

UNM - Uvod

February 25, 2021

Problem

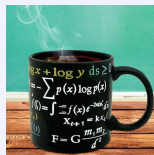
$$y = F(x), x \in A_1, y \in A_2$$



+



+



+



⇒

Problem

$$y^* = F^*(x^*), x^* \in A_1^*, y^* \in A_2^*$$



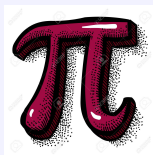
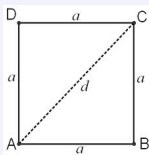
Greska???



Pojam i vrsta greške

- Neotklonjiva greška
- Greška metode
- Računska greška

Približna vrednost broja?



	fiksni zarez	pokretni zarez
zapis	$n = n_1 + n_2$	mantisa · baza ^{eksponent}
	0031 20700	3120700 002
mana	zapis malih/velikih brojeva	nije jednoznačno
	10000, 0.000001	03120700 003

Da li je greška bitna?

Primer

Izračunati $\sum_{k=1}^{10^6} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{10^6}$.

4 decimale	6 decimala
$S = \text{round}(S+1/i, 4);$	$S = \text{round}(S+1/i, 6);$
10.7509	14.356262

Primer

Rešiti $1.22x^2 + 3.32x + 2.28 = 0$ (3 cifre, 2 decimalna mesta).

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 = 11.2$$

$$4ac = 11.1$$

$$\sqrt{b^2 - 4ac} = 0.32$$

$$x_{1,2} = \{-1.24, -1.50\}$$

$$x_{1,2} = \{-1.30, -1.44\}$$

Da li je greška (zaista) bitna?

- **Marsov klimatski orbiter (Mars Climate Orbiter)**
`https://solarsystem.nasa.gov/missions/mars-climate-orbiter/in-depth/`
- **Arijana 5 (Ariane 5)**
`http://www-users.math.umn.edu/~arnold/disasters/ariane.html`
- **Patriot raketa**
`https://embeddedgurus.com/barr-code/2014/03/lethal-software-defects-patriot-missile-failure/`
- ...

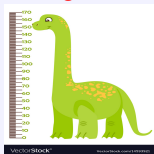
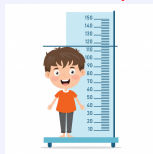
Greške broja

x - tačna vrednost, x^* - približna vrednost

Stvarna greška $\Delta = x - x^*$

Apsolutna greška $\Delta x^* = |x - x^*|$

Granica apsolutne greške $\Delta x^* = |x - x^*| \leq Ax^*$



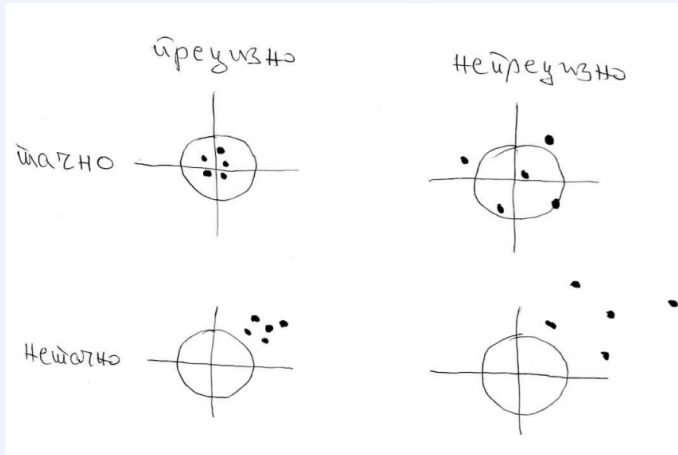
Relativna greška $\delta x^* = \frac{\Delta x^*}{|x|}$, $x \neq 0$

Granica relativne greške $\delta x^* = \frac{\Delta x^*}{|x|} \leq Rx^* = \frac{Ax^*}{|x^*|}$

Procentualna greška $100 \cdot \delta x^*$

Promilna greška $1000 \cdot \delta x^*$

Precizno ili tačno?



