TE plasmas

1. Maxwell distribution for velocities (momentum, kinetic energy),
2. Boltzmann distribution for internal energy states (excitation states),
3. Saha equilibrium distribution for ionization states,
4. Planck's distribution for radiation

TE plasmas

Binary collisions for 1-3

Interaction between radiation and matter (thermal absorption/emission) 4

Collisions regulate the establishment of equilibrium distributions of plasma particle velocities, excitation states, and ionization states

Equilibrium distribution of radiation is realized through the interactions of photons with different particles (via thermal or true absorption/emission processes)

In deep and hot interior of normal stars, particles and radiation have nearly the TE distributions determined by the local temperature

In the solar photosphere, the particles can still be considered to have equilibrium distributions at the local temperature (the mean free path of the particles is of the order of or less than 1 cm, while the mean temperature gradient is only about 10^{-4} K cm^{-1})

On the other hand, the mean free path of photons in the photosphere is so large (order of hundreds of kilometers) that on that length scale temperature changes can be considerable ($\sim$ 3000 K in the case of the solar photosphere)

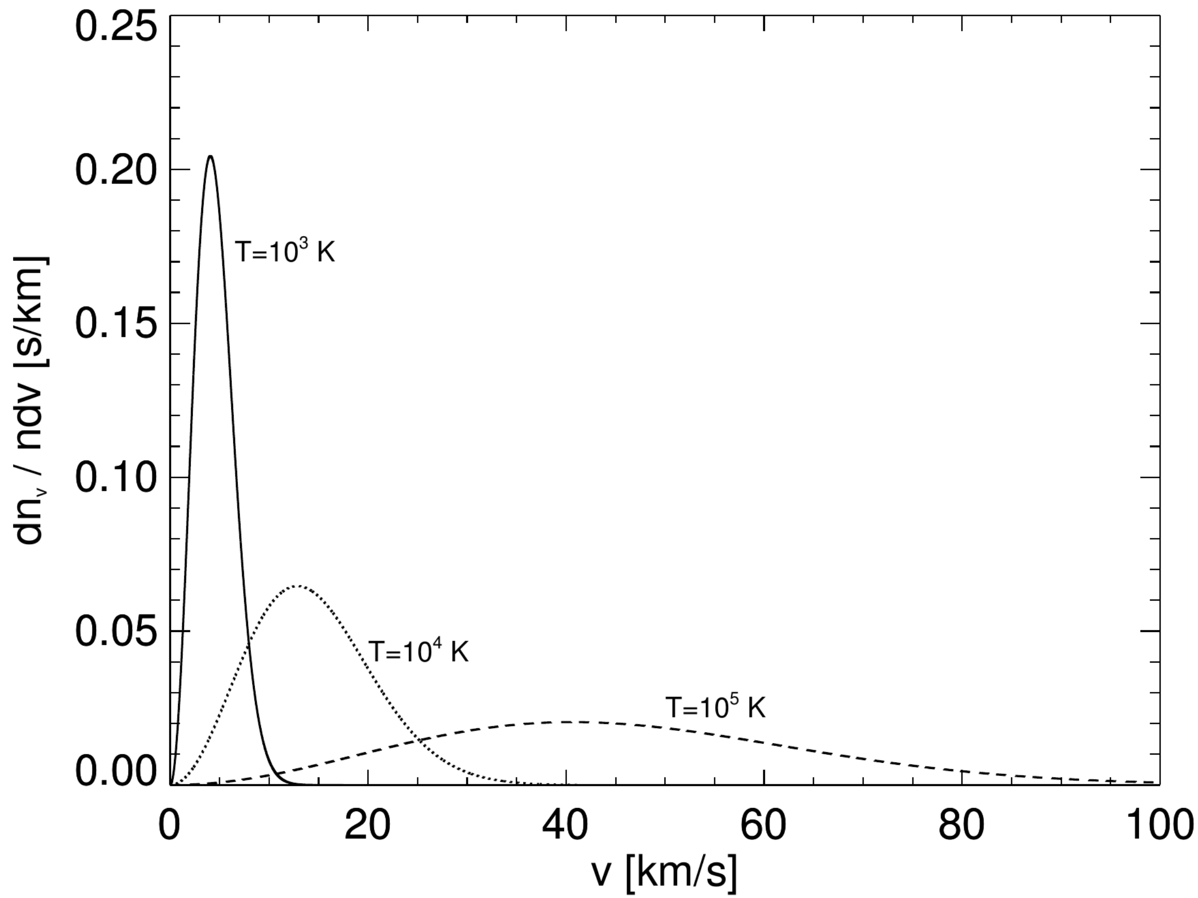
For this reason, it can then be considered that the particles have local, equilibrium distributions, but that this is not the case with radiation

It can be assumed that Kirchhoff's law is valid at the local temperature (approximation of local TE, so-called LTE), while the radiation spectrum needs to be determined by solving the radiation transfer equation

