

sa prošlog časa:

$$(*) \quad S''(x) = M_i \cdot \frac{x_{i+1} - x}{h_{i+1}} + M_{i+1} \frac{x - x_i}{h_{i+1}}$$

$$(**) \quad S'(x) = -M_i \frac{(x_{i+1} - x)^2}{2h_{i+1}} + M_{i+1} \frac{(x - x_i)^2}{2h_{i+1}} + A_i$$

$$A_i = \frac{f(x_{i+1}) - f(x_i)}{h_{i+1}} - \frac{h_{i+1}}{6} (M_{i+1} - M_i)$$

$$x \in [x_i, x_{i+1}]: \quad S(x) = d_i + \beta_i (x - x_i) + \gamma_i (x - x_i)^2 + \delta_i (x - x_i)^3$$

$$\left. \begin{array}{l} S(x_i) = f(x_i) \quad (\text{u skladu sa def.}) \\ S(x_i) = d_i \end{array} \right\} \quad \boxed{d_i = f(x_i)}$$

$$S'(x) = \beta_i + 2\gamma_i (x - x_i) + 3\delta_i (x - x_i)^2$$

$$S'(x_i) = \beta_i$$

$$(**): \quad S'(x_i) = -M_i \cdot \frac{h_{i+1}}{2} + 0 + \underbrace{\frac{f(x_{i+1}) - f(x_i)}{h_{i+1}} - \frac{h_{i+1}}{6} (M_{i+1} - M_i)}_{A_i}$$

$$= \frac{f(x_{i+1}) - f(x_i)}{h_{i+1}} - \frac{h_{i+1}}{6} (2M_i + M_{i+1})$$

$$\Rightarrow \boxed{\beta_i = \frac{f(x_{i+1}) - f(x_i)}{h_{i+1}} - \frac{h_{i+1}}{6} (2M_i + M_{i+1})}$$

$$S''(x) = 2\gamma + 6\delta_i (x - x_i)$$

$$S''(x_i) = 2\gamma$$

$$S''(x_i) = M_i$$

$$\left. \begin{array}{l} S''(x_i) = 2\gamma \\ S''(x_i) = M_i \end{array} \right\} \quad \boxed{\gamma_i = \frac{1}{2} M_i}$$

$$S'''(x) = 6\delta_i$$

$$(*): \quad S'''(x) = -\frac{M_i}{h_{i+1}} + \frac{M_{i+1}}{h_{i+1}}$$

$$\left. \begin{array}{l} S'''(x) = 6\delta_i \\ S'''(x) = -\frac{M_i}{h_{i+1}} + \frac{M_{i+1}}{h_{i+1}} \end{array} \right\} \quad \boxed{\delta_i = \frac{1}{6h_{i+1}} (M_{i+1} - M_i)}$$

$$\lim_{x \rightarrow x_i^-} S'(x) = \lim_{x \rightarrow x_i^-} S'(x) \quad , i=1, \dots, n-1$$

$$x \in [x_{i-1}, x_i] :$$

$$(*) : S'(x) = -M_{i-1} \frac{(x_i - x)^2}{2\ell_i} + M_i \frac{(x - x_{i-1})^2}{2\ell_i} + \frac{f(x_i) - f(x_{i-1})}{\ell_i} - \frac{\ell_i}{6} (M_i - M_{i-1})$$

$$x \in [x_i, x_{i+1}]$$

$$(*) : S'(x) = -M_i \frac{(x_{i+1} - x)^2}{2\ell_{i+1}} + M_{i+1} \frac{(x - x_i)^2}{2\ell_{i+1}} + \frac{f(x_{i+1}) - f(x_i)}{\ell_{i+1}} - \frac{\ell_{i+1}}{6} (M_{i+1} - M_i)$$

