АГ (ГП) - КОЛОКВИУМ — I ГРУПА

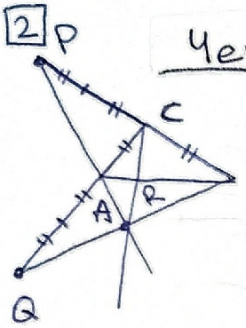
1 Т тежиште ABCD, S тежиште ABC

$\Rightarrow \vec{AT} = \frac{1}{4}(\underbrace{\vec{AA}}_{=\vec{0}} + \vec{AB} + \vec{AC} + \vec{AD})$, $\vec{AS} = \frac{1}{3}(\vec{AA} + \vec{AB} + \vec{AC})$

$$\vec{ST} = \vec{SA} + \vec{AT} = -\frac{1}{3}\vec{AB} - \frac{1}{3}\vec{AC} + \frac{1}{4}\vec{AB} + \frac{1}{4}\vec{AC} + \frac{1}{4}\vec{AD}$$

$$= \boxed{-\frac{1}{12}}\vec{AB} + \boxed{\frac{-1}{12}}\vec{AC} + \boxed{\frac{1}{4}}\vec{AD}$$

2



Чева $\frac{\vec{AR}}{\vec{RB}} \cdot \frac{\vec{BP}}{\vec{PC}} \cdot \frac{\vec{CQ}}{\vec{QA}} = 1$ $\quad \vec{CB} = -\frac{1}{3}\vec{BP}$, $\vec{CA} = -\frac{1}{3}\vec{QC}$

$\vec{BP} = \vec{BC} + \vec{CP} = \frac{1}{3}\vec{BP} - \vec{PC} \Rightarrow \frac{2}{3}\vec{BP} = -\vec{PC} \Rightarrow \frac{\vec{BP}}{\vec{PC}} = \boxed{-\frac{3}{2}}$

$\vec{CQ} = \vec{CA} + \vec{AQ} = -\frac{1}{3}\vec{QC} - \vec{QA} \Rightarrow \frac{2}{3}\vec{CQ} = -\vec{QA} \Rightarrow \frac{\vec{CQ}}{\vec{QA}} = \boxed{-\frac{3}{2}}$

$\frac{1}{3}\vec{CQ}$

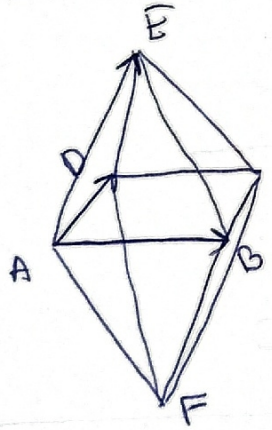
$$\Rightarrow \frac{\vec{AR}}{\vec{RB}} \cdot \frac{-3}{2} \cdot \frac{-3}{2} = 1 \Rightarrow \frac{\vec{AR}}{\vec{RB}} = \boxed{\frac{4}{9}}$$

3 $3 = \vec{a} \circ (3\vec{a} - 2\vec{b}) = 3\|\vec{a}\|^2 - 2\|\vec{a}\| \cdot \|\vec{b}\| \cos \sphericalangle(\vec{a}, \vec{b}) = 3\|\vec{a}\|^2 - 8\|\vec{a}\|$

$3\|\vec{a}\|^2 - 8\|\vec{a}\| - 3 = 0 \Rightarrow \|\vec{a}\|_{1/2} = \frac{8 \pm \sqrt{64+36}}{6} = \frac{8 \pm 10}{6}$ → $-1/3$, → $\boxed{3}$

4 $V_{\text{тетр. ABCD}} = \frac{1}{6}\left|\left[\vec{AB}, \vec{AC}, \vec{AD}\right]\right| = \frac{1}{6}\left|\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -9 \\ 1 & 0 & 1 \end{vmatrix}\right| = \frac{1}{6}\left|\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -11 \\ 1 & 0 & 0 \end{vmatrix}\right| = \frac{1}{6}|-24| = \boxed{4}$

5



$[\epsilon]_{Ae} = [\vec{AC}]_e = (1,1,0)$ $\quad -1 \downarrow \quad +$

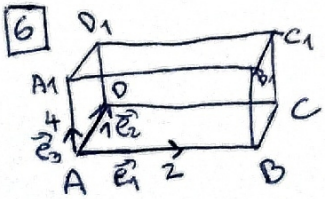
$[\vec{f_1}]_e = [\vec{CE}]_e = [\vec{CA} + \vec{AE}]_e = (-1,-1,1)$

$[\vec{f_2}]_e = [\vec{CD}]_e = [-\vec{AB}]_e = (-1,0,0)$

$[\vec{f_3}]_e = [\vec{CF}]_e = [-\vec{AE}]_e = (0,0,-1)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