Značajno ili sigurno?

$$x = \pm(a_1 10^n + a_2 10^{n-1} + \dots + a_m 10^{n-m+1} + \dots),$$

$$x^* = \pm(a_1 10^n + a_2 10^{n-1} + \dots + a_m 10^{n-m+1}),$$

$$a_i \in \{0, 1, \dots, 9\}, a_1 \neq 0, n \in \mathbb{Z}, k \in \mathbb{Z}$$

Značajna cifra

1.2345

0.12340

0.00102030

0.012300

Sigurna cifra a_k (uži, širi smisao) $Ax^* \leq w \cdot 10^{n-k+1}$, $0 < w \leq 1$.

$\pi =$ 3.141592653589793...

$\pi^* = \frac{22}{7} =$ 3.142857142857143... (precizno, ali ne i tačno)

Veza grešaka broja i njegovih cifara

Apsolutna greška \rightarrow da li je cifra sigurna?

Relativna greška \rightarrow koliko cifara je sigurno?

Lema

Između broja sigurnih cifara i relativne greške postoji sledeća veza:

$$\frac{w}{(a_1 + 1) \cdot 10^k} < Rx^* \leq \frac{w}{a_1 \cdot 10^{k-1}}$$

Dokaz:

$$x^* = \pm(a_1 10^n + a_2 10^{n-1} + \dots + a_m 10^{n-m+1})$$

Ako je a_k poslednja sigurna cifra $\Rightarrow w \cdot 10^{n-k} < Ax^* \leq w \cdot 10^{n-k+1}$

$$\Rightarrow \frac{w \cdot 10^{n-k}}{|x^*|} < \frac{Ax^*}{|x^*|} \leq \frac{w \cdot 10^{n-k+1}}{|x^*|}$$

$$\Rightarrow \frac{w \cdot 10^{n-k}}{|a_1 10^n + a_2 10^{n-1} + \dots + a_m 10^{n-m+1}|} < Rx^* \leq \frac{w \cdot 10^{n-k+1}}{|a_1 10^n + a_2 10^{n-1} + \dots + a_m 10^{n-m+1}|}$$

Pošto je $0 \leq a_2 10^{n-1} + \dots + a_m 10^{n-m+1} < 10^n$

$$\Rightarrow \frac{w \cdot 10^{n-k}}{a_1 10^n + 10^n} < Rx^* \leq \frac{w \cdot 10^{n-k+1}}{a_1 10^n}$$

$$\Rightarrow \frac{w}{(a_1 + 1) \cdot 10^k} < Rx^* \leq \frac{w}{a_1 \cdot 10^{k-1}}$$

Stabilnost



Stabilan algoritam/zadatak
Nestabilan algoritam/zadatak

algoritam zadatak	stabilan	nestabilan
stabilan	✓	? ! ? • • •
nestabilan	!!! • • •	✓

Da li je stabilnost bitna?

Primer

$$I_n = \int_0^1 \frac{x^n}{x+10} dx, \quad n = 0, 1, 2, \dots$$

$$\frac{x}{x+10} = 1 - \frac{10}{x+10} \quad \setminus \cdot x^{n-1}$$

$$\frac{x^n}{x+10} = x^{n-1} - \frac{10x^{n-1}}{x+10} \quad \setminus \int$$

$$\int_0^1 \frac{x^n}{x+10} dx = \int_0^1 x^{n-1} dx - 10 \int_0^1 \frac{x^{n-1}}{x+10} dx$$

$$I_n = \frac{1}{n} - 10I_{n-1} \quad (\star)$$

$$I_{n-1} = \frac{1}{10} \left(\frac{1}{n} - I_n \right) \quad (\heartsuit)$$

	(*) (15 decimala)	(*) (5 decimala)	(♡) (5 decimala)
0	0.09531	0.09531	0.09531
1	0.04690	0.04690	0.04690
2	0.03102	0.03100	0.03102
3	0.02315	0.02333	0.02315
4	0.01846	0.01670	0.01847
5	0.01535	0.03300	0.01535
6	0.01314	-0.16333	0.01314
7	0.01148	1.77616	0.01148
8	0.01019	-17.63660	0.01019
9	0.00917	176.47711	0.00917
10	0.00833	1764.67110	0.00833
11	0.00762		0.00763
12	0.00711		0.00704
13	0.00578		0.00653
14	0.01358		0.00610
15	-0.06912		0.00571
16	0.75372		0.00537
17	-7.47839		0.00508
18	74.83944		0.00475
19	-748.34180		0.00500
20	7483.46802		0