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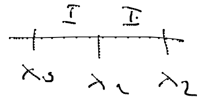
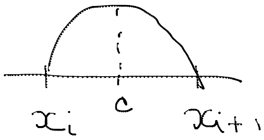
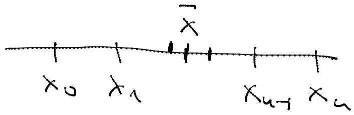




$[x_i, x_{i+1}]$, $\exists c : F'(c) = 0$



$n+2$ tačke

$\Rightarrow n+1$ interval

$\Rightarrow F'(c_i) = 0 \qquad i = 0, \dots, n \qquad F'$ ima $n+1$ nulu

$n+1 \Rightarrow n$ intervala, F' zadovoljava Rolovu teoremu

$\Rightarrow F''(x)$ ima n nula

$\vdots$

$F^{(n+1)}(x)$ ima bar 1 nulu ξ na $[y_1, y_2]$

$y_1 = \min\{\bar{x}, x_0, \dots, x_n\}$

$y_2 = \max\{\bar{x}, x_0, \dots, x_n\}$

$\Rightarrow F^{(n+1)}(\xi) = 0$

$$\underset{0}{\underset{\shortparallel}{F^{(n+1)}(\xi)}} = f^{(n+1)}(\xi) - \underbrace{L_n^{(n+1)}(\xi)}_{\substack{=0 \\ st(L_n) = n}} - K \cdot \underbrace{\omega_{n+1}^{(n+1)}(\xi)}_{(n+1)!}$$

$$0 = f^{(n+1)}(\xi) - K \cdot (n+1)!$$

$$\Rightarrow K = \frac{f^{(n+1)}(\xi)}{(n+1)!}$$

$$F(\bar{x}) = 0 = f(\bar{x}) - L_n(\bar{x}) - \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \omega_{n+1}(\bar{x})$$

$$\Rightarrow \boxed{f(\bar{x}) - L_n(\bar{x}) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \omega_{n+1}(\bar{x})}, \quad \xi \in [y_1, y_2]$$

$\boxtimes$

$$\max_{x \in [y_1, y_2]} |f^{(n+1)}(x)| = M_{n+1}$$

$$|R_n(x)| = |f(x) - L_n(x)| \le \frac{M_{n+1}}{(n+1)!} \cdot |\omega_{n+1}(x)|$$

Njutnov int. poly sa podeljenim razlikama

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$[x_{i_0}]$

$f[x_i] = f(x_i)$ PR reda 0

$[x_{i_0}, x_{i_1}]$

$$f[x_i, x_{i+1}] = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$ PR reda 1

$$f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$$ reda 2

$\vdots$

$$f[x_i, \dots, x_{i+k}] = \frac{f[x_{i+1}, \dots, x_{k+i}] - f[x_i, \dots, x_{i+k-1}]}{x_{i+k} - x_i}$$ reda k

$x_0, \dots, x_n$ nebitan redosled

(*) PR reda k se pomoću vrednosti funkcije u čvorovima kojima je određena izračunava formulom:

$$f[x_0, \dots, x_k] = \sum_{i=0}^{k} \frac{f(x_i)}{\prod\limits_{\substack{j=0 \\ j \neq i}}^{k} (x_i - x_j)}$$

Dokaz:

$k=1$: $$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_0)}{x_0 - x_1} + \frac{f(x_1)}{x_1 - x_0}$$

$$= \sum_{i=0}^{1} \frac{f(x_i)}{\prod\limits_{\substack{j=0 \\ j \neq i}}^{1} (x_i - x_j)} \quad \checkmark$$

$k \leq n-1 \overset{?}{\Rightarrow} k = n$

$$f[x_0, \dots, x_n] = \frac{\overbrace{f[x_1, \dots, x_n] - f[x_0, \dots, x_{n-1}]}^{\text{PR reda } n-1}}{x_n - x_0}$$

$$= \frac{1}{x_n - x_0} \left[\sum_{i=1}^{n} \frac{f(x_i)}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{n} (x_i - x_j)} - \sum_{i=0}^{n-1} \frac{f(x_i)}{\prod\limits_{\substack{j=0 \\ j \neq i}}^{n-1} (x_i - x_j)} \right]$$

$$= \frac{1}{x_n - x_0} \left[\frac{f(x_n)}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{n-1} (x_n - x_j)} + \sum_{i=1}^{n-1} \frac{f(x_i)}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{n} (x_i - x_j)} - \frac{f(x_0)}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{n-1} (x_0 - x_j)} - \sum_{i=1}^{n-1} \frac{f(x_i)}{\prod\limits_{\substack{j=0 \\ j \neq i}}^{n-1} (x_i - x_j)} \right]$$