6



$A(0,0,0)$ $\quad B(2,0,0)$

$C(2,1,0)$ $\quad D_1(0,1,4)$

$$d(AC, BD_1) = \frac{|[\vec{AC}, \vec{BD_1}, \vec{AB}]|}{\|\vec{AC} \times \vec{BD_1}\|} = \frac{1\begin{vmatrix} 2 & 1 & 0 \\ -2 & 1 & 4 \\ 2 & 0 & 0 \end{vmatrix}\|}{\left\|\begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ -2 & 1 & 4 \end{vmatrix}\right\|} = \frac{|2 \cdot 4|}{\sqrt{4^2 + (-8)^2 + 4^2}}$$

$$= \frac{8}{\sqrt{96}} = \frac{8}{4\sqrt{6}} = \frac{2}{\sqrt{6}} \overset{\cdot\sqrt{6}}{=} \frac{\sqrt{6}}{3}$$

7 $A(r_A, \phi_A)$, $B(r_B, \phi_B)$

$$d(A,B) = \sqrt{r_A^2 + r_B^2 - 2r_A r_B \cos(\phi_A - \phi_B)} \quad (*)$$

$A(12, \frac{\pi}{6})$, $B(5, \frac{2\pi}{3})$

$\Rightarrow d(A,B) = \sqrt{12^2 + 5^2 - 2 \cdot 12 \cdot 5 \cos\left(\frac{\pi}{6} - \frac{2\pi}{3}\right)} = \sqrt{169} = \boxed{13}$

$\cos\left(-\frac{\pi}{2}\right) = 0$

8 $a \cap b$: $x + y + 2 = 0$ / $-2x - y - 5 = 0$ $+$

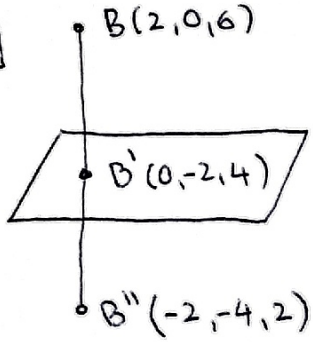
$-x - 3 = 0 \Rightarrow \boxed{x = -3}$, $\boxed{y = 1}$

$c: 3x + y + 2023 = 0$

$\vec{n}_c = (3,1) = \vec{p}$

$\Rightarrow p: \frac{x + \boxed{3}}{3} = \frac{y - \boxed{1}}{\boxed{1}}$

9



$\alpha: x + y + z - 2 = 0$

1) $h \perp \alpha$, $B \in h \Rightarrow h: \begin{cases} x = t + 2 \\ y = t \\ z = t + 6 \end{cases}$, $t \in \mathbb{R}$

2) $B' = h \cap \alpha: t + 2 + t + t + 6 - 2 = 0 \Rightarrow t = -2$

$B'(0,-2,4)$ $\quad$ $B''(-2,-4,2)$

$\boxed{C(0,-2,4)}$

10 $p: \begin{cases} x = 1 \\ y - 2z - 2 = 0 \end{cases} \Leftrightarrow p: \begin{cases} x = 1 \\ y = 2t + 2 \\ z = t \end{cases}$, $t \in \mathbb{R} \Rightarrow \vec{p} = (0,2,1)$

$P(1,2,0) \in p \subset \beta$ ✓

$\alpha: x + y - z + 1 = 0$

$p \subset \beta \Rightarrow \vec{n}_\beta \perp \vec{p}$, $\beta \perp \alpha \Rightarrow \vec{n}_\beta \perp \vec{n}_\alpha$

$$\vec{n}_\beta = \vec{p} \times \vec{n}_\alpha = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 1 & 1 & -1 \end{vmatrix} = (-3,1,-2) \parallel \boxed{(3,-1,2) = \vec{n}_\beta}$$

$P(1,2,0) \in \beta$

$\vec{n}_\beta = (3,-1,2)$ $\Rightarrow \beta: 3(x-1) - 1(y-2) + 2(z-0) = 0$

$\beta: \boxed{3}x + \boxed{-1}y + 2z + \boxed{-1} = 0$

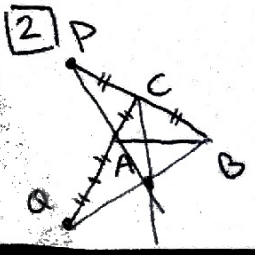
АГ (Г1) – КОЛОКВИУМ | **II ГРУПА**

1 T тежиште $ABCD$, S теж. ABD

$\Rightarrow \vec{AT} = \frac{1}{4}(\vec{AB} + \vec{AC} + \vec{AD})$, $\vec{AS} = \frac{1}{3}(\vec{AB} + \vec{AD})$

$\vec{ST} = \vec{SA} + \vec{AT} = \boxed{-\frac{1}{12}}\vec{AB} + \boxed{\frac{1}{4}}\vec{AC} + \boxed{\frac{-1}{12}}\vec{AD}$

2



$\frac{\vec{AR}}{\vec{RB}} \cdot \frac{\vec{BP}}{\vec{PC}} \cdot \frac{\vec{CQ}}{\vec{QA}} = 1$ $\quad$ $\vec{CB} = -\frac{1}{2}\vec{BP}$, $\vec{CA} = -\frac{1}{3}\vec{QC}$