$$\begin{cases} \lim_{x \rightarrow x_i^-} S'(x) = 0 + M_i \cdot \frac{\ell_i^2}{2\ell_i} + \frac{f(x_i) - f(x_{i-1})}{\ell_i} - \frac{\ell_i}{6} (M_i - M_{i-1}) \\ \lim_{x \rightarrow x_i^+} S'(x) = -M_i \frac{\ell_{i+1}^2}{2\ell_{i+1}} + 0 + \frac{f(x_{i+1}) - f(x_i)}{\ell_{i+1}} - \frac{\ell_{i+1}}{6} (M_{i+1} - M_i) \end{cases}$$

$$\frac{\ell_i}{6} M_{i-1} + \left(\frac{\ell_i}{2} - \frac{\ell_i}{6} + \frac{\ell_{i+1}}{2} - \frac{\ell_{i+1}}{6} \right) M_i + \frac{\ell_{i+1}}{6} M_{i+1} =$$

$$\frac{f(x_{i+1}) - f(x_i)}{\ell_{i+1}} - \frac{f(x_i) - f(x_{i-1})}{\ell_i}$$

$$\frac{\ell_i}{6} M_{i-1} + \left(\frac{\ell_i + \ell_{i+1}}{3} \right) \cdot M_i + \frac{\ell_{i+1}}{6} M_{i+1} = -11 - \quad / \cdot \frac{6}{\ell_{i+1} + \ell_i}$$

$$\underbrace{\frac{\ell_i}{\ell_{i+1} + \ell_i}}_{\mu_i} \cdot M_{i-1} + 2M_i + \underbrace{\frac{\ell_{i+1}}{\ell_{i+1} + \ell_i}}_{\nu_i = 1 - \mu_i} M_{i+1} = \frac{6}{\ell_{i+1} - \ell_i} (-11 -)$$

$$\mu_i + \nu_i = 1$$

$$\frac{6}{\ell_i + \ell_{i+1}} \left[\frac{f(x_{i+1}) - f(x_i)}{\ell_{i+1}} - \frac{f(x_i) - f(x_{i-1})}{\ell_i} \right]$$

$\downarrow$ $\quad \quad \quad \downarrow$
 $x_{i+1} - x_i \quad \quad \quad x_i - x_{i-1}$
 $\underbrace{\hspace{1.5cm}} \quad \quad \quad \underbrace{\hspace{1.5cm}}$
 $f[x_i, x_{i+1}] - f[x_{i-1}, x_i]$

$$= 6 \cdot \frac{f[x_i, x_{i+1}] - f[x_{i-1}, x_i]}{x_{i+1} - x_{i-1}} = 6 f''[\underbrace{x_{i-1}, x_i, x_{i+1}}_{\lambda_i}]$$

$$\begin{cases} M_i \cdot M_{i-1} + 2M_i + M_i M_{i+1} = \lambda_i, & i=1, \dots, n-1 \\ M_i = \frac{h_i}{a_i + h_{i+1}}, & V_i = 1 - M_i, & \lambda_i = 6 f[x_{i-1}, x_i, x_{i+1}] \end{cases}$$

→ sistem $n+1$ lín. pú sa nepoznávan $M_0, \dots, M_n$ ($n+1$)

a) $M_0 = 0, M_n = 0$ ($M_0 = V_0 = \lambda_0 = 0$) S''
($M_n = V_n = \lambda_n = 0$)

b) $M_0 = M_n, M_1 = M_{n+1}$

c) $S'(a) = f'(a), S'(b) = f'(b)$ ($M_0 = 0, V_0 = 1$
 $M_n = 1, V_n = 0$)

$x_0 = a, i=0:$

$$S'(a)_{x_0}^{(*)} = -M_0 \frac{h_1}{2} + \frac{f(x_1) - f(x_0)}{x_1 - x_0} - \frac{h_1}{6} (M_1 - M_0) = f'(a)_{x_0}$$

$$\frac{h_1}{3} M_0 + \frac{h_1}{6} M_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0} - f'(a)$$

$x_n = b, i=n:$

$$S'(b)_{x_n}^{(*)} = \dots = f'(b)_{x_n}$$

$$\frac{h_n}{6} M_{n-1} + \frac{h_n}{3} M_n = f'(b) - \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} - f'(b)$$

a), b), c) ⇒ systém $n+1$ lín. pú sa $n+1$ nepoznávan $M_0, \dots, M_n$



jednoduché riešenie? (⇔) jednoduchý spline?