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$$= \frac{1}{x_n - x_0}\left[\frac{f(x_n)}{\prod\limits_{\substack{j=1\\ j\neq i}}^{n-1}(x_n - x_j)} - \frac{f(x_0)}{\prod\limits_{\substack{j=1\\ j\neq i}}^{n-1}(x_0 - x_j)} + \sum_{i=1}^{n-1}\frac{\overbrace{(\cancel{x_i} - x_0) - (\cancel{x_i} - x_n)}^{-x_0 + x_n}}{\prod\limits_{\substack{j=0\\ j\neq i}}^{n}(x_i - x_j)}\cdot f(x_i)\right]$$

$$= \frac{f(x_n)}{\prod\limits_{\substack{j=0\\ j\neq i}}^{n-1}(x_n - x_j)} + \underbrace{\frac{f(x_0)}{\prod\limits_{\substack{j=1\\ j\neq i}}^{n}(x_0 - x_j)}} + \sum_{i=1}^{n-1}\frac{f(x_i)}{\prod\limits_{\substack{j=0\\ j\neq i}}^{n}(x_i - x_j)}$$

$$= \sum_{i=0}^{n}\frac{f(x_i)}{\prod\limits_{\substack{j=0\\ j\neq i}}^{n}(x_i - x_j)}$$

☒

1) pr lin. operator

$$(\alpha_1 f_1 + \alpha_2 f_2)[x_0, \dots, x_n] = \alpha_1 \cdot f_1[x_0, \dots, x_n] + \alpha_2 f_2[x_0, \dots, x_n]$$

2) nebitan je redosled čvorova

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Lagrange
$$\begin{cases} L_n(x) = \sum_{i=0}^{n} \dfrac{\omega_{n+1}(x)}{(x-x_i)\cdot \omega_{n+1}'(x)} \cdot f(x_i) \\ \omega_{n+1}(x) = \prod_{j=0}^{n} (x-x_j) \\ \omega_{n+1}'(x) = \sum_{i=0}^{n} \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j) \end{cases}$$

$$f(x) - L_k(x) = f(x) - \omega_{k+1}(x) \cdot \sum_{i=0}^{k} \frac{f(x_i)}{(x-x_i)\cdot \prod_{\substack{j=0\\ j\neq i}}^{k} (x_i - x_j)}$$

$$= \omega_{k+1}(x) \underbrace{\left[\frac{f(x)}{\prod_{j=0}^{k} (x-x_j)} + \underbrace{\sum_{i=0}^{k} \frac{f(x_i)}{(x-x_i)\cdot \prod_{\substack{j=0\\ j\neq i}}^{k} (x_i - x_j)}}_{= f[x_0, \dots, x_k]} \right]}_{f[x, x_0, \dots, x_k]}$$

$$= \omega_{k+1}(x) \cdot f[x, x_0, \dots, x_k]$$

$$L_n(x) = L_0(x) + \underbrace{(L_1(x) - L_0(x))}_{} + \dots + \underbrace{(L_n(x) - L_{n-1}(x))}_{}$$

L_n - ip $x_0, \dots, x_n$

$L_n(x) - L_{n-1}(x)$ - stepena n

$x_0, \dots, x_{n-1}$: $L_n(x_j) = f(x_j) = L_{n-1}(x_j)$

$$L_n(x) - L_{n-1}(x) = a_n \cdot \omega_n(x)$$

$$\omega_n(x) = \prod_{j=0}^{n-1} (x - x_j)$$

$x = x_n$:

$$L_n(x_n) - L_{n-1}(x_n) = a_n \, \omega_n(x_n)$$

$$f(x_n) - L_{n-1}(x_n) = \omega_n(x_n) \cdot f[x_0, \dots, x_n]$$

$$\Rightarrow a_n = f[x_0, \dots, x_n]$$

$$\Rightarrow L_n(x) - L_{n-1}(x) = f[x_0, \dots, x_n] \cdot \omega_n(x)$$

Njutn sa podeljenim

$$\Rightarrow \boxed{L_n(x) = f(x_0) + f[x_0, x_1] \cdot (x - x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) + \dots + f[x_0, \dots, x_n](x-x_0) \cdot \ldots \cdot (x - x_{n-1})}$$

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(*) Veza izvoda i PR

$$f(x) - L_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \omega_{n+1}(x) \qquad \text{(Lagranž)}$$

$$f(x) - L_n(x) = \omega_{n+1}(x)\, f[x, x_0, \ldots, x_n] \qquad \text{(sada)}$$

$$\Rightarrow f[x, x_0 \ldots x_n] = \frac{f^{(n+1)}(\xi)}{(n+1)!}$$

(*) $f(x) = P_n(x)$

$L_m(x) = ?$

$\Big\}\ L_m = P_n, \quad m \geq n$

$m < n : \quad L_m \neq P_n$

(*)

x	x_0	...	x_n
f	f_0	...	f_n

$\rightarrow L_n, \ N_n$

x	x_0	...	x_n	x_{n+1}
f	f_0	...	f_n	f_{n+1}

$\rightarrow L_{n+1}$ od početka

$\rightarrow N_n$ + samo 1 sabirak

	[0]	[1]	[2]	..		
x_0	f_0	$f[\ ]$	*		*	*
x_1	f_1	⋮	*		*	
⋮	⋮	*	*			
x_n	f_n	*				
x_{n+1}	f_{n+1}					

x	0	0.2	0.5
f	1	1.221403	1.648721

$f(x) = e^x$

Lagranž $L_2(x) = 0.634757\,x^2 + \ldots\ldots$

Njutn : $L_2(x) = 0.634756\,x^2 + \ldots\ldots$

greška računa