$$\sqrt{8kT/\pi m}$$

$$\sqrt{2kT/m}$$

$$\sqrt{3kT/m}$$

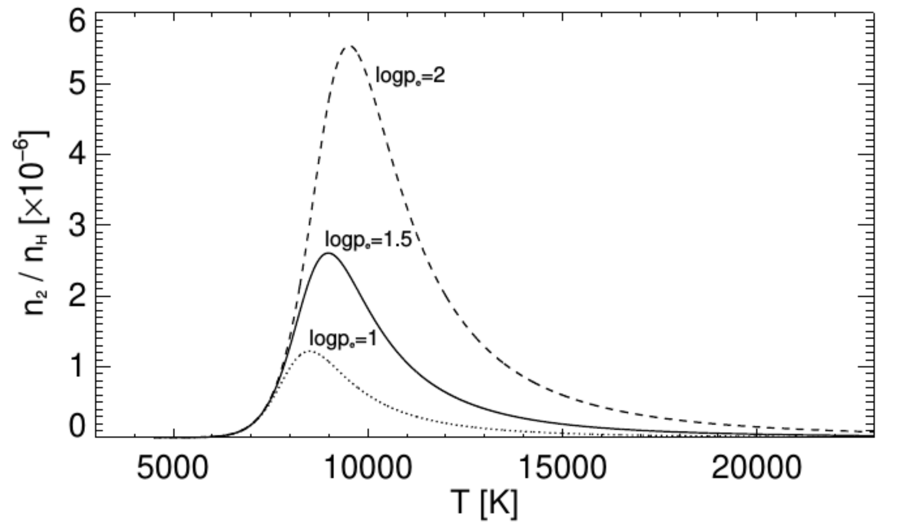


$$\sqrt{kT/m}$$

$$dn_v = 4\pi n \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$$

$$\frac{n_{\boldsymbol{n}}}{n_{\boldsymbol{n}'}} = \frac{g_{\boldsymbol{n}}}{g_{\boldsymbol{n}'}}\, e^{-\frac{\mathcal{E}_{\boldsymbol{n}\boldsymbol{n}'}}{kT}} \qquad g_{\boldsymbol{n}} = 2\boldsymbol{n}^2$$

How much Balmer lines are important in stellar spectra?



For H plasma:

$$\frac{\alpha^2}{1-\alpha^2} = \frac{(2\pi m_{\mathrm{e}})^{3/2}}{h^3}\frac{(kT)^{5/2}}{p} e^{-\frac{\mathcal{E}_{\mathrm{jon}}}{kT}}$$

$$\alpha \equiv n_{\mathrm{H\,II}}/(n_{\mathrm{H\,I}} + n_{\mathrm{H\,II}})$$

$$\mathcal{E}_{\mathrm{jon}} = 13.598 \text{ eV}$$

$$p = (n_{\mathrm{H\,I}} + n_{\mathrm{H\,II}} + n_{\mathrm{e}})kT$$

$$p = (1+\alpha)p_{\mathrm{e}}/\alpha$$

$$n_{\mathrm{H\,II}} = n_{\mathrm{e}}$$

$$n_2/n_{\mathrm{H}} = (1-\alpha)n_2/n_{\mathrm{H\,I}}$$

$$n_2/n_{\mathrm{H\,I}} = g_2 e^{-\mathcal{E}_{21}/(kT)}/\,\mathcal{U}(T)$$

$$\mathcal{U}(T) = \textstyle\sum_{\boldsymbol{n}} g_{\boldsymbol{n}} e^{-\mathcal{E}_{\boldsymbol{n}1}/(kT)} =$$

$$g_1 + g_2 e^{-\mathcal{E}_{21}/(kT)} + g_3 e^{-\mathcal{E}_{31}/(kT)} + \cdots$$

