vec{r}}(t)}_{\vec{a}(t)} = \vec{F}(\vec{r}(t), t) \Rightarrow \vec{a}(t) = \frac{1}{m}\vec{F}(\vec{r}(t), t)$

$$\vec{r}(t+\Delta t) + \vec{r}(t-\Delta t) = 2\vec{r}(t) + \vec{a}(t)(\Delta t)^2 \Rightarrow$$ (POKRATE SE ČLANOVI DO $(\Delta t)^4$)

$$\underline{\vec{r}(t+\Delta t) = 2\vec{r}(t) - \vec{r}(t-\Delta t) + \frac{1}{m}\vec{F}(\vec{r}(t), t)(\Delta t)^2}$$

$$\vec{r}(t+\Delta t) - \vec{r}(t-\Delta t) = 2\vec{v}(t)\Delta t \Rightarrow$$ (OSTALO SE ZANEMARI)

$$\underline{\vec{r}(t-\Delta t) = \vec{r}(t+\Delta t) - 2\vec{v}(t)\Delta t}$$

$$\vec{r}(t+\Delta t) = 2\vec{r}(t) - \vec{r}(t+\Delta t) + 2\vec{v}(t)\Delta t + \frac{1}{m}\vec{F}(\vec{r}(t), t)(\Delta t)^2 \Rightarrow$$

$$\underline{\vec{r}(t+\Delta t) = \vec{r}(t) + \vec{v}(t)\Delta t + \frac{1}{2m}\vec{F}(\vec{r}(t), t)(\Delta t)^2}$$

$$\vec{v}(t) = \frac{\vec{r}(t+\Delta t) - \vec{r}(t-\Delta t)}{2\Delta t} \Rightarrow \vec{v}(t+\Delta t) = \frac{\vec{r}(t+2\Delta t) - \vec{r}(t)}{2\Delta t}$$

SLIČNO, $\vec{r}(t+2\Delta t) = 2\vec{r}(t+\Delta t) - \vec{r}(t) + \frac{1}{m}\vec{F}(\vec{r}(t+\Delta t), t+\Delta t)(\Delta t)^2 \Rightarrow$

$$\vec{v}(t+\Delta t) = \frac{1}{2\Delta t}\left(2\vec{r}(t+\Delta t) - \vec{r}(t) + \frac{1}{m}\vec{F}(\vec{r}(t+\Delta t), t+\Delta t)(\Delta t)^2 - \vec{r}(t)\right)$$

$$\vec{v}(t+\Delta t) = \frac{1}{2\Delta t}\left(\cancel{2\vec{r}(t)} + 2\vec{v}(t)\Delta t + \frac{1}{m}\vec{F}(\vec{r}(t), t)(\Delta t)^2 - \cancel{2\vec{r}(t)} + \frac{1}{m}\vec{F}(\vec{r}(t+\Delta t), t+\Delta t)(\Delta t)^2\right)$$

$$\underline{\vec{v}(t+\Delta t) = \vec{v}(t) + \frac{1}{2m}\left(\vec{F}(\vec{r}(t), t) + \vec{F}(\vec{r}(t+\Delta t), t+\Delta t)\right)\Delta t}$$

$$\Rightarrow \boxed{\begin{aligned} \vec{r}_{n+1} &= \vec{r}_n + \vec{v}_n\Delta t + \frac{1}{2m}\vec{F}_n(\Delta t)^2 \\ \vec{v}_{n+1} &= \vec{v}_n + \frac{1}{2m}(\vec{F}_n + \vec{F}_{n+1})\Delta t \end{aligned}}$$

$\vec{F}(\vec{r}(t+\Delta t), \vec{v}(t+\Delta t), t+\Delta t)$

AKO SILA ZAVISI OD $\vec{v}$ ONDA NIJE EKSPLICITNO, ALI MOŽE DA SE TRANSFORMIŠE ZA LORENCOVU SILU U EKSPLICITAN OBLIK

$\Rightarrow$

MAGNETNI VERLEOV METOD

NEKA JE $\vec{F}_n = q\vec{E}_n + q\vec{v}_n \times \vec{B}_n \Rightarrow$

$\vec{F}_{n+1} = q\vec{E}_{n+1} + q\vec{v}_{n+1} \times \vec{B}_{n+1}$

$$\vec{r}_{n+1} = \vec{r}_n + \vec{v}_n\Delta t + \frac{1}{2m}(q\vec{E}_n + q\vec{v}_n \times \vec{B}_n)(\Delta t)^2$$