! Spline ⇒ zadavajúca systém



da li řešení ž?

(a), (c):

$$B \begin{pmatrix} 2 & \nu_0 & 0 & \dots & 0 \\ M_1 & 2 & \nu_1 & 0 & \dots & 0 \\ 0 & M_2 & 2 & \nu_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & M_n & 2 \end{pmatrix} \begin{pmatrix} M_0 \\ \vdots \\ M_n \end{pmatrix} = \begin{pmatrix} \lambda_0 \\ \vdots \\ \lambda_n \end{pmatrix} \quad (\square)$$

(b):

$$\begin{pmatrix} 2 & \nu_1 & 0 & \dots & 0 & M_1 \\ M_2 & 2 & \nu_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & M_{n-1} & 2 & \nu_{n-1} \\ \nu_n & 0 & \dots & 0 & M_n & 2 \end{pmatrix} \begin{pmatrix} M_1 \\ \vdots \\ M_n \end{pmatrix} = (\lambda_1, \dots, \lambda_n)^T \quad (\square\square)$$

(T) Sistemi eni, qua (□) i (□□) imajo jedinstveno rešenje pri pozitivnoj podeli intervala [a, b].

□: A regularna?

Lema: $Az = w \Rightarrow \max_{0 \leq i \leq n} |z_i| \leq \max_{0 \leq i \leq n} |w_i|$, $z = (z_0, \dots, z_n)^T$, $w = (w_0, \dots, w_n)^T$

Dokaz leme:

$$|z_m| = \max_{0 \leq i \leq n} |z_i|$$

m-vrsta:

$$Az = w \Rightarrow M_m z_{m-1} + 2z_m + \nu_m z_{m+1} = w_m$$

$$2z_m = w_m - M_m z_{m-1} - \nu_m z_{m+1}$$

$$|2z_m| = |w_m - M_m z_{m-1} - \nu_m z_{m+1}|$$

$$\leq |w_m| + M_m |z_{m-1}| + \nu_m |z_{m+1}|$$

$$|w_m| \geq |2z_m| - M_m |z_{m-1}| - \nu_m |z_{m+1}|$$

$$\max_{0 \leq i \leq n} |w_i| \geq |w_m| \geq |2z_m| - \underbrace{M_m |z_{m-1}|}_{\leq |z_m|} - \underbrace{\nu_m |z_{m+1}|}_{\leq |z_m|}$$

$$\geq |2z_m| - M_m |z_m| - \nu_m |z_m|$$

$$= (2 - M_m - \nu_m) |z_m|$$

$$= 1 \cdot |z_m| = \max_{0 \leq i \leq n} |z_i|$$

Dokaz Teoreme: $Az = w$

$$Az = 0$$

P.S. A singularna $\Rightarrow \exists z \neq 0$

$$Az = 0 \quad (\text{netrivitalno})$$

$$0 \leq \max |z_i| \leq \max |w_i| = 0 \Rightarrow z_i = 0, \forall i \Rightarrow z = (0, \dots, 0)^T$$