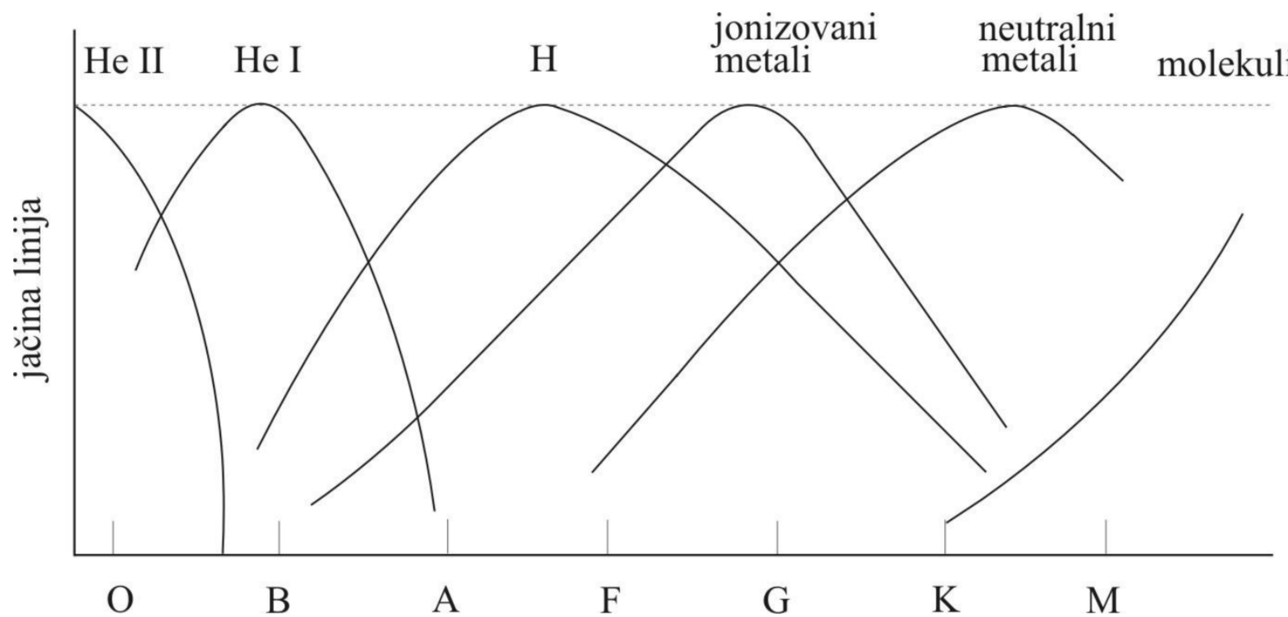
in reality, a finite number of states

Solar core – Saha is not OK
ionization by pressure

Solar photosphere $\alpha \approx 10^{-4}$

$$1.57 \times 10^7 \text{ K}$$

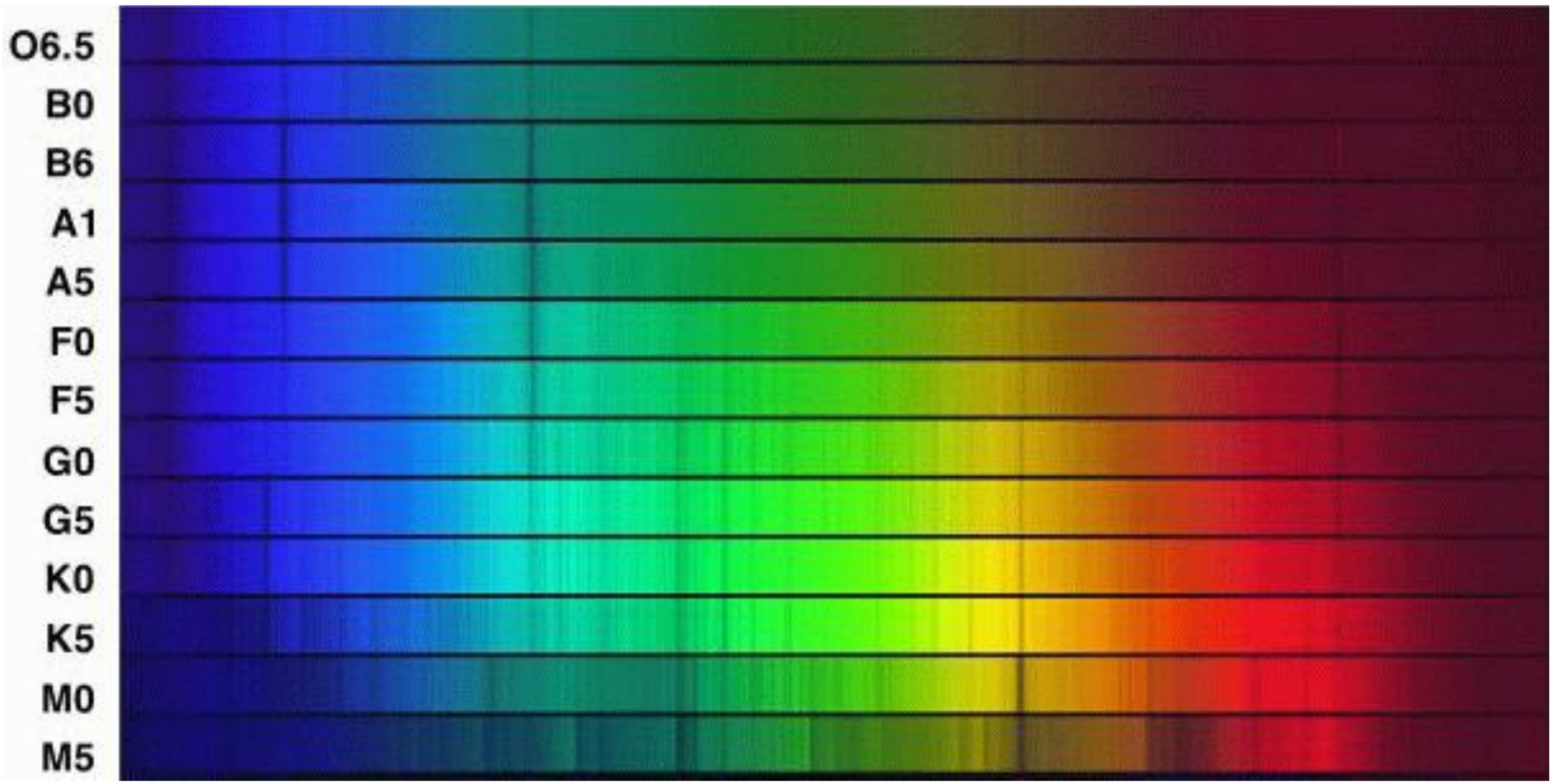
$$2.33 \times 10^{16} \text{ Pa} = 2.33 \times 10^{17} \text{ dyn/cm}^2$$


He II
He I
H
jonizovani metali
neutralni metali
molekuli
jačina linija
O
B
A
F
G
K
M

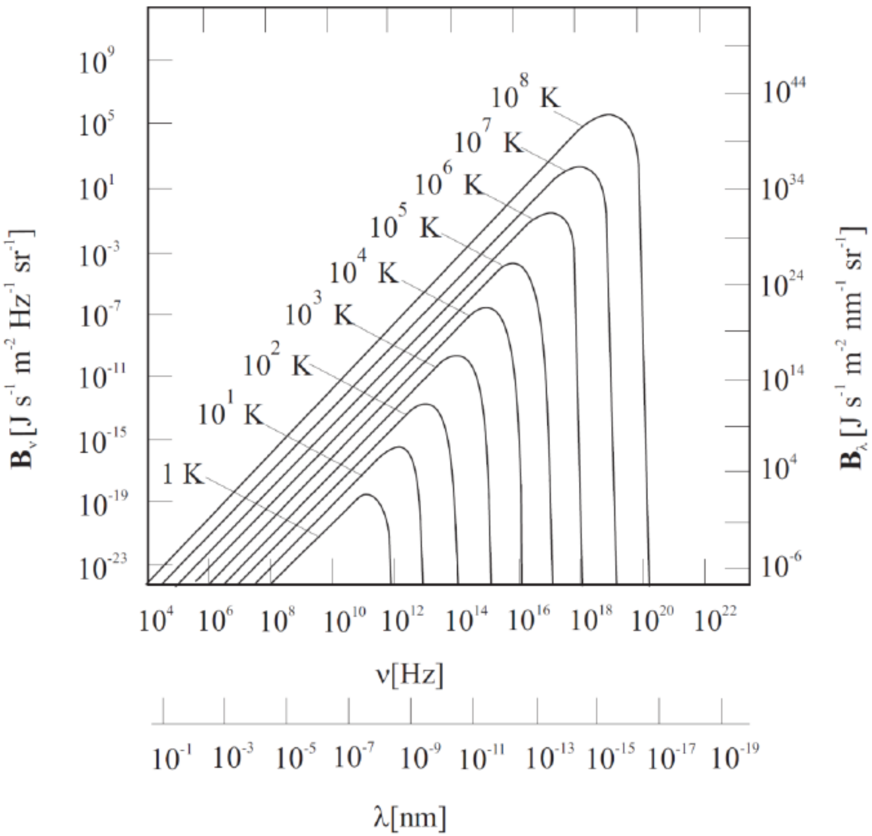
Klasifikacija Annie Jump Cannon

O – B – A – F – G – K – M

(Oh, Be A Fine Girl Kiss Me)