$$\vec{p}' \equiv \frac{q\Delta t}{2m}(\vec{E}_n + \vec{v}_n \times \vec{B}_n) \Rightarrow \vec{r}_{n+1} = \vec{r}_n + \vec{v}_n\Delta t + \vec{p}'\Delta t \rightarrow \vec{r}_{n+1} = \vec{r}_n + \underbrace{(\vec{v}_n + \vec{p}')}_{\vec{p}}\Delta t \Rightarrow$$

$$\boxed{\begin{aligned} \vec{r}_{n+1} &= \vec{r}_n + \vec{p}\Delta t \\ \vec{p} &= \vec{v}_n + \frac{q\Delta t}{2m}(\vec{E}_n + \vec{v}_n \times \vec{B}_n) \end{aligned}}$$

$$\vec r_{n+1} = \vec r_n + \vec p\,\Delta t, \quad \vec p = \vec v_n + \frac{q\Delta t}{2m}(\vec E_n + \vec v_n \times \vec B_n)$$

SA DRUGE STRANE, $\vec v_{n+1} = \vec v_n + \frac{1}{2m}(\vec F_n + \vec F_{n+1})\Delta t$

$$\vec v_{n+1} = \vec v_n + \frac{1}{2m}(q\vec E_n + q\vec v_n\times\vec B_n + q\vec E_{n+1} + q\vec v_{n+1}\times \vec B_{n+1})\Delta t$$

$$\vec v_{n+1} = \underbrace{\vec v_n + \frac{q\Delta t}{2m}(\vec E_n + \vec v_n\times\vec B_n)}_{\vec p} + \frac{q\Delta t}{2m}\vec E_{n+1} + \frac{q\Delta t}{2m}\vec v_{n+1}\times\vec B_{n+1}$$

$$\vec v_{n+1} = \underbrace{\vec p + \frac{q\Delta t}{2m}\vec E_{n+1}}_{\vec s} + \underline{\frac{q\Delta t}{2m}\vec v_{n+1}\times\vec B_{n+1}} \Rightarrow \vec v_{n+1} = \vec s + \frac{q\Delta t}{2m}\vec v_{n+1}\times\vec B_{n+1} \;\Big/ \cdot\vec B_{n+1},\ \times\vec B_{n+1} \Rightarrow$$

$$\vec v_{n+1}\cdot\vec B_{n+1} = \vec s\cdot\vec B_{n+1}$$

$$\vec v_{n+1}\times\vec B_{n+1} = \vec s\times\vec B_{n+1} + \frac{q\Delta t}{2m}\left(\vec B_{n+1}(\vec v_{n+1}\cdot\vec B_{n+1}) - \vec v_{n+1}(B_{n+1})^2\right) \Rightarrow$$

$$\vec v_{n+1}\times\vec B_{n+1} = \vec s\times\vec B_{n+1} + \frac{q\Delta t}{2m}\vec B_{n+1}(\vec s\cdot\vec B_{n+1}) - \frac{q\Delta t}{2m}\vec v_{n+1}B_{n+1}^2 \Rightarrow$$

$$\vec v_{n+1} = \vec s + \frac{q\Delta t}{2m}\left(\vec s\times\vec B_{n+1} + \frac{q\Delta t}{2m}\vec B_{n+1}(\vec s\cdot\vec B_{n+1}) - \frac{q\Delta t}{2m}\vec v_{n+1}B_{n+1}^2\right) \Rightarrow$$

$$\boxed{\vec v_{n+1} = \frac{1}{1+\left(\frac{q\Delta t}{2m}\right)^2 B_{n+1}^2}\left(\vec s + \frac{q\Delta t}{2m}\vec s\times\vec B_{n+1} + \left(\frac{q\Delta t}{2m}\right)^2\vec B_{n+1}(\vec s\cdot\vec B_{n+1})\right)}$$

$$\vec a\times\vec b = \begin{vmatrix}\vec e_x & \vec e_y & \vec e_z\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} = \begin{pmatrix}a_y b_z - a_z b_y\\ a_z b_x - a_x b_z\\ a_x b_y - a_y b_x\end{pmatrix}$$

IDEJNI ALGORITAM:

1. ZADATI POČETNE USLOVE ZA $q, m, x, y, z, v_x, v_y, v_z, \vec E, \vec B$
2. $\vec p = \vec v_n + \frac{q\Delta t}{2m}(\vec E_n + \vec v_n\times\vec B_n)$
3. $\vec r_{n+1} = \vec r_n + \vec p\,\Delta t$
4. $\vec E_{n+1}, \vec B_{n+1}$ ZA $\vec r_{n+1}$ i $t+\Delta t$ (AKO ZAVISE OD $\vec r$ i t)
5. $\vec s = \vec p + \frac{q\Delta t}{2m}\vec E_{n+1}$
6. $\vec v_{n+1} = \frac{1}{1+\left(\frac{q\Delta t}{2m}\right)^2 B_{n+1}^2}\left(\vec s + \frac{q\Delta t}{2m}\vec s\times\vec B_{n+1} + \left(\frac{q\Delta t}{2m}\right)^2\vec B_{n+1}(\vec s\cdot\vec B_{n+1})\right)$

* OBIČNO JE POGODNIJE RADITI SA BEZDIMENZIONIM J-NAMA, ŠTO OVDE NE RADIMO

Z1 ISPITATI KRETANJE NERELATIVISTIČKIH e^-, p^+, α-ČESTICA U $\vec B = B\vec e_z$, $B = 5\,\mu G = const.$ KAKO SE PONAŠA KINETIČKA ENERGIJA TOKOM VREMENA? KOLIKO JE $\left|\frac{E_{fin}-E_{in}}{E_{in}}\right|$? NEKA JE $x_0 = y_0 = z_0 = 0$, $v_x = 0$, v_y, v_z = RANDOM $\ll c$

POTREBNO JE PRILAGODITI T-sim, dt, T-sup, d (U GRAFIČKOM PRIKAZU)

Z2 $m = 1\,kg$, $q = 1\,C$, $B_0 = 1\,T$, $A = 1\,T/m$

$\vec B = (Ax + B_0)\vec e_z$

$x_0 = y_0 = z_0 = 0$, $\vec v_0 = (1\,m/s)\vec e_y$

PRILAGODITI T-sim, dt, T-sup, d → NPR. T-sim = 3000 s, T-sup = 0.1 s, dt = 0.01 s, d = 1

23 $m = 1\,kg$, $\mathcal{L} = 1\,C$, $A = 0{,}1$ $x_0 = y_0 = z_0 = 0$ NPR.

$\vec{B} = A z \vec{e_x} + (1 + A x)\vec{e_z}$ $\vec{v_0} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ (m/s) $T_{sim} = 3000\,s$, $dt = 0{,}01\,s$, $T_{sup} = 0{,}1\,s$

NERELATIVISTIČKA J-NA KRETANJA – RUNGE-KUTA 4 (RK4) –

KRETANJE U POSTOJANOM POLJU MAGNETNOG DIPOLA (KONSTANTNO U VREMENU)

$\vec{B} = \frac{\mu_0}{4\pi r^3}\left(3(\vec{M}\cdot\vec{e_r})\vec{e_r} - \vec{M}\right)$, $\vec{r} = x\vec{e_x} + y\vec{e_y} + z\vec{e_z}$, $\vec{r} = r\vec{e_r}$, $r^2 = x^2 + y^2 + z^2$

AKO JE $\underline{\vec{M} = -M\vec{e_z}}$ =>

$$\vec{B} = \frac{\mu_0}{4\pi r^3}\left(-3M(\vec{e_z}\cdot\vec{e_r})\vec{e_r} + M\vec{e_z}\right) =$$

$$= \frac{\mu_0}{4\pi r^3}\left(-3M(\vec{e_z}\cdot\vec{r})\frac{\vec{r}}{r^2} + M\vec{e_z}\right) =$$

$$= \frac{\mu_0 M}{4\pi r^5}\left(-3z\vec{r} + r^2\vec{e_z}\right) = \frac{\mu_0 M}{4\pi r^5}\begin{pmatrix} -3zx \\ -3zy \\ -3z^2 + (x^2+y^2+z^2) \end{pmatrix}$$

$$\vec{B} = -\frac{\mu_0 M}{4\pi r^5}\begin{pmatrix} 3zx \\ 3zy \\ 2z^2 - x^2 - y^2 \end{pmatrix}$$

ZA $x = R_\oplus$, $y = 0$, $z = 0$ =>

$\vec{B} \equiv \vec{B_0} = \frac{\mu_0 M}{4\pi R_\oplus^5} R_\oplus^2 \vec{e_z}$ => $B_0 = \frac{\mu_0 M}{4\pi R_\oplus^3}$

$R_\oplus = 6378137\,m$ => $\frac{\mu_0 M}{4\pi} = B_0 R_\oplus^3$ =>

$B_0 = 3{,}07\cdot 10^{-5}\,T$

$$\vec{B} = -\frac{B_0 R_\oplus^3}{r^5}\begin{pmatrix} 3zx \\ 3zy \\ 2z^2 - x^2 - y^2 \end{pmatrix}$$

$B_0 R_\oplus^3 = 7{,}965626\cdot 10^{15}\,Tm^3$

IPAK, MAGNETNA OSA ZAKLAPA UGAO $\varphi = 11{,}5$ SA OSOM ZEMLJINE ROTACIJE KOJA SE MOŽE, U OVOM PRIMERU, UZETI DA SE POKLAPA SA Z-OSOM

$$\vec{M} = -M\begin{pmatrix} 0 \\ \sin\varphi \\ \cos\varphi \end{pmatrix} \Rightarrow \vec{B} = \frac{-\mu_0 M}{4\pi r^5}\left(3(y\sin\varphi + z\cos\varphi)\begin{pmatrix} x \\ y \\ z \end{pmatrix} - (x^2+y^2+z^2)\begin{pmatrix} 0 \\ \sin\varphi \\ \cos\varphi \end{pmatrix}\right) \Rightarrow$$

$$\vec{B_0} = -\frac{\mu_0 M}{4\pi R_\oplus^5}\begin{pmatrix} 0 \\ -R_\oplus^2\sin\varphi \\ -R_\oplus^2\cos\varphi \end{pmatrix}$$

$$\vec{B} = -\frac{B_0 R_\oplus^3}{r^5}\begin{pmatrix} 3yx\sin\varphi + 3zx\cos\varphi \\ 2y^2\sin\varphi - z^2\sin\varphi + 3yz\cos\varphi - x^2\sin\varphi \\ 2z^2\cos\varphi - x^2\cos\varphi - y^2\cos\varphi + 3yz\sin\varphi \end{pmatrix}$$

=> $B_0 = \frac{\mu_0 M}{4\pi R_\oplus^3}$

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x, y, z IZRAŽAVAMO U $[R_\oplus]$

R.K.4:

AKO IMAMO $\frac{dy}{dt} = f(y, t)$, $y(t_0) = y_0$ ONDA (BEZ IZVOĐENJA) SLEDI

$$y_{n+1} = y_n + \frac{dt}{6}(k_1 + 2k_2 + 2k_3 + k_4), \text{ GDE SU}$$

$$k_1 = f(y_n, t_n)$$

$$k_2 = f\left(y_n + \frac{k_1}{2}dt, t_n + \frac{dt}{2}\right)$$

$$k_3 = f\left(y_n + \frac{k_2}{2}dt, t_n + \frac{dt}{2}\right)$$

$$k_4 = f(y_n + k_3\,dt, t_n + dt)$$

$$\left.\begin{aligned}\frac{d\vec{v}}{dt}&=\vec{a}\\ \frac{d\vec{r}}{dt}&=\vec{v}\end{aligned}\right| \Rightarrow \quad \begin{aligned}\vec{v}_{n+1}&=\vec{v}_n+\frac{dt}{6}\left(\vec{k_{1a}}+2\vec{k_{2a}}+2\vec{k_{3a}}+\vec{k_{4a}}\right)\\ \vec{r}_{n+1}&=\vec{r}_n+\frac{dt}{6}\left(\vec{k_{1v}}+2\vec{k_{2v}}+2\vec{k_{3v}}+\vec{k_{4v}}\right)\end{aligned}$$

GDE SU:

$$\vec{r}_1=\vec{r}_n$$
$$\vec{k_{1v}}=\vec{v}_n$$
$$\vec{k_{1a}}=\vec{a}\left(\vec{k_{1v}},\vec{B}(\vec{r}_1)\right)$$
$$\vec{r}_2=\vec{r}_n+\frac{1}{2}\vec{k_{1v}}\,dt$$
$$\vec{k_{2v}}=\vec{v}_n+\frac{1}{2}\vec{k_{1a}}\,dt$$
$$\vec{k_{2a}}=\vec{a}\left(\vec{k_{2v}},\vec{B}(\vec{r}_2)\right)$$
$$\vec{r}_3=\vec{r}_n+\frac{1}{2}\vec{k_{2v}}\,dt$$
$$\vec{k_{3v}}=\vec{v}_n+\frac{1}{2}\vec{k_{2a}}\,dt$$
$$\vec{k_{3a}}=\vec{a}\left(\vec{k_{3v}},\vec{B}(\vec{r}_3)\right)$$
$$\vec{r}_4=\vec{r}_n+\vec{k_{3v}}\,dt$$
$$\vec{k_{4v}}=\vec{v}_n+\vec{k_{3a}}\,dt$$
$$\vec{k_{4a}}=\vec{a}\left(\vec{k_{4v}},\vec{B}(\vec{r}_4)\right)$$

U NAŠEM PROBLEMU JE

$$\vec{a}=\frac{\mathcal{L}}{m}\vec{v}\times\vec{B}$$

$$\vec{B}=\vec{B}_{dipol}=\vec{B}(\vec{r})$$

U OPŠTEM SLUČAJU JE

$$\vec{a}=\frac{\mathcal{L}}{m}(\vec{E}+\vec{v}\times\vec{B})$$

$$\vec{E}=\vec{E}(\vec{r},t)$$

$$\vec{B}=\vec{B}(\vec{r},t)$$

(SVE VREME PODRAZUMEVAMO ORBITALNI METOD – NEMA INTERAKCIJE MEĐU ČESTICAMA, ZRAČENJE, ...)

RELATIVISTIČKA JEDNAČINA KRETANJA

$$\frac{d\vec{p}}{dt}=\underbrace{\vec{F}(\vec{r},\vec{v},t)}_{\text{TROVEKTOR SILE}}, \quad \vec{p}=\underbrace{\gamma m_0}_{m}\vec{v}$$

TROVEKTOR IMPULSA

$$m=\gamma m_0, \quad \gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$v^2=v_x^2+v_y^2+v_z^2$$

$$E=mc^2=\gamma m_0 c^2 \quad \perp\!\!\perp$$

$$\gamma m_0\frac{d\vec{v}}{dt}+m_0\frac{d\gamma}{dt}\vec{v}=\vec{F}(\vec{r},\vec{v},t)$$

$$\frac{dE}{dt}=\frac{d}{dt}(\gamma m_0 c^2)=\vec{F}\cdot\vec{v}\Rightarrow$$

$$m_0c^2\frac{d\gamma}{dt}=\vec{F}\cdot\vec{v} \quad \perp\!\!\perp$$

$$\frac{d\vec{v}}{dt}=\frac{1}{\gamma m_0}\left(\vec{F}-m_0\frac{d\gamma}{dt}\vec{v}\right)$$

$$\Rightarrow \boxed{\frac{d\vec{v}}{dt}=\frac{1}{\gamma m_0}\left(\vec{F}-\frac{(\vec{F}\cdot\vec{v})\vec{v}}{c^2}\right)}$$

AKO JE $\vec{F}=\mathcal{L}\vec{E}+\mathcal{L}\vec{v}\times\vec{B} \Rightarrow (\vec{F}\cdot\vec{v})\vec{v}=(\mathcal{L}\vec{E}\cdot\vec{v})\vec{v}+\cancel{(\mathcal{L}\vec{v}\times\vec{B})\cdot\vec{v}}^{\,0}\,\vec{v}$

AKO SE IZUZME $\vec{E}\Rightarrow$

$$\frac{d\vec{v}}{dt}=\frac{1}{\gamma m_0}\vec{F}=\underbrace{\frac{1}{\gamma m_0}\mathcal{L}\vec{v}\times\vec{B}}_{\vec{a}}$$