□ leme

* Konstrukcija kubni splajna za fiv zadatu tablicam

x	0	1	2
f	1	2	3

$$f'(0) = 0, f'(2) = -1$$

↑
uvrštavaju

$$\left. \begin{aligned} x=0: & \quad \frac{h_1}{3} M_0 + \frac{h_1}{6} M_1 = \frac{f(x_1) - f(x_0)}{a_1} - f'(0) \\ x=1: & \quad M_1 M_0 + 2M_1 + \frac{1}{2} M_2 = \lambda_1 \\ x=2: & \quad \frac{h_2}{6} M_1 + \frac{h_2}{3} M_2 = f'(2) - \frac{f(x_2) - f(x_1)}{a_2} \end{aligned} \right\} \begin{aligned} h_1 &= h_2 = 1 \\ M_1 &= \frac{h_1}{h_1 + h_2} = \frac{1}{2} \\ \lambda_1 &= 1 - M_1 = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \lambda_1 &= 6 f[x_0, x_1, x_2] = 6 \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \\ &= \frac{6}{x_2 - x_0} \left[\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0} \right] \\ &= \frac{6}{2} \left[\frac{1}{1} - \frac{1}{1} \right] = 3 \cdot 0 = 0 \end{aligned}$$

$$\left. \begin{aligned} \frac{1}{3} M_0 + \frac{1}{6} M_1 &= 1 - 0 = 1 \\ \frac{1}{2} M_0 + 2M_1 + \frac{1}{2} M_2 &= 0 \\ \frac{1}{6} M_1 + \frac{1}{3} M_2 &= -1 - 1 = -2 \end{aligned} \right\} \begin{aligned} A \setminus b \\ M_0 &= \frac{5}{2} \\ M_1 &= 1 \\ M_2 &= -\frac{13}{2} \end{aligned}$$

$$S(x) = \alpha_i + \beta_i(x - x_i) + \gamma_i(x - x_i)^2 + \delta_i(x - x_i)^3, \quad x \in [x_i, x_{i+1}]$$

$$\begin{aligned} x \in [0, 1], \quad i=0: & \quad \alpha_0 = f(x_0) = 1 \\ & \quad \beta_0 = \dots = 1 - \frac{1}{6}(5+1) = 0 \\ & \quad \gamma_0 = \frac{1}{2} M_0 = \frac{5}{4} \\ & \quad \delta_0 = \frac{1}{6 \cdot 1} \left(1 - \frac{5}{2} \right) = -\frac{1}{4} \end{aligned}$$

$$x \in [1, 2], \quad i=1: \quad \alpha_1 = 2, \quad \beta_1 = \frac{7}{4}, \quad \gamma_1 = \frac{1}{2}, \quad \delta_1 = -\frac{5}{4}$$

$$S(x) = \begin{cases} 1 + 0 \cdot (x-0) + \frac{5}{4} (x-0)^2 - \frac{1}{4} (x-0)^3 = 1 + \frac{5}{4} x^2 - \frac{1}{4} x^3, & x \in [0, 1] \\ 2 + \frac{7}{4} (x-1) + \frac{1}{2} (x-1)^2 - \frac{5}{4} (x-1)^3 = 2 - 3x + \frac{17}{4} x^2 - \frac{5}{4} x^3, & x \in [1, 2] \end{cases}$$

* Odraditi kubni spline periodične po zadate tablicem
sa periodom jednakim 3

x	0	1	2	3	4
f	1	3	2	1	3

$\forall i, G_i = 1, M_i = 1/2, V_i = 1/2$

$M_0 = M_3$

$M_1 = M_4$

$\lambda_1 = G f[x_0, x_1, x_2] = \dots = -9$

$\lambda_2 = G f[x_1, x_2, x_3] = \dots = 0$

$\lambda_3 = G f[x_2, x_3, x_4] = \dots = 9$

$$\left. \begin{aligned} \frac{1}{2} M_0 + 2 M_1 + \frac{1}{2} M_2 &= -9 \\ \frac{1}{2} M_1 + 2 M_2 + \frac{1}{2} M_3 &= 0 \\ \frac{1}{2} M_2 + 2 M_3 + \frac{1}{2} M_4 &= 9 \end{aligned} \right\} \begin{aligned} M_0 &= 6 \\ M_1 &= -6 \\ M_2 &= 0 \end{aligned}$$