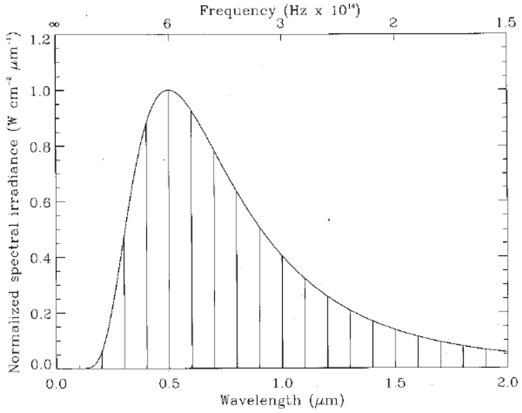
Plankove krive za razne vrednosti temperature



$$\nu I_\nu = \lambda I_\lambda = \frac{2k^4T^4}{h^3c^2}\frac{x^4}{e^x - 1}$$

$$x \equiv h\nu/(kT)$$

$$hc/(\lambda kT)$$



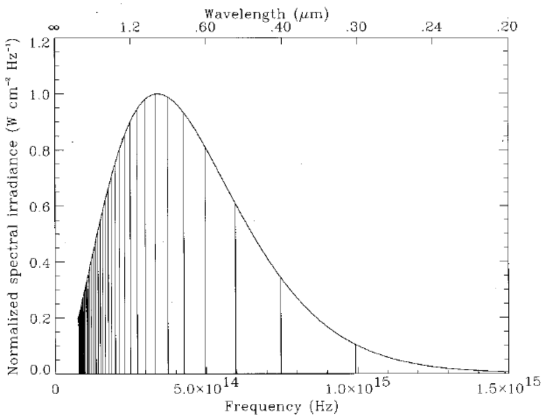


Fig. 4. The same Planck function and wavelength intervals as Fig. 3 transformed into frequency intervals. Note that the frequency intervals are not equal.

Distributions: for unit interval

$$p = nkT\left(1 - \frac{1}{18N_\mathrm{D}}\right) \approx nkT$$

equilibrium radiation pressure

$$p = nkT\left(1 - \frac{1}{18N_\mathrm{D}}\right) + \frac{4\sigma T^4}{3c} \approx nkT + \frac{a}{3}T^4$$

$$\boxed{p_\mathrm{g} = nkT = \rho\mathcal{R}T, \quad \mathcal{R} = R_\mathrm{g}/\mu, \quad R_\mathrm{g} = kN_\mathrm{A}}$$

$$N_\mathrm{A} \approx 6.022 \times 10^{23}\ \mathrm{mol}^{-1}$$
$$1/N_\mathrm{A} \approx 1.660 \times 10^{-24}\ \mathrm{g} = m_\mathrm{u}$$
$$m_\mathrm{H} \approx 1.0078\ m_\mathrm{u}$$

$$R_\mathrm{g} \approx 8.314\ \mathrm{J\ K^{-1}\ mol^{-1}} = 8.314 \times 10^7\ \mathrm{erg\ K^{-1}\ mol^{-1}}$$
$$a = 4\sigma/c \approx 7.566 \times 10^{-16}\ \mathrm{J\ m^{-3}\ K^{-4}} = 7.566 \times 10^{-15}\ \mathrm{erg\ cm^{-3}\ K^{-4}}$$

$$\mu = \rho N_\mathrm{A}/n$$
$$\mu m_\mathrm{u} = \sum n_i m_i / \sum n_i$$

Fully ionized gas

$$\mu \approx \left(2\mathcal{X} + \frac{3}{4}\mathcal{Y} + \frac{1}{2}\mathcal{Z}\right)^{-1} = \left(\frac{1}{2} + \frac{3}{2}\mathcal{X} + \frac{1}{4}\mathcal{Y}\right)^{-1}$$

$$\mathcal{X} + \mathcal{Y} + \mathcal{Z} = 1$$

Relative abundances per mass

Solar interior

$$\mathcal{X} \approx 0.70,\ \mathcal{Y} \approx 0.28,\ \mathcal{Z} \approx 0.02$$
$$\mu \approx 0.6$$

H plasma, HII regions

$$\mu = 0.5$$

Solar photosphere

$$\mu \approx 1.3$$

HI

$$\mu = 1$$

H_2

$$\mu = 2$$

Plasma dynamics

If total number of particles is N, we will have N nonlinear coupled differential equations of motion to solve simultaneously

A self-consistent formulation must be used, since the fields and the particle trajectories are intrinsically coupled, that is, the internal fields associated with the presence and motion of the plasma particles influence their motions which, in turn, modify the internal fields

Although the self-consistent approach is conceivable in principle, it cannot be carried out in practice without introducing some averaging scheme, in view of the extremely large number of variables involved

According to the laws of classical mechanics, in order to determine the position and velocity of each particle in the plasma as a function of time under the action of known forces, it is necessary to know the initial position and velocity of each particle

For a system consisting of a very large number of interacting particles these initial conditions are obviously unknown

Furthermore, in order to explain and predict the macroscopic phenomena observed in nature and in the laboratory, it is not of interest to know the detailed individual motion of each particle, since the observable macroscopic properties of a plasma are due to the average collective behavior of a large number of particles

We must discard, therefore, the possibility of analytically solving the set of simultaneous equations of motion for a large number of interacting particles

Depending on the character of the system, as well as the specific process in the plasma that is to be explained, appropriate models, i.e. methods of analysis, are chosen

It is especially important to keep in mind the limits of applicability of the specific method (approximation) used in the description of a plasma phenomenon

It is necessary to know both the values of the basic parameters of the studied plasma and the characteristic space/time scale of the considered processes