-2 $\quad$ $-\frac{3}{2}$

$\Rightarrow 3\frac{\vec{AR}}{\vec{RB}} = 1 \Rightarrow \frac{\vec{AR}}{\vec{RB}} = \boxed{\frac{1}{3}}$

може и са снимке или као прва група

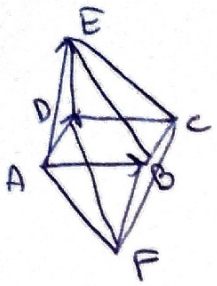
3 $6 = \vec{a} \circ (2\vec{a} - \vec{b}) = 2\|\vec{a}\|^2 - \|\vec{a}\| \cdot \|\vec{b}\| \cos\angle(\vec{a},\vec{b}) = 2\|\vec{a}\|^2 - \|\vec{a}\|$ ($\|\vec{b}\| = 2$, $\|\vec{a}\| = 2$)

$\Rightarrow 2\|\vec{a}\|^2 - \|\vec{a}\| - 6 = 0 \Rightarrow \|\vec{a}\|_{1/2} = \frac{1 \pm \sqrt{1+48}}{4} = \frac{1 \pm 7}{4}$ → $-3/2$, → $\boxed{2}$

4 $V = \frac{1}{6}\left|\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -9 \\ 2 & 0 & 2 \end{vmatrix}\right| = \frac{1}{6} \cdot 48 = \boxed{8}$

→ 2 пута детерминанта из I групе

5



$[\vec{c}] = [\vec{AC}]_e = (1,1,0)$

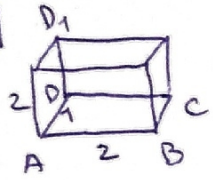
$[\vec{f_1}] = [\vec{CB}]_e = [-\vec{AD}]_e = (0,-1,0)$

$[\vec{f_2}] = [\vec{CE}]_e = [\vec{CA} + \vec{AE}] = (-1,-1,1)$

$[\vec{f_3}] = [\vec{CF}]_e = [-\vec{AE}]_e = (0,0,-1)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

6



$A(0,0,0)$ $B(2,0,0)$

$C(2,1,0)$ $D_1(0,1,2)$

$$d(AC, BD_1) = \frac{|[\vec{AC}, \vec{BD_1}, \vec{AB}]|}{\|\vec{AC} \times \vec{BD_1}\|} = \frac{1\left|\begin{vmatrix} 2 & 1 & 0 \\ -2 & 1 & 2 \\ 2 & 0 & 0 \end{vmatrix}\right|}{\left\|\begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ -2 & 1 & 2 \end{vmatrix}\right\|} = \frac{|2 \cdot 2|}{\sqrt{2^2 + (-4)^2 + 4^2}} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

7 $A(4, \frac{\pi}{3})$, $B(3, \frac{5\pi}{6})$

$$d(A,B) = \sqrt{4^2 + 3^2 - 2 \cdot 4 \cdot 3 \underbrace{\cos\left(\frac{\pi}{3} - \frac{5\pi}{6}\right)}_{=0}} = \sqrt{25} = \boxed{5}$$

8 $a: x - y + 2 = 0$

$b: -2x + y + 1 = 0$ +

$-x + 3 = 0 \Rightarrow \boxed{x = 3}$, $\boxed{y = 5}$

$c: 3x - y + 2023 = 0$

$\vec{n}_c = (3,-1) = \vec{p}$

$$\Rightarrow \boxed{p: \frac{x + \boxed{-3}}{3} = \frac{y - \boxed{5}}{\boxed{1}}}$$

9 из групе I $\Rightarrow \boxed{C = B''(-2,-4,2)}$

10 $p: \begin{cases} x - y + z + 1 = 0 \\ x + y - z + 1 = 0 \end{cases}$ +

$P: \begin{matrix} x = -1 \\ -y + z = 0 \end{matrix} \Rightarrow P: \begin{matrix} x = -1 \\ y = t \\ z = t \end{matrix}, t \in \mathbb{R} \Rightarrow \boxed{\begin{matrix} P(-1,0,0) \in p \\ \vec{p} = (0,1,1) \end{matrix}}$

$\alpha: 2x - y + 5z - 3 = 0$

$P(-1,0,0) \in p \subset \beta$ ✓

$$\vec{n}_\beta = \vec{n}_\alpha \times \vec{p} = \begin{vmatrix} i & j & k \\ 2 & -1 & 5 \\ 0 & 1 & 1 \end{vmatrix} = (-6,-2,2) \parallel (3,1,-1) = \vec{n}_\beta \checkmark$$

$\Rightarrow \beta: 3(x+1) + 1 \cdot y - 1 \cdot z = 0$

$$\boxed{\beta: \boxed{3}x + \boxed{1}y - z + \boxed{3} = 0}$$