$x \in [0, 1] : \alpha_0 = 1, \beta_0 = 1, \gamma_0 = 3, \delta_0 = -2$

$x \in [1, 2] : \alpha_1 = 3, \beta_1 = 1, \gamma_1 = -3, \delta_1 = 1$

$x \in [2, 3] : \alpha_2 = 2, \beta_2 = -2, \gamma_2 = 0, \delta_2 = 1$

$$S(x) = \begin{cases} 1 + 1 \cdot x + 3x^2 - 2x^3, & x \in [0, 1] \\ 3 + 1(x-1) - 3(x-1)^2 + 1(x-1)^3, & x \in [1, 2] \\ 2 - 2(x-2) + 0(x-2)^2 + 1(x-2)^3, & x \in [2, 3] \end{cases}$$

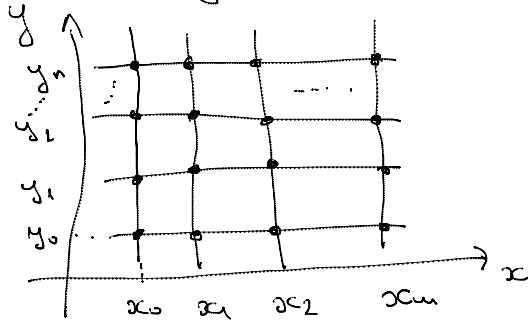
* Prirodni spline $\Rightarrow M_0 = M_n = 0$

Dvodomenzionalna interpolacija

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11:30 AM

$$f(x, y) \leadsto L_n(x, y)$$

$$f(x_i, y_j) = L_n(x_i, y_j) \quad , \quad i = 0, \dots, m, \quad j = 0, \dots, n$$



I) ~~više~~ puta jednost. int.

$$f(\tilde{x}, \tilde{y}) = ?$$

$$u+1 : f(\tilde{x}, y_j) \quad , \quad j = 0, \dots, n$$

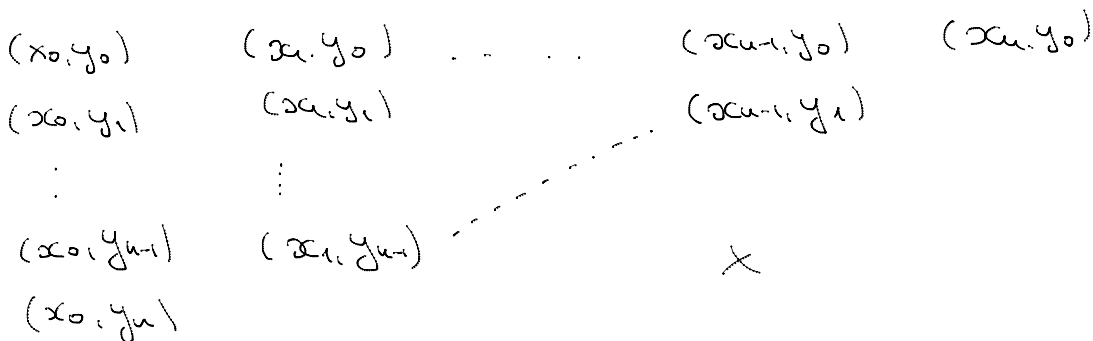
$$f(\tilde{x}, \tilde{y})$$

$$\text{II)} \quad L_n(x, y) = \boxed{C_{00}} + \boxed{C_{10}x + C_{01}y} + \boxed{C_{20}x^2 + C_{11}xy + C_{02}y^2} + \dots$$

$$+ C_{n0}x^n + C_{n-1,1}x^{n-1}y + \dots + C_{1,n-1}x \cdot y^{n-1} + C_{0n}y^n$$

$$1 + 2 + 3 + 4 + \dots + (u+1) = \frac{1}{2}(u+1)(u+2) \text{ nepoznatih}$$

toliko uslova interpolacije



$$\frac{1}{2}(u+1)(u+2) \text{ uslova}$$

$$f(\tilde{x}, \tilde{y}) = ?$$

