Odrediti na $[0,1]$ polinom najbolje ravnomerne aproksimacije prvog stepena za $f(x)=\sqrt{1+x^2}$. Računati na 4 decimale.

$\boxed{\text{PRIMENA REMEZOVOG ALGORITMA}}$

KORAK 0: Biramo neke 3 tačke iz $[0,1]$, npr: $x_0=0$, $x_1=\frac{1}{2}$, $x_1=1$.
(Mogao se i formirati $T_3^{[0,1]}(x)$ pa se njegove nule uzeti za x_0,x_1,x_2)

KORAK 1: Formiramo sistem oblika $b_0+b_1\cdot x_i+b_2x_i^2+\dots+b_nx_i^n+(-1)^i\cdot E=f(x_i)$, $i=0,\dots,n+1$

$n=1$, te imamo:

$$\left.\begin{aligned} b_0+b_1\cdot x_0+E&=f(x_0)\\ b_0+b_1\cdot x_1-E&=f(x_1)\\ b_0+b_1\cdot x_2+E&=f(x_2)\end{aligned}\right\} \quad \left.\begin{aligned} b_0+b_1\cdot 0+E&=1\\ b_0+b_1\cdot\tfrac{1}{2}-E&=1.1180\\ b_0+b_1\cdot 1+E&=\sqrt{2}\end{aligned}\right\} \quad \begin{aligned} b_0&=0.9554\\ b_1&=0.4142\\ E&=0.0446\end{aligned}$$

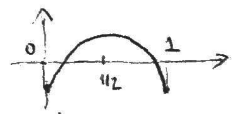
KORAK 2: $P(x)=b_0+b_1x=0.9554+0.4142x$

KORAK 3: Za svako x_i proveravamo da li je $P(x_i)-f(x_i)$ jednako $+E$ ili $-E$

x_0: $P(x_0)-f(x_0)=0.9554-1=-0.0446=-E \Rightarrow$ tražimo min f-je $P(x)-f(x)$ u okolini tačke $x_0=0$

$$\left.\begin{aligned} F(x)&=P(x)-f(x)=0.9554+0.4142x-\sqrt{1+x^2}\\ F'(x)&=0.4142-\frac{x}{\sqrt{1+x^2}}\\ F''(x)&=-\frac{1}{(1+x^2)\sqrt{1+x^2}}<0\end{aligned}\right\}$$

$F(x)$ je konkavna



svoj min u okolini nule dostiže na $[0,1]$ na rubu tj. $\overline{x_0}=0$

x_1: $P(x_1)-f(x_1)=+E \Rightarrow$ tražimo max f-je $F(x)$ u okolini $x_1=\frac{1}{2}$

$F'(x)=0.4142-\frac{x}{\sqrt{1+x^2}}=0 \Longrightarrow$ (nekom metodom sa UNM) $\overline{x_1}=0.4551$

x_2: $P(x_2)-f(x_2)=-E \Rightarrow$ tražimo min f-je $F(x)$ u okolini $x_2=1$

Opet zbog konkavnosti, svoj min dostiže za $\overline{x_2}=1$

Računamo:
$$\left.\begin{aligned} z_0&=P(\overline{x_0})-f(\overline{x_0})=-0.0446\\ z_1&=P(\overline{x_1})-f(\overline{x_1})=0.0452\\ z_2&=P(\overline{x_2})-f(\overline{x_2})=-0.0446\end{aligned}\right\}$$

nije se poklopilo modulo na 4 decimale (ovo treba da bude $E_n(f)=L$)

$\max_{i=0,1,2}|z_i|-\min_{i=0,1,2}|z_i|=0.0006>10^{-4}$

KORAK 4: $x_0=\overline{x_0}$, $x_1=\overline{x_1}$, $x_2=\overline{x_2}$

Idemo na KORAK 1!

DRUGA ITERACIJA

KORAK 1:

$$\left.\begin{array}{l} b_0 + b_1 \cdot \cancel{x_0} + E = f(x_0) \\ b_0 + b_1 \cdot x_1 - E = f(x_1) \\ b_0 + b_1 \cdot x_2 + E = f(x_1) \end{array}\right\} \quad \begin{array}{l} b_0 + E = 1 \\ b_0 + b_1 \cdot 0.4551 - E = 1.0987 \\ b_0 + b_1 \cdot 1 + E = \sqrt{2} \end{array} \quad \left.\right\} \quad \begin{array}{l} b_0 = 0.9551 \\ b_1 = 0.4142 \\ E = 0.0449 \end{array}$$

KORAK 2: $P(x) = b_0 + b_1 x = 0.9551 + 0.4142x$

KORAK 3:

x_0: tražimo min $\overline{x_0} = 0$ (isto kao u prvoj iteraciji)

x_1: tražimo max $\overline{x_1} = 0.4551$ (isto kao u prvoj iteraciji, $F'(x)$ je isto)

x_2: tražimo min $\overline{x_2} = 1$ (-||-)

$$\left.\begin{array}{l} z_0 = p(\overline{x_0}) - f(\overline{x_0}) = -0.0449 \\ z_1 = p(\overline{x_1}) - f(\overline{x_1}) = 0.0449 \\ z_2 = p(\overline{x_2}) - f(\overline{x_2}) = -0.0449 \end{array}\right\}$$

(moduli) se poklapaju na 4 decimale

kriterijum zaustavljanja ispunjen

$\Downarrow$

$$\left.\begin{array}{l} x_0 = 0 \\ x_1 = 0.4551 \\ x_2 = 1 \end{array}\right\}$$ su tačke Čebiševljeve alt.

$Q_0(x) = p(x) = 0.9551 + 0.4142x$

$E_n(f) = E = 0.0449$