b) $$T_n(x) = \frac{(x-\sqrt{x^2-1})^4 + (x+\sqrt{x^2-1})^4}{2}$$

$$T_n(x) = \cos(n \arccos x)$$

$$(f, T_{2k}) = \int_{-1}^{1} \overset{p(x)}{\cdot}\,|x| \cdot T_{2k}(x)\,dx = 2\int_0^1 p(x)\cdot x \cdot T_{2k}(x)\,dx$$

$$(f, T_{2k+1}) = 0$$

$$c_0 = \frac{(f,T_0)}{(T_0,T_0)} = 2\cdot\underbrace{\frac{1}{\pi}}\cdot\int_0^1 \frac{x}{\sqrt{1-x^2}}\cdot\underbrace{T_0}_{=1}\,dx \overset{t=1-x^2}{=} \dots = 0{,}6366$$

$$(T_i,T_j) = \begin{cases} 0, & i\neq j \\ \pi, & i=j=0 \\ \pi/2, & i=j\neq 0 \end{cases}$$

$$c_1 = 0$$

$$c_2 = \frac{(f,T_2)}{(T_2,T_2)} = 2\cdot\frac{2}{\pi}\int_0^1 \frac{x}{\sqrt{1-x^2}}\cdot T_2\,dx = \dots = 0{,}4244$$

$$T_2 = 2x^2-1$$

$$c_3 = 0$$

$$c_4 = \frac{(f,T_4)}{(T_4,T_4)} = 2\cdot\frac{2}{\pi}\int_0^1 \frac{x}{\sqrt{1-x^2}}\, T_4\,dx = \dots = -0{,}0849$$

$$T_4 = 8x^4 - 8x^2 + 1$$

$$c_5 = 0$$

$$Q(x) = c_0\cdot T_0 + \cancelto{0}{c_1T_1} + c_2T_2 + \cancelto{0}{c_3T_3} + c_4T_4 + \cancelto{0}{c_5T_5}$$

$$= 0{,}6366\cdot 1 + 0{,}424\cdot(2x^2-1) - 0{,}0849\cdot(8x^4-8x^2+1)$$

$$= -0{,}6792x^4 + 1{,}5280x^2 + 0{,}1273$$

$$[0,1] \to [a,b]$$

$$(g_k,g_j) = \dots = \frac{x^{k+j+1}}{k+j+1}\Big|_a^b = \frac{b^{k+j+1}}{k+j+1} - \frac{a^{k+j+1}}{k+j+1}$$

$$\underbrace{\text{fun}}_{\text{string}}\ \underbrace{.*\,x\,\hat{}}_{\text{string}}\ \underbrace{j}_{\text{broj}}$$

$$L_i(x) = \frac{2i-1}{i} \cdot x \cdot L_{i-1}(x) - \frac{i-1}{i} \cdot L_{i-2}(x), \quad L_0(x) = 1, \; L_1(x) = x$$

$$T_i(x) = 2x\, T_{i-1}(x) - T_{i-2}(x), \quad T_0(x) = 1, \; T_1(x) = x$$

$P_k(x)$, $k = 0, 1, \ldots$, prebrojivo mnogo ortonormiran sistem

$$f \sim \sum_{i=0}^{\infty} c_i p_i(x), \quad c_i = \int_a^b p(x) f(x) p_i(x)\, dx$$

$\{P_k(x)\}$: $1, x, x^2, \ldots, x^4, \ldots$ potpun $\Rightarrow$ Furijeov red konvergira ka $f(x)$

$$\lim_{n \to \infty} \int_a^b p(x) \left[f(x) - \sum_{i=0}^{n} c_i p_i(x) \right]^2 dx = 0$$

f periodicna $\rightsquigarrow$ φ periodicna

$\{g_k\}$: $1, \sin x, \cos x, \sin 2x, \cos 2x, \ldots$ biće ortogonalan na $[-\pi, \pi]$

$$(\sin kx, \sin mx) = \begin{cases} k = m \neq 0: \displaystyle\int_{-\pi}^{\pi} \sin^2 kx\, dx = \int_{-\pi}^{\pi} \frac{1 - \cos 2kx}{2}\, dx = \ldots = \pi \\ k \neq m: \displaystyle\int_{-\pi}^{\pi} \sin(kx) \cdot \sin(mx)\, dx = \frac{1}{2} \int_{-\pi}^{\pi} (\cos(k+m)x - \cos(k-m)x)\, dx \\ = 0 \end{cases}$$

$$(\cos kx, \cos mx) = \begin{cases} k = m = 0: \displaystyle\int_{-\pi}^{\pi} 1 \cdot 1\, dx = 2\pi \\ k = m \neq 0: \displaystyle\int_{-\pi}^{\pi} \cos^2 kx\, dx = \int_{-\pi}^{\pi} \frac{1 + \cos 2kx}{2}\, dx \\ = \pi \\ k \neq m: \displaystyle\int_{-\pi}^{\pi} \cos kx \cdot \cos mx\, dx = \frac{1}{2} \int_{-\pi}^{\pi} (\cos(k+m)x + \cos(k-m)x)\, dx \\ = 0 \end{cases}$$

$$(\sin kx, \cos mx) = \begin{cases} k = m \neq 0: \displaystyle\int_{-\pi}^{\pi} \sin kx \cdot \cos kx\, dx = \frac{1}{2} \int_{-\pi}^{\pi} \sin 2kx\, dx = 0 \\ k \neq m: \displaystyle\int_{-\pi}^{\pi} \sin kx \cdot \cos mx\, dx = \frac{1}{2} \int_{-\pi}^{\pi} [\sin(k+m)x + \sin(k-m)x]\, dx \\ = \ldots = 0 \end{cases}$$

$$Q(x) = \sum \underset{\sim}{c_i} g_i$$

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$$Q(x) = \underbrace{\frac{a_0}{2} + \sum_{k=1}^{\infty} (\underset{\sim}{a_k} \cdot \cos kx + \underset{\sim}{b_k} \sin kx)}$$

trigonometrijski Furijeov red

$$\frac{(f, g_k)}{(g_k, g_k)}$$

$$\frac{a_0}{2} = \frac{(f, 1)}{(1, 1)} = \frac{\int_{-\pi}^{\pi} f \cdot 1 \, dx}{\int_{-\pi}^{\pi} 1 \cdot 1 \, dx} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \, dx$$

$$a_k = \frac{(f, \cos kx)}{(\cos kx, \cos kx)} = \frac{1}{\pi} (f, \cos kx) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos kx \, dx$$

$$b_k = \frac{(f, \sin kx)}{(\sin kx, \sin kx)} = \frac{\int_{-\pi}^{\pi} f \cdot \sin kx \, dx}{\int_{-\pi}^{\pi} \sin^2 kx} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx \, dx$$

$\{g_k\}$ potpun $\Rightarrow$ Red (F) ka $f(x)$

$$\lim_{n \to \infty} \int_{-\pi}^{\pi} \left(f - \frac{a_0}{2} - \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx)\right)^2 dx = 0$$