Numerical simulations + analytical study

Orbital method – kinetic theory – fluid models

CHARGED PARTICLE MOTION IN CONSTANT AND UNIFORM MAGNETIC FIELD – helical motion

Gyro-frequency or cyclotron frequency $$\omega_{\mathrm{c}} = \frac{|q|B}{m}$$

Gyro-radius or cyclotron radius $$r_{\mathrm{c,th}} = \frac{v_{\mathrm{th}}}{\omega_{\mathrm{c}}}$$

$$\omega_{\mathrm{ce}}/\omega_{\mathrm{cp}} = m_{\mathrm{p}}/m_{\mathrm{e}} \approx 1836$$

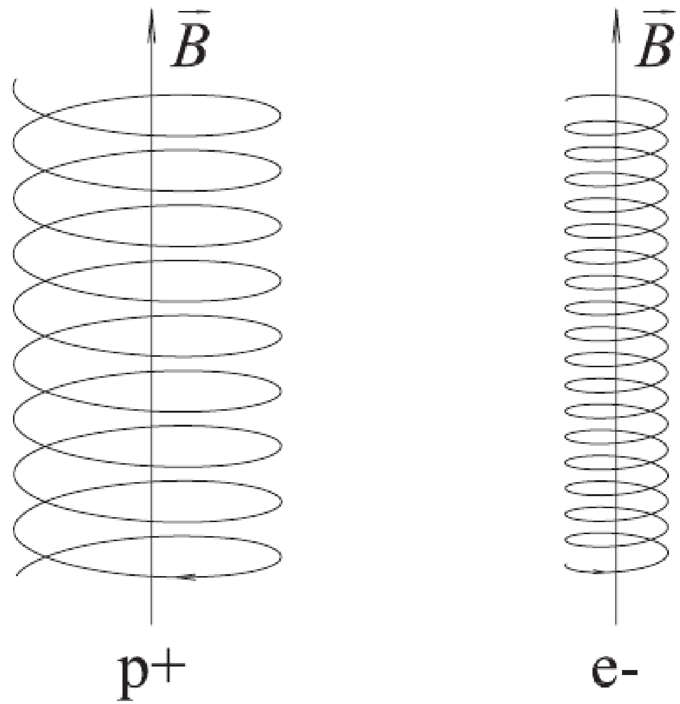
$$r_{\mathrm{ce}}/r_{\mathrm{cp}} = \sqrt{m_{\mathrm{e}}/m_{\mathrm{p}}} \approx 1/43$$

Anisotropy due to B field

Magnetized plasma $$\omega_{\mathrm{c}} \gg \omega_{\mathrm{coll}} \qquad r_{\mathrm{c,th}} \ll r_{\mathrm{mfp}}$$

$$r_{\mathrm{c,th}} \ll L$$

$\beta = 2\mu_0 \, nkT / B^2$



$\vec{B}$
$\vec{B}$
p+
e-

Tip	n_e [m^{-3}]	T_e [K]	r_{De} [m]	ω_{pe} [MHz]	N_{De}	B [T]	ω_{ce} [MHz]	$r_{ce,th}$ [m]	β_e
1	10^{20}	6000	5×10^{-7}	9×10^4	60	0.1	3×10^3	2×10^{-5}	2×10^{-3}
2	10^{16}	10^6	7×10^{-4}	900	10^7	3×10^{-2}	800	7×10^{-4}	4×10^{-4}
3	10^{14}	10^6	7×10^{-3}	90	10^8	10^{-4}	3	0.2	0.3
4	10^{11}	1200	8×10^{-3}	3	2×10^5	3×10^{-5}	0.8	0.03	5×10^{-6}
5	10^8	5000	0.5	0.09	5×10^7	5×10^{-10}	10^{-5}	3×10^3	70
6	10^7	10^5	7	0.03	10^{10}	10^{-8}	3×10^{-4}	700	0.3
7	10^3	10^6	2×10^3	3×10^{-4}	4×10^{13}	5×10^{-10}	10^{-5}	4×10^4	0.1

1 - Solar photosphere; 2 – Solar coronal loops; 3 – Solar corona; 4 – Earth's ionosphere; 5 – HII regions; 6 – Solar wind at 1 au; 7 – Hot ionized ISM

The orbital method in plasma dynamics implies that the nature of the entire collective is studied solely on the basis of knowledge of the movement of isolated particles of different types in known, external fields

The electromagnetic field of charged particles, in principle, changes the external field in which the particles move, so such an effective field can no longer be considered known

Only in the case of sufficiently rarefied magnetized plasma it is reasonable to expect that a collective of charged particles will behave approximately according to the same laws as an isolated particle

Interplanetary, interstellar, intergalactic plasmas – at least qualitative explanation