SA $\vec{E}\Rightarrow$

$$\frac{d\vec{v}}{dt}=\frac{1}{\gamma m_0}\left(\mathcal{L}\vec{E}+\mathcal{L}\vec{v}\times\vec{B}-\frac{(\mathcal{L}\vec{E}\cdot\vec{v})\vec{v}}{c^2}\right)$$

$$\frac{d\vec{v}}{dt}=\underbrace{\frac{\mathcal{L}}{m_0}\frac{1}{\gamma}\left(\vec{E}+\vec{v}\times\vec{B}-\frac{(\vec{E}\cdot\vec{v})\vec{v}}{c^2}\right)}_{\vec{a}}$$

$$\vec{a}=\frac{\sqrt{1-v^2/c^2}}{m_0}\mathcal{L}\left(\vec{E}+\vec{v}\times\vec{B}-\frac{(\vec{E}\cdot\vec{v})\vec{v}}{c^2}\right)$$

SKALARNO:

$$\begin{pmatrix}a_x\\a_y\\a_z\end{pmatrix}=\frac{\mathcal{L}}{m_0}\sqrt{1-\frac{v_x^2+v_y^2+v_z^2}{c^2}}\begin{pmatrix}E_x+v_yB_z-v_zB_y-\frac{1}{c^2}(E_xv_x+E_yv_y+E_zv_z)v_x\\ E_y+v_zB_x-B_zv_x-1/c^2(E_xv_x+E_yv_y+E_zv_z)v_y\\ E_z+v_xB_y-v_yB_x-1/c^2(E_xv_x+E_yv_y+E_zv_z)v_z\end{pmatrix}$$

SKALARNO ILI VEKTORSKI SE MOŽE RADITI

U PRIMERU JE SKALARNO

$E_k = \gamma m_0 c^2 - m_0 c^2 \Rightarrow \gamma m_0 c^2 = E_k + m_0 c^2 \Rightarrow \gamma^2 = \left(\frac{E_k + m_0 c^2}{m_0 c^2}\right)^2 \Rightarrow$

$(m_0 c^2)^2 = (E_k + m_0 c^2)^2 - \frac{v^2}{c^2}(E_k + m_0 c^2)^2 \Rightarrow$

$v^2 \frac{(E_k + m_0 c^2)^2}{c^2} = (E_k + m_0 c^2)^2 - (m_0 c^2)^2 \Rightarrow v^2 = \frac{((E_k + m_0 c^2)^2 - (m_0 c^2)^2)c^2}{(E_k + m_0 c^2)^2} \Rightarrow$

$v = c\sqrt{1 - \left(\frac{m_0 c^2}{m_0 c^2 + E_k}\right)^2}$, AKO JE $E_k = 10\,MeV$ ZA PROTON $\Rightarrow$

$v = 0{,}145\,c$

$m_p = 1{,}6726219 \cdot 10^{-27}\,kg$

AKO JE $E_k = 250\,MeV$ ZA PROTON $\Rightarrow$

$v = 0{,}616\,c$

NEKA JE NAGIBNI UGAO NPR. U POČETKU $\alpha = 30^\circ \Rightarrow$

$v_x = 0,\ v_y = v \sin\alpha,\ v_z = v\cos\alpha$

ZA $\vec{B}_{dipol}(\vec{r})$ JE $m_0 c^2 \frac{d\gamma}{dt} = \vec{F} \cdot \vec{v} = q(\vec{v} \times \vec{B}_{dipol}(\vec{r})) \cdot \vec{v} = 0 \Rightarrow \gamma = const,\ v^2 = const$

$\Rightarrow$ MAGNETNI VERLEOV ALGORITAM SE MOŽE PRIMENITI I ZA RELATIVISTIČKU JEDNAČINU KRETANJA

VERLEOV ALGORITAM JE SIMPLEKTIČNI ŠTO UKAZUJE NA STABILNOST TOKOM DUŽEG VREMENA INTEGRACIJE, ZA RAZLIKU OD RK4 ALGORITMA (EKSPLICITNOG, KOJI JE PRIKAZAN OVDE)

Z4 ZA PROTONE SA 10 MeV i 250 MeV $x_0 = 2{,}5 - 4\,R_\oplus,\ y_0 = z_0 = 0$, $\vec{v}_0 = \begin{pmatrix} 0 \\ (0{,}145 - 0{,}616) \cdot 0{,}5c \\ (0{,}145 - 0{,}616) \cdot 0{,}866c \end{pmatrix}$

UPOREDITI MAGNETNI VERLE I RK4 ZA POSTOJANI MAGNETNI DIPOL OD RANIJE

KORIGOVATI T-sim, dt i T-snap