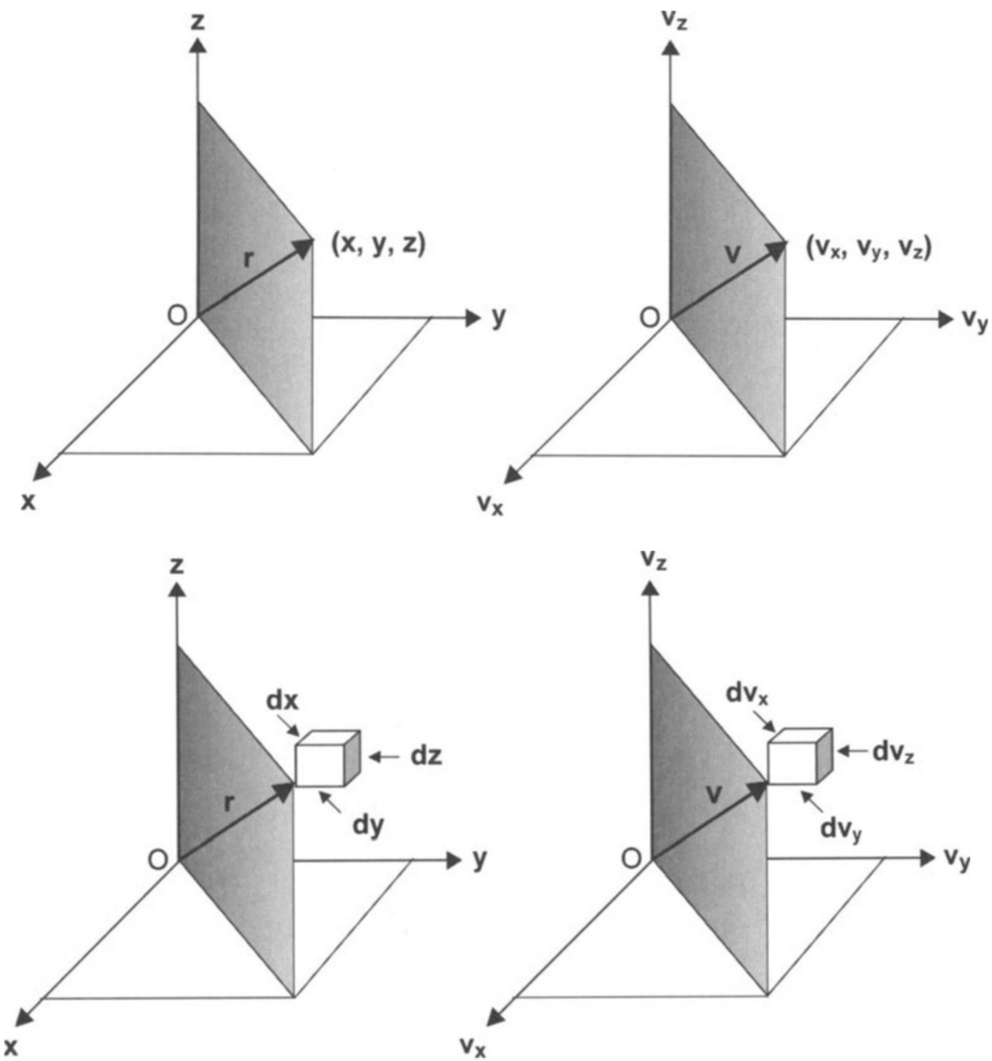
When analyzing the dynamics of a system made up of a very large number of particles, it is convenient to use the appropriate statistical tools provided by the kinetic theory

The current dynamic state of the particle is determined by the radius vector of the position (a point in the configuration space) as well as the velocity (a point in the velocity space; or momentum)

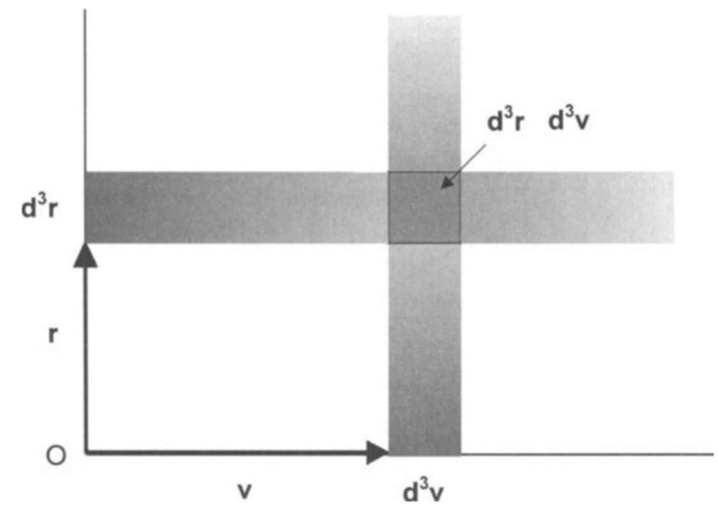
Single-particle, six-dimensional phase space - independent coordinates **r** and **v** (or **p**)

The current dynamical state of a particle is then determined by a phase point, while at each moment of time, the dynamical state of a system of N particles is represented by N phase points in one such space

The evolution of the entire system is represented by characteristic trajectories in the phase space



$$d^3\vec{r}d^3\vec{v} \equiv dxdydzdv_xdv_ydv_z$$



$$d^6\mathcal{N}_\alpha(\vec{r},\vec{v},t) = f_\alpha(\vec{r},\vec{v},t)d^3\vec{r}d^3\vec{v}$$

According to Gibbs, the (statistical) ensemble is such a set of identical systems (copies) that differ only by initial conditions (each of the copies represent one possible state of a real system)

We can associate one phase space with each of the individual copies from the ensemble

In each of those spaces, the same volume element can be observed

The number of phase points in that volume element (the same for all systems in the ensemble), generally differs, so one can find the mean number of phase points in the volume element per ensemble – the ensemble averaged phase density

A volume element in configuration space is usually defined as a volume small enough compared to the characteristic scales of change of the relevant physical quantities that describe the observed system but still large enough to contain a sufficient number of particles to warrant statistical treatment

When defining such a volume element, it is common to consider length scales significantly larger than the interparticle distance, but still of the order of magnitude or less than the Debye radius

The averaged phase density (the so-called distribution function) does not describe the movement of individual particles in the plasma

The coordinates **r** and **v** identify certain elements of the phase volume at the moment of time t

The positions of individual particles in the phase space are undetermined up to that phase volume

The choice of the appropriate phase volume element, suitable for statistical analysis, is followed by the assumption that the averaged phase density is a continuous function of its arguments

The number density of phase points does not change abruptly from one to another, adjacent volume element

Also, the averaged phase density is a positive, finite quantity at every instant of time (the mass, momentum, and energy of the system are finite)

The phase density must therefore tend to zero when v tends to higher and higher values (there are no particles with infinite speed)

$$f_\alpha^{\mathrm{M}}(\vec{r},v,t) = n_\alpha(\vec{r},t)\left(\frac{m_\alpha}{2\pi k T_\alpha(\vec{r},t)}\right)^{3/2} e^{-\frac{m_\alpha(\vec{v}-\vec{u}_\alpha(\vec{r},t))^2}{2kT_\alpha(\vec{r},t)}}$$

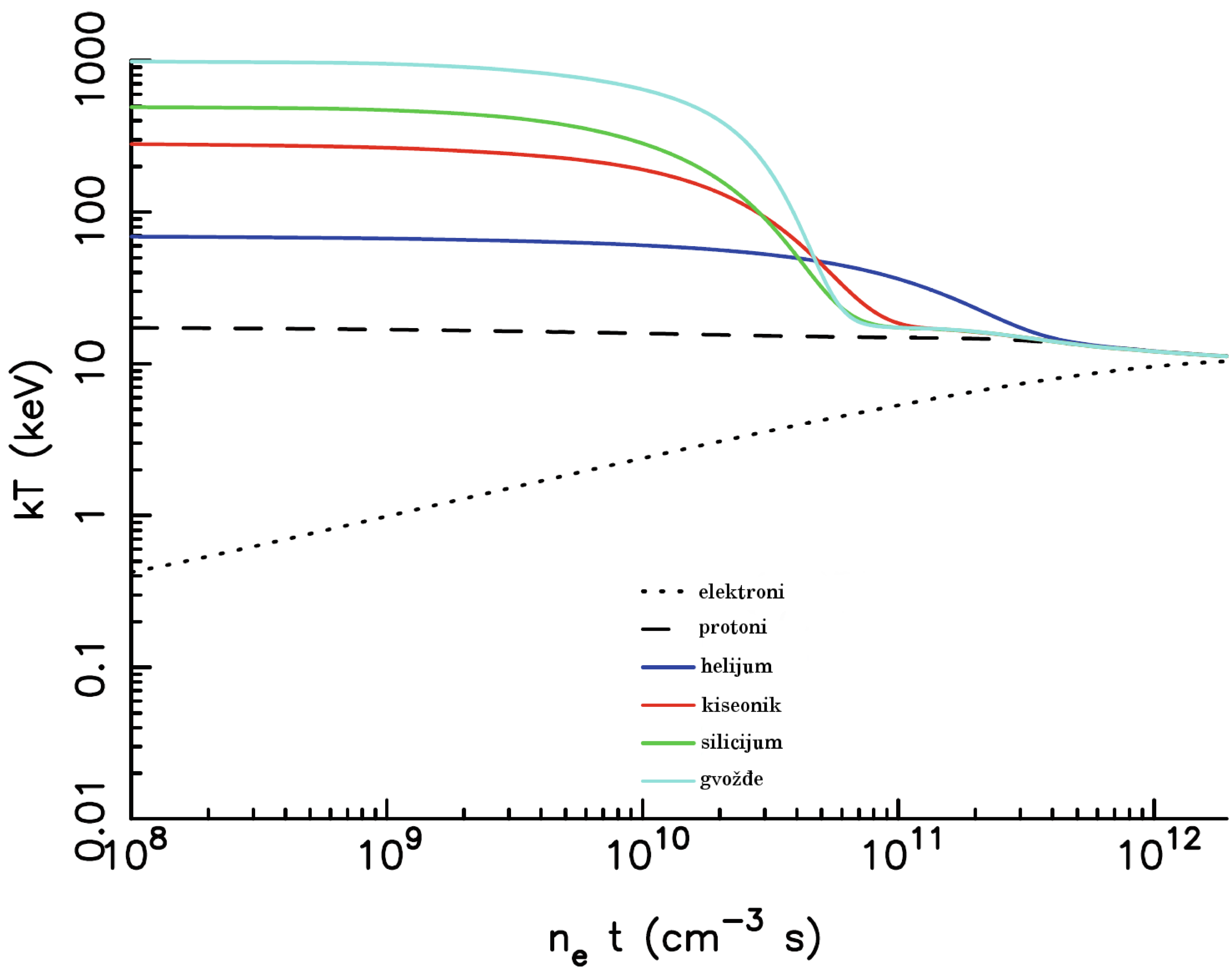
$$f_\alpha^{\mathrm{M}}(v) = n_\alpha\left(\frac{m_\alpha}{2\pi k T}\right)^{3/2} e^{-\frac{m_\alpha v^2}{2kT}}$$

$$f_\alpha^{\mathrm{M}}(v) = n_\alpha\left(\frac{m_\alpha}{2\pi k T_\alpha}\right)^{3/2} e^{-\frac{m_\alpha v^2}{2kT_\alpha}}$$

$$f_\alpha^{\mathrm{bM}}(\vec{v}) = n_\alpha\left(\frac{m_\alpha}{2\pi k}\right)^{3/2} \frac{1}{T_{\alpha\perp}\sqrt{T_{\alpha\parallel}}} e^{-\frac{m_\alpha v_\perp^2}{2kT_{\alpha\perp}}-\frac{m_\alpha v_\parallel^2}{2kT_{\alpha\parallel}}}$$

$$f_\alpha^{\mathrm{M}\mathcal{E}_{\mathrm{pot}}}(\vec{r},v) = n_\alpha(\vec{r})\left(\frac{m_\alpha}{2\pi k T}\right)^{3/2} e^{-\frac{m_\alpha v^2}{2kT}}, n_\alpha(\vec{r}) = n_{\alpha 0} e^{-\frac{\mathcal{E}_{\mathrm{pot}}(\vec{r})}{kT}}$$

Temperature equilibration in supernova remnants



$$n_\alpha(\vec{r},t) = \int_{V_{\vec{v}}} f_\alpha(\vec{r},\vec{v},t) d^3\vec{v} \qquad \int_{V_{\vec{v}}} d^3\vec{v} \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dv_x dv_y dv_z$$

$$\rho_\alpha = m_\alpha n_\alpha$$

$$\langle \mathcal{G}(\vec{r},\vec{v},t)\rangle_\alpha(\vec{r},t) = \frac{1}{n_\alpha(\vec{r},t)} \int_{V_{\vec{v}}} \mathcal{G}(\vec{r},\vec{v},t) f_\alpha(\vec{r},\vec{v},t) d^3\vec{v}$$

$$\vec{u}_\alpha(\vec{r},t) \equiv \langle \vec{v}\rangle_\alpha(\vec{r},t) = \frac{1}{n_\alpha(\vec{r},t)} \int_{V_{\vec{v}}} \vec{v} f_\alpha(\vec{r},\vec{v},t) d^3\vec{v}$$

$$\vec{w}_\alpha = \vec{v} - \vec{u}_\alpha$$

$$T_\alpha(\vec{r},t) = \frac{m_\alpha}{3k n_\alpha(\vec{r},t)} \int_{V_{\vec{v}}} w_\alpha^2 f_\alpha(\vec{r},\vec{v},t) d^3\vec{v} = \frac{m_\alpha}{3k}\langle w_\alpha^2\rangle_\alpha$$

$$\hat{\mathrm{P}}_\alpha = m_\alpha \int_{V_{\vec{v}}} \vec{w}_\alpha \vec{w}_\alpha f_\alpha(\vec{r},\vec{v},t) d^3\vec{v} = \rho_\alpha \langle \vec{w}_\alpha \vec{w}_\alpha\rangle_\alpha \qquad \hat{\mathrm{P}}_\alpha = p_\alpha \hat{\mathrm{I}} + \hat{\pi}$$

$$p_\alpha = \frac{1}{3}\mathrm{Tr}\hat{\mathrm{P}}_\alpha = \frac{1}{3}\rho_\alpha \langle w_\alpha^2\rangle_\alpha \qquad p_\alpha = n_\alpha k T_\alpha$$

$$\frac{\partial f_\alpha(\vec{r},\vec{v},t)}{\partial t} + \vec{v}\cdot\nabla f_\alpha(\vec{r},\vec{v},t) + \frac{q_\alpha}{m_\alpha}\left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot\nabla_v f_\alpha(\vec{r},\vec{v},t) = \mathcal{I}_\alpha$$

long range, smooth, interactions – average internal fields

short range interactions: particle-particle, particle-wave, wave-wave

$$\nabla\cdot\vec{E}(\vec{r},t) = \frac{\rho^{\mathrm{el}}(\vec{r},t)}{\varepsilon_0}, \quad \nabla\times\vec{E}(\vec{r},t) = -\frac{\partial\vec{B}(\vec{r},t)}{\partial t},$$

$$\nabla\cdot\vec{B}(\vec{r},t) = 0, \quad \nabla\times\vec{B}(\vec{r},t) = \mu_0\vec{j}(\vec{r},t) + \frac{1}{c^2}\frac{\partial\vec{E}(\vec{r},t)}{\partial t},$$

$$\rho^{\mathrm{el}}(\vec{r},t) = \sum_\alpha q_\alpha\int_{V_{\vec{v}}} f_\alpha(\vec{r},\vec{v},t)d^3\vec{v}, \quad \vec{j}(\vec{r},t) = \sum_\alpha q_\alpha\int_{V_{\vec{v}}} \vec{v}f_\alpha(\vec{r},\vec{v},t)d^3\vec{v}.$$

Kinetic theory:

Plasmas out off equilibrium

Plasma waves and instabilities on the finest scale, when velocity structure is needed

Wave-particle and wave-wave interaction

Charged particle acceleration

Fluid models

Multicomponent models

$$\tau_{\mathrm{HD}} \gg \tau_{\mathrm{eq},\alpha}$$

Two-fluid model

Magnetohydrodynamics

$$\tau_{\mathrm{HD}} \gg \tau_{\mathrm{eq}}$$

$$\omega_{\mathrm{coll,ee}} \sim \omega_{\mathrm{coll,ep}} \sim \omega^{\mathcal{E}}_{\mathrm{coll,ee}} \sim \sqrt{m_{\mathrm{p}}/m_{\mathrm{e}}}\,\omega_{\mathrm{coll,pp}} \sim \sqrt{m_{\mathrm{p}}/m_{\mathrm{e}}}\,\omega^{\mathcal{E}}_{\mathrm{coll,pp}} \sim (m_{\mathrm{p}}/m_{\mathrm{e}})\,\omega_{\mathrm{coll,pe}} \sim (m_{\mathrm{p}}/m_{\mathrm{e}})\,\omega^{\mathcal{E}}_{\mathrm{coll,pe}} \sim (m_{\mathrm{p}}/m_{\mathrm{e}})\,\omega^{\mathcal{E}}_{\mathrm{coll,ep}}$$

Fluid models

Phenomenology (define fluid(s) with certain characteristics)

Kinetic theory → average over velocity space → fluid models

Fluid dynamics + Electrodynamics

Lagrange vs. Euler view

field theory

velocity field
density field
pressure field
temperature field
magnetic field
…

Neutral ideal fluids

equation of motion

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = \vec{f} - \frac{1}{\rho}\nabla p$$

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla\cdot(\rho\vec{u}\vec{u}) = \rho\vec{f} - \nabla p$$

continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$

$$\frac{\partial \rho}{\partial t} + (\vec{u}\cdot\nabla)\rho + \rho\nabla\cdot\vec{u} = 0$$

adiabatic processes

$$\frac{\partial e}{\partial t} + (\vec{u}\cdot\nabla)e + (\gamma_{\mathrm{g}} - 1)e\nabla\cdot\vec{u} = 0$$

$$\frac{\partial p}{\partial t} + (\vec{u}\cdot\nabla)p + \gamma_{\mathrm{g}}p\nabla\cdot\vec{u} = 0$$

internal energy per unit mass

$$e = c_V T = \frac{1}{\gamma_{\mathrm{g}} - 1}\frac{p}{\rho}$$

ideal gas

Ideal fluid + Electrodynamics = Ideal MHD

Dissipative MHD

Different kinetic and fluid models:

different length/time scales – different processes

Hybrid models: kinetic